Topological fractional pumping with *ultracold atoms (in synthetic ladders)*

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Acknowledgments

L. Fallani (LENS - Firenze)

S. Barbarino, L. Taddia, D. Rossini, L. Mazza, R. Fazio, Nat. Commun. **6**, 8134 (2015)
S. Barbarino, L. Taddia, D. Rossini, L. Mazza, R. Fazio, New J. Phys. **18**, 035010 (2016)
M. Calvanese Strinati, E. Cornfeld, D. Rossini, S. Barbarino, M. Dalmonte, R. Fazio, E. Sela, L. Mazza, Phys. Rev. X. **7**, 021033 (2017)
L. Taddia, E. Cornfeld, D. Rossini, L. Mazza, E. Sela, R. Fazio, Phys. Rev. Lett. **118**, 230402 (2017)
S. Barbarino, M. Dalmonte, R. Fazio, and G. Santoro, arXiv:1708.02929

Pumping

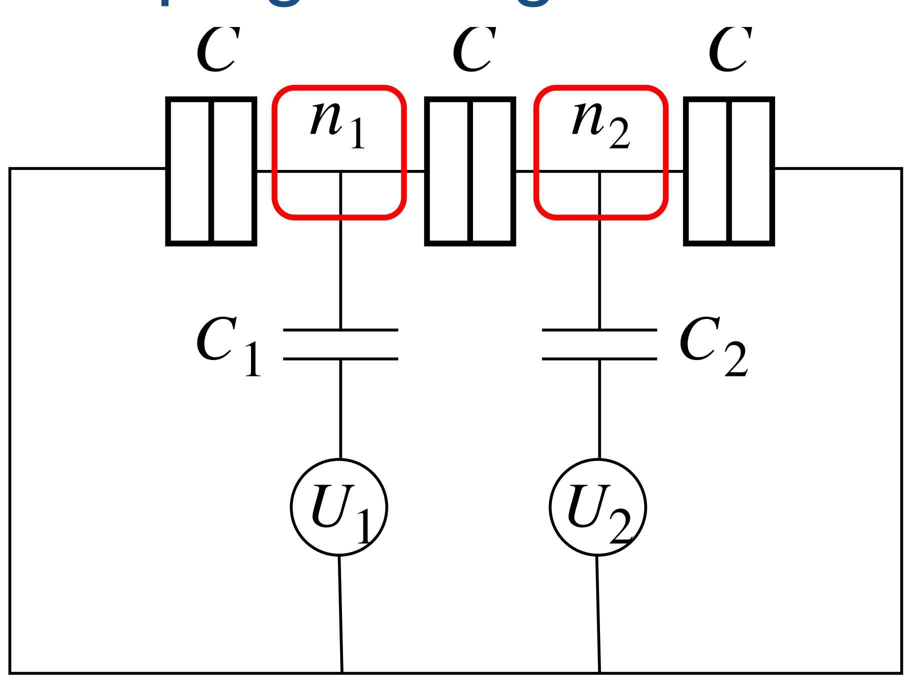
Matter and energy can be transported, or pumped, without imposing any external bias, by a periodic modulation of some system parameters.

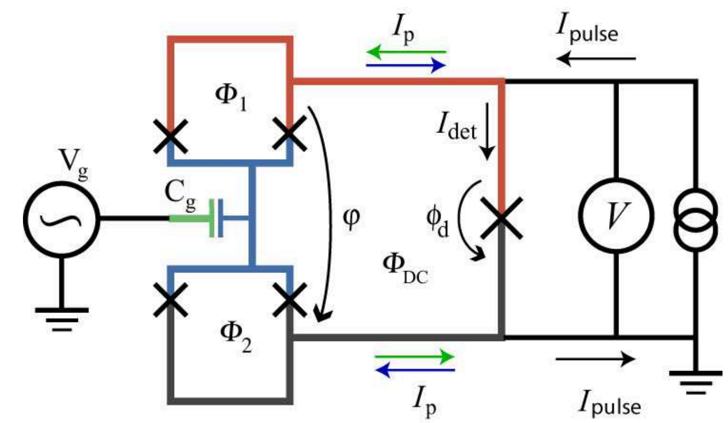
Quantum Pumping

- Definition of novel current standards
- Study of quantum coherent transport in nano-systems
- Diagnostics of many-body quantum states

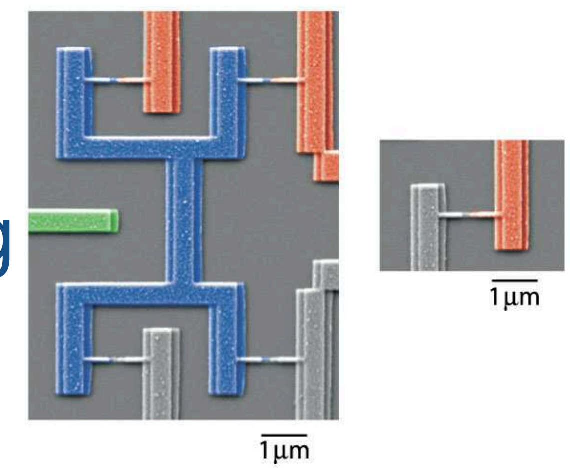
Quantum Pumping in cond-mat (exps)

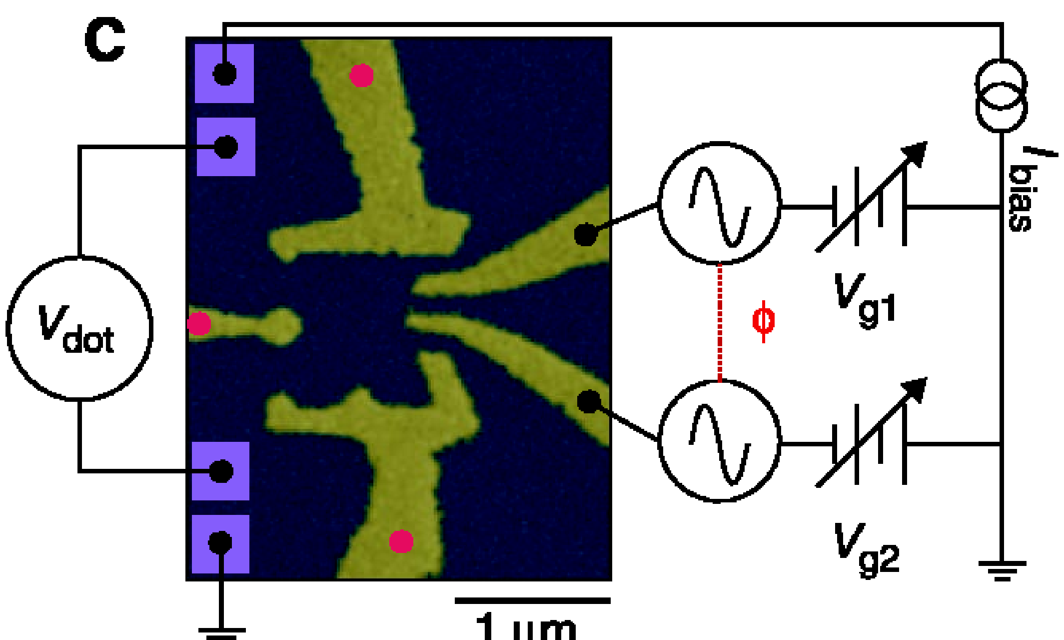
Pumping through a double dot system (Saclay)

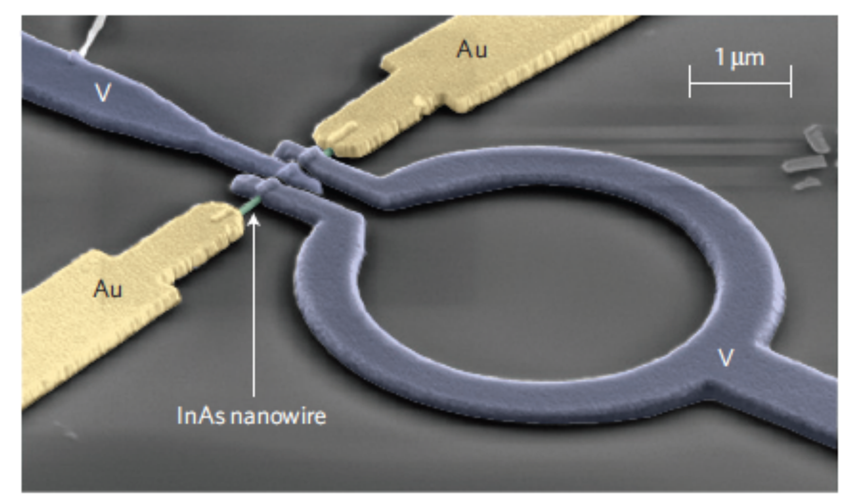




Cooper pair pumping (Helsinki)







Pumping through an open dot (Harvard)

Josephson pump (PISA)

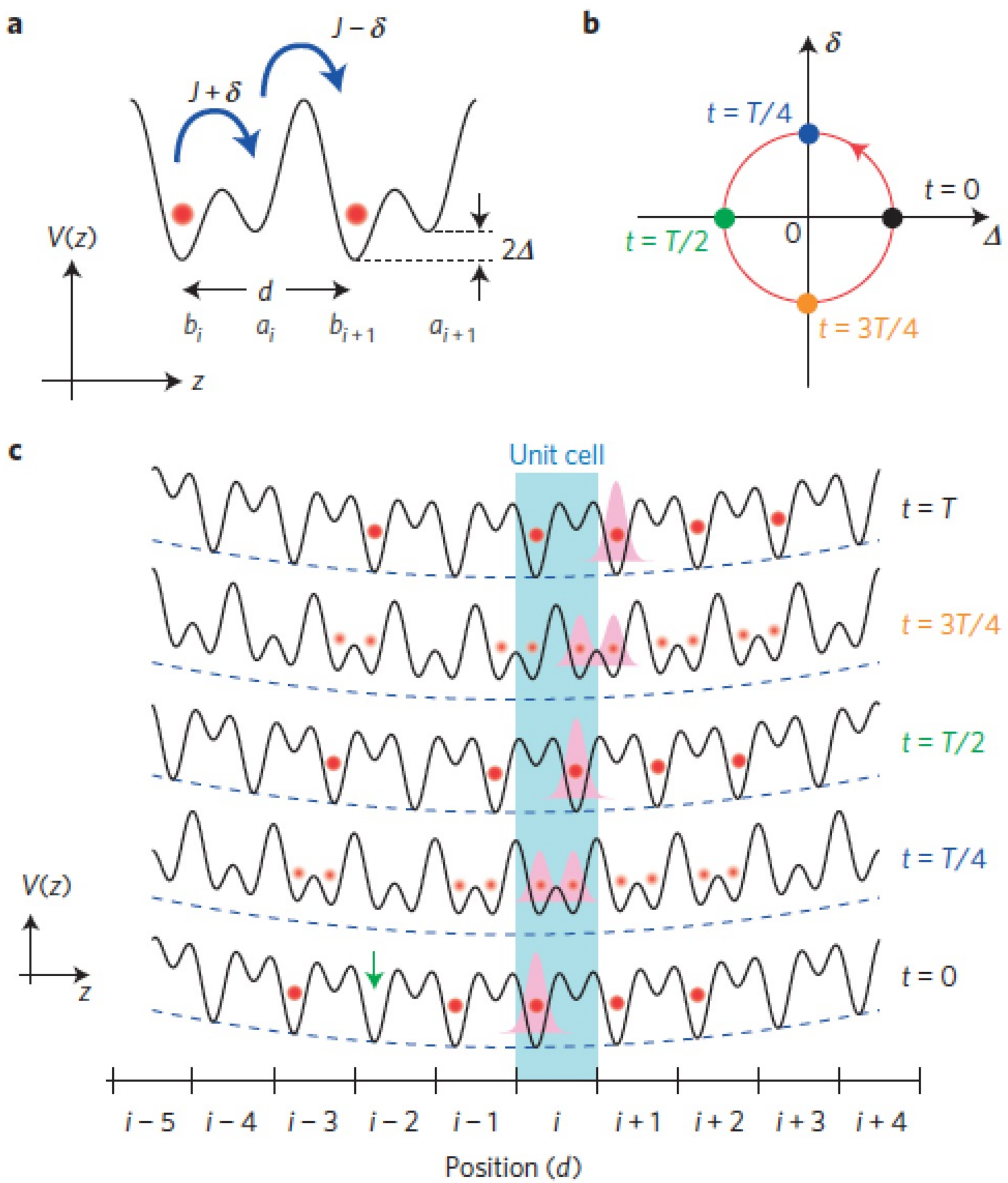
Quantum Pumping

In quantum systems pumping is a **coherent process** connected to the time-evolution of the wave-function **under a cyclic modulation**

In the ***adiabatic*** limit, quantum pumping becomes ***geometric*** , meaning that it is related to the Berry phase (or its non-Abelian generalisation) accumulated during the cycle

In some cases the ***pump*** can be of ***topological nature***, i.e. the transported charge/mass in a cycle can be quantised and robust to perturbation

Thouless pump



Thouless in 1983 showed that in some one-dimensional insulating systems the pumped charge may be quantised to an integer number.

Experiments in 2016 using cold atoms in optical lattices

S. Nakajima, T. Tomita, S. Taie, T. Ichinose, H. Ozawa, L. Wang, M. Troyer, and Y. Takahashi, Nat. Phys. **12**, 296 (2016).

M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, Nat. Phys. **12**, 350 (2016).

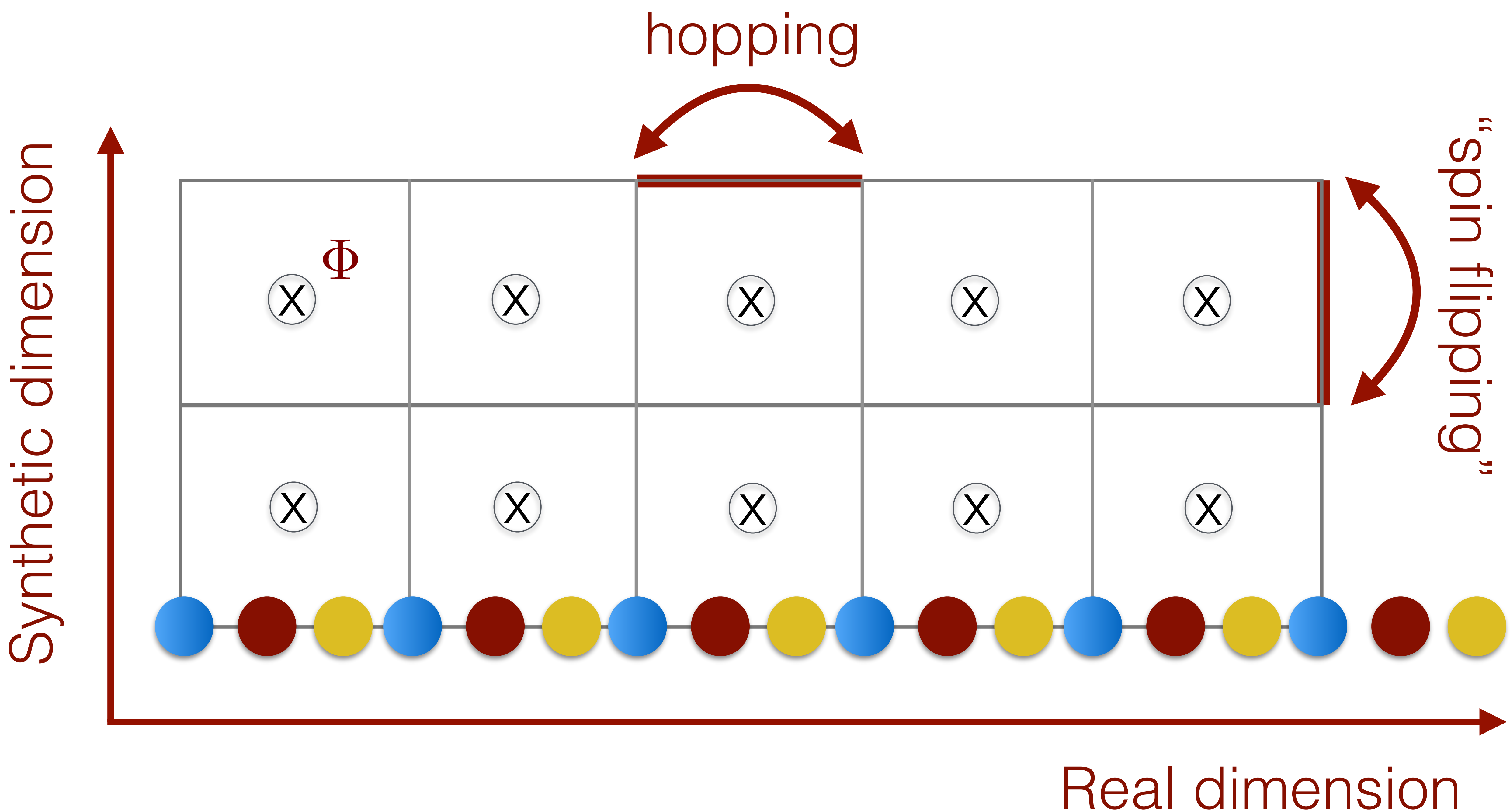
Search for fractional topological pumps (in cold atoms)

Alkaline-earth(-like) atoms, trapped in optical lattices and in the presence of an external gauge field, can stabilise insulating states at given fractional fillings.

By exploiting these properties, it is possible to realise a topological fractional pump.

F. Grusdt and M. Honing, Phys. Rev. A 90, 053623 (2014).
T.-S. Zeng, C. Wang, and H. Zhai, Phys. Rev. Lett. 115, 095302 (2015).
E.G. Dalla Torre, E. Berg, and E. Altman, Phys. Rev. Lett. 97, 260401 (2006).
D. Rossini, M. Gibertini, V. Giovannetti, and R. Fazio, Phys. Rev. B 87, 085131 (2013).
…

Higher spins and synthetic dimension

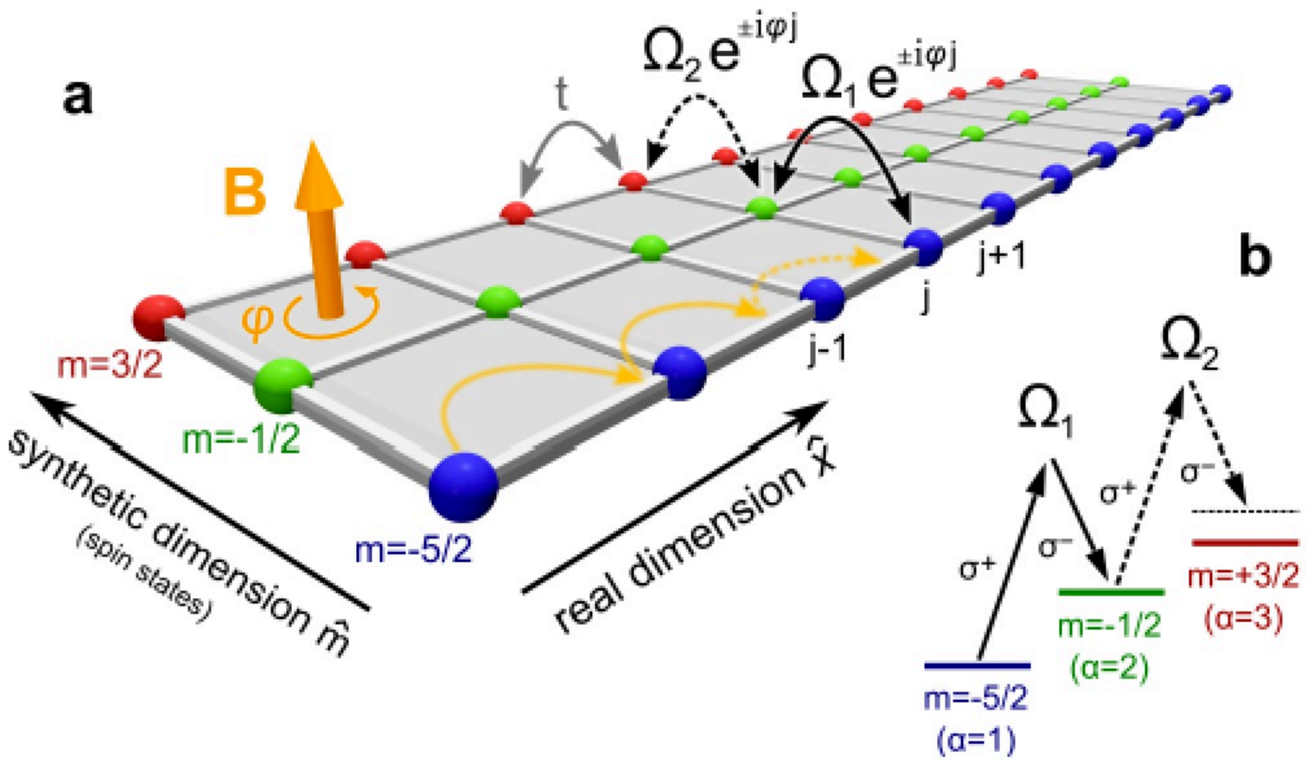


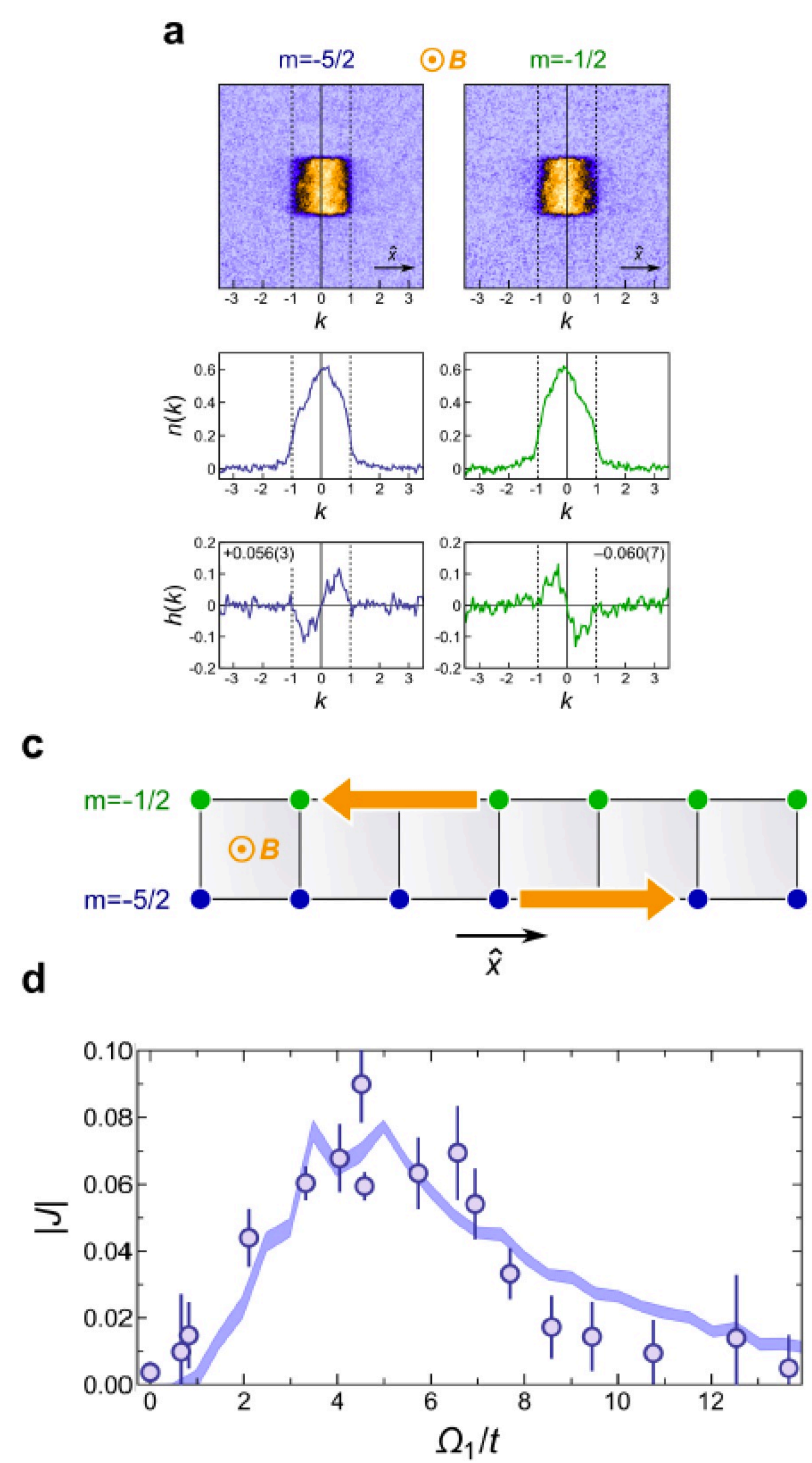
Note: The interaction is highly anisotropic

A. Celi *et al*, Phys. Rev. Lett. **112**, 043001 (2014)
C. Boada *et al*, Phys. Rev. Lett. **108** 133001 (2012)

Higher spins and synthetic dimension

A. Mancini *et al*, Science **349,** 1510 (2015)





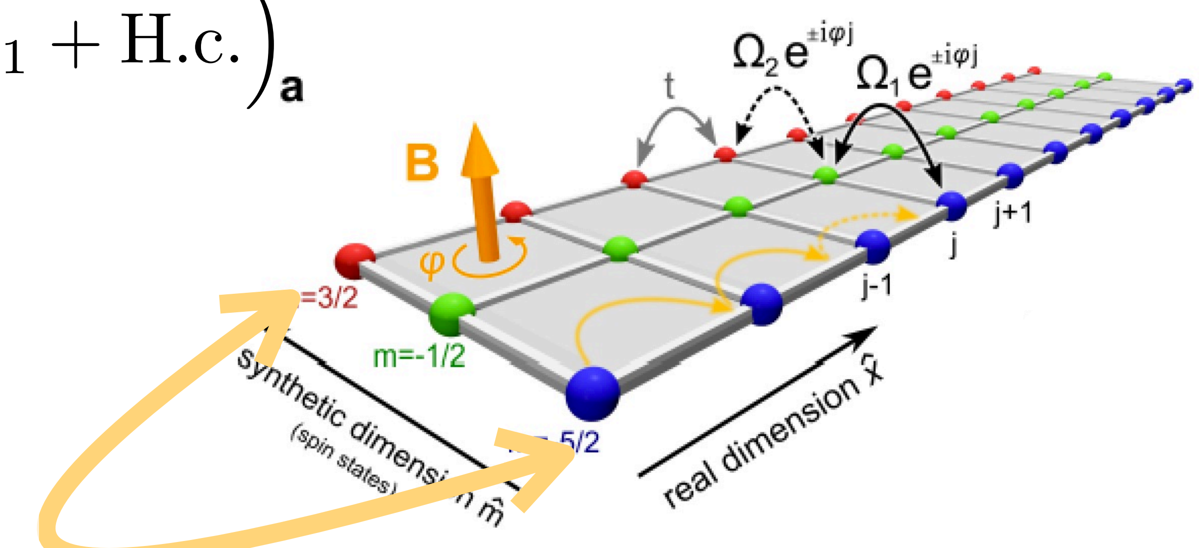
Measurement of the circulating currents flowing in the ladder

The model

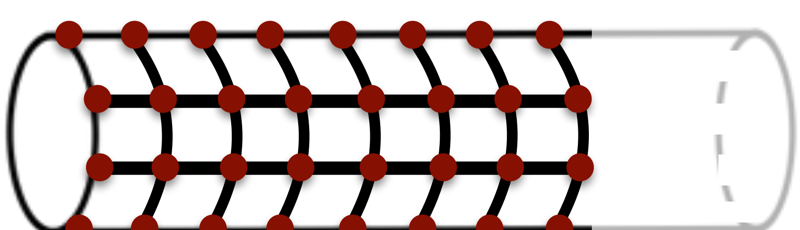
$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2$$

$$\mathcal{H}_0 = -t \sum_j \sum_{m=-\mathcal{I}}^{\mathcal{I}} \left(\hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + \text{H.c.} \right) + U \sum_j \sum_{m<m'} \hat{n}_{j,m} \hat{n}_{j,m'}$$

$$\mathcal{H}_1 = \sum_j \sum_{m=-I}^{I-1} \left(\Omega_m e^{-i\,\varphi\, j} \hat{c}^\dagger_{j,m} \hat{c}_{j,m+1} + \text{H.c.} \right)$$

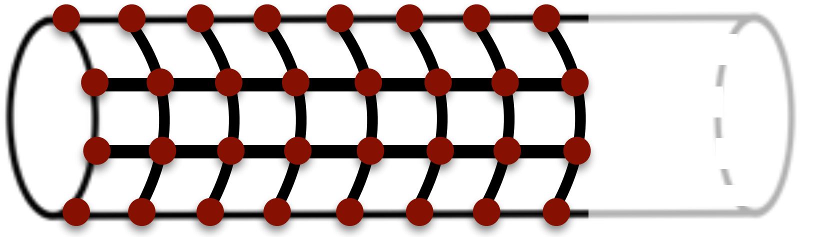


$$\mathcal{H}_2 = \sum_j \left(\Omega\, {}^{-i2k_{\mathrm{SO}}j} \hat{c}^\dagger_{j,I} \hat{c}_{j,-I} + \text{H.c.} \right)$$



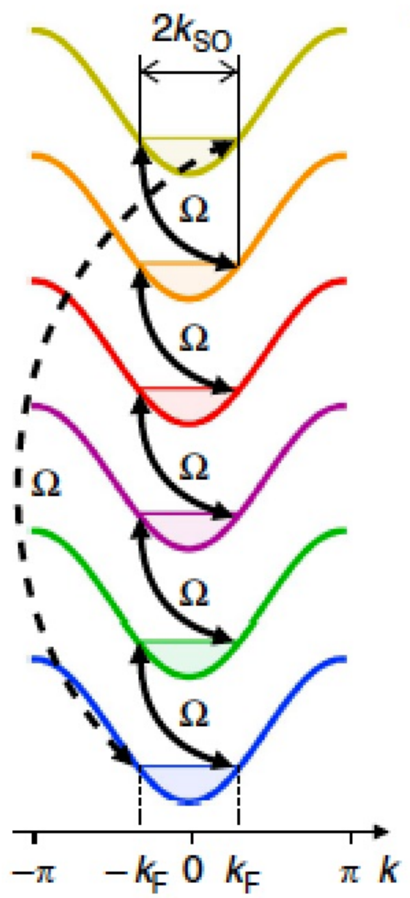
Fractional fillings

$$\nu \equiv \frac{\pi N}{k_{\mathrm{SO}}(2I+1)L} = \frac{p}{q}$$



Gapped phases at fractional fillings

Non-interacting system



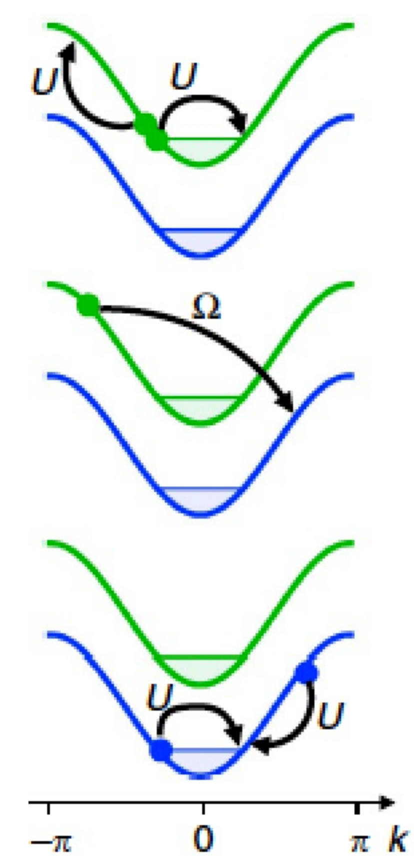
Fermions with momentum difference $\Delta k = \pm 2k_{so}$ (and neighbouring spin) are coupled.

When $k_F = k_{so}$ the system becomes gapped.

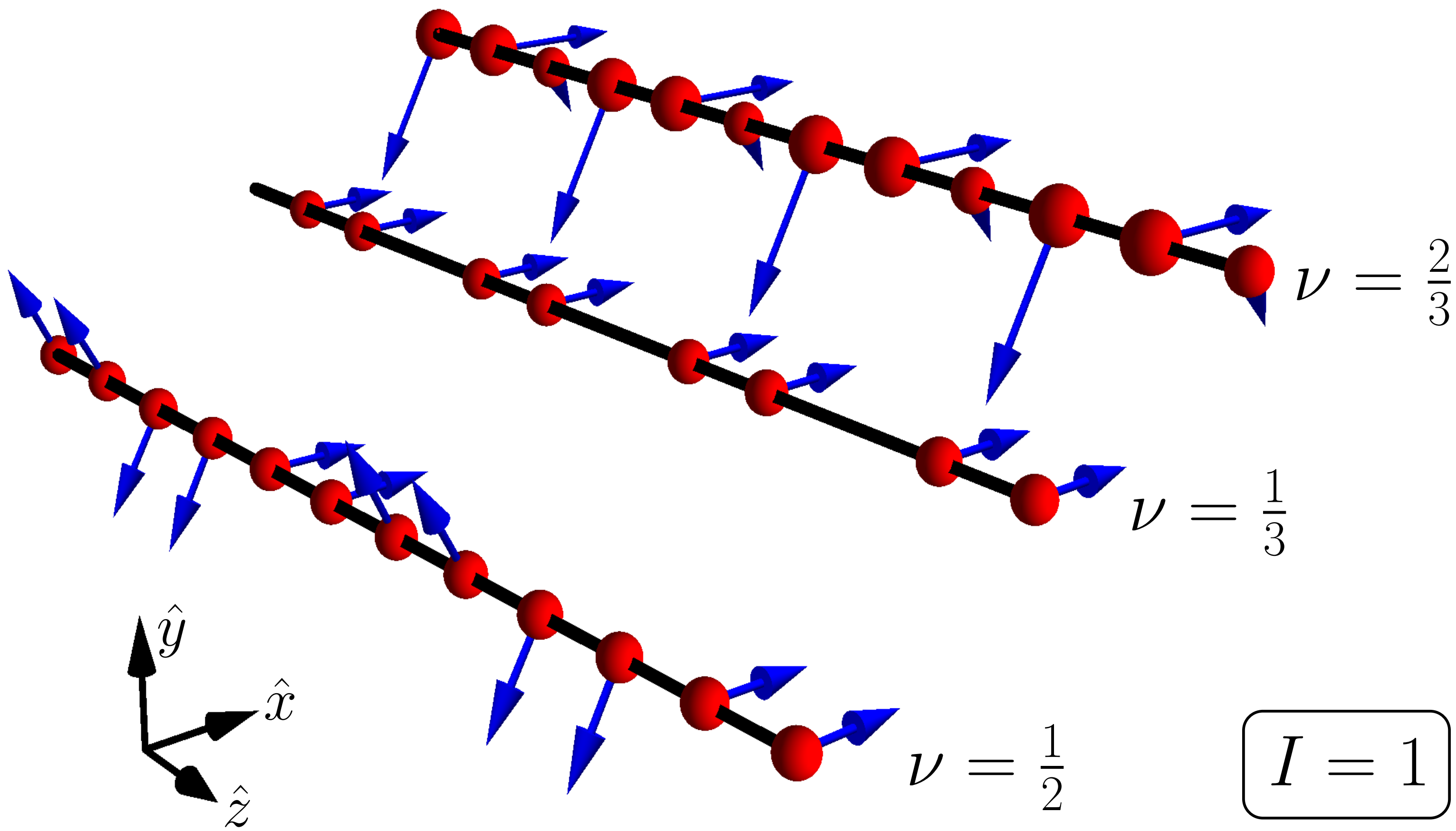
In the presence of interactions the system can develop a gap for lower fillings via higher-order scattering terms.

Below three intermediate processes that generate a coupling between two Fermi edges with neighbouring spins

$$3k_F = k_{so}$$



Magnetic crystals

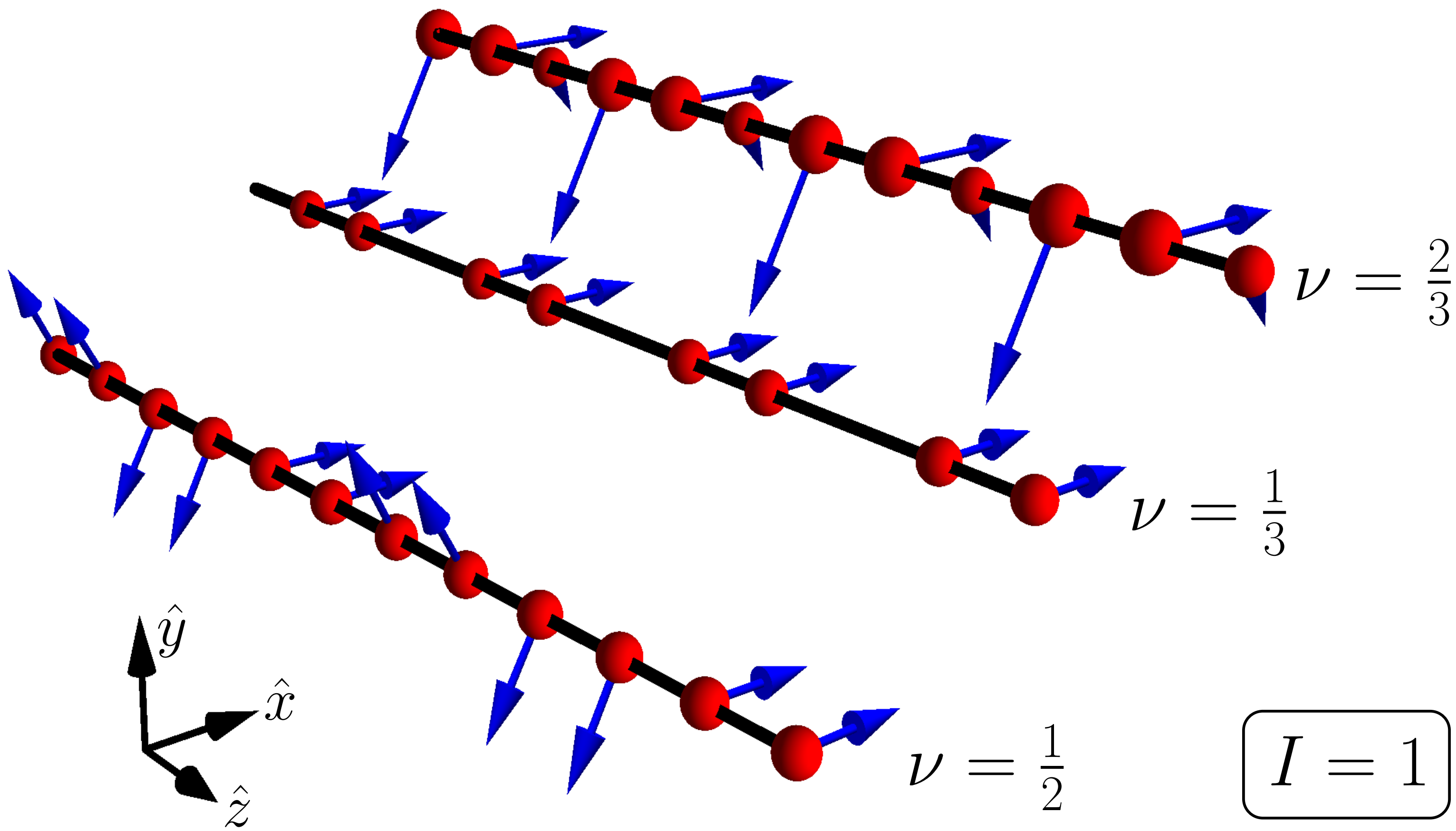


$\boxed{I = 1}$

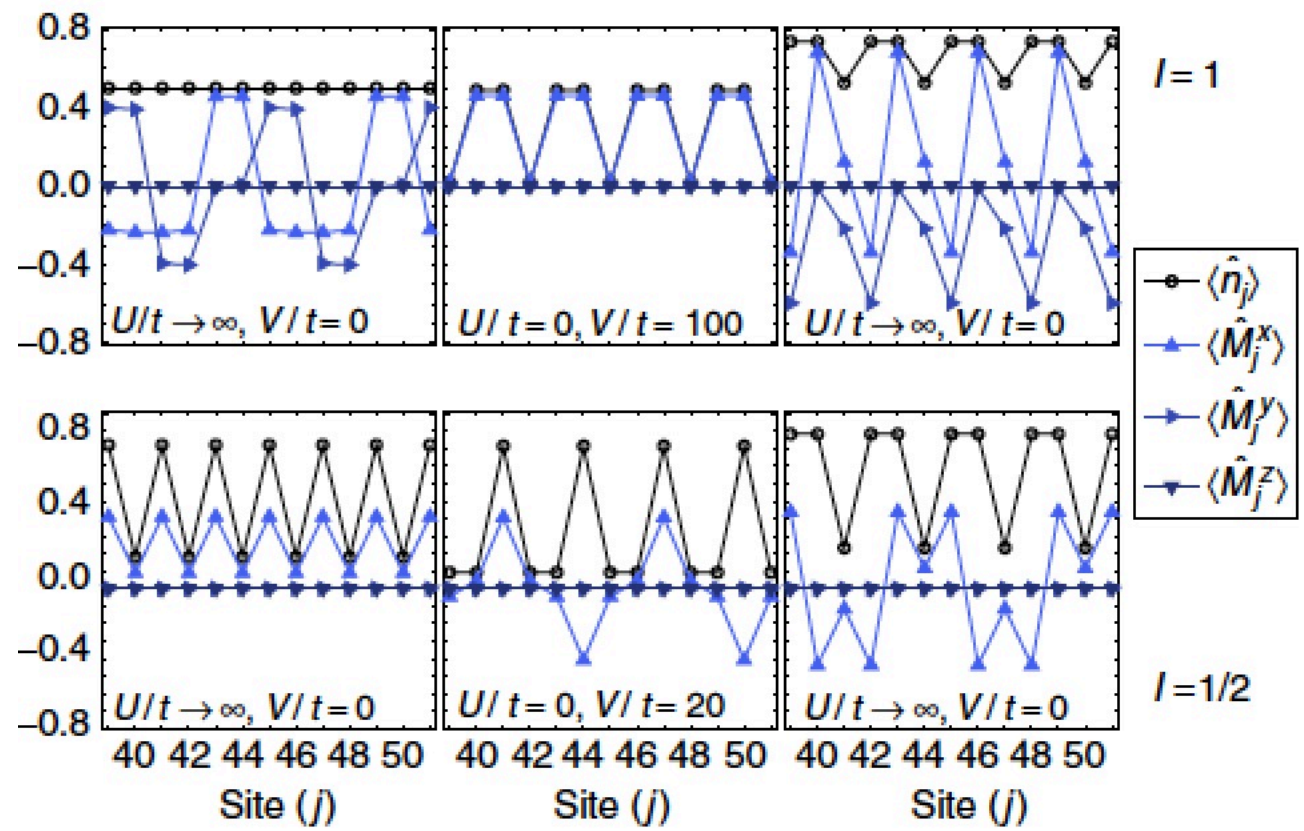
The ground state of a magnetic insulator is q-fold degenerate

- Finite range interactions stabilise charge ordering at densities smaller than the inverse range.
- Crystals show magnetic ordering

Magnetic crystals

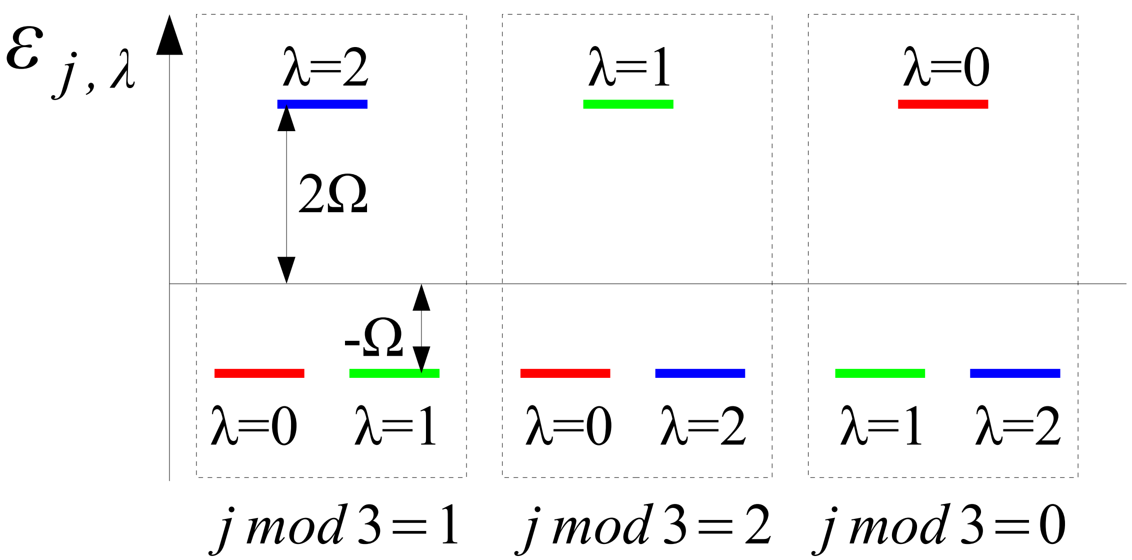


$$\boxed{I = 1}$$



Magnetic crystals

$$\Omega/t, \quad \Omega/U \gg 1$$

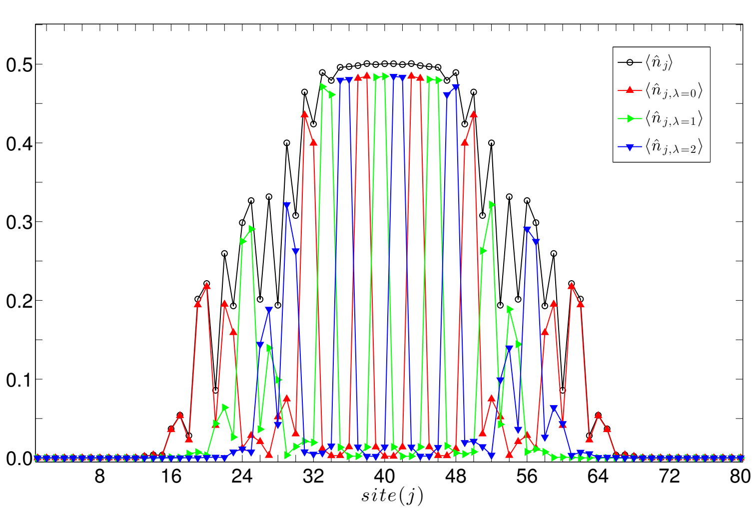


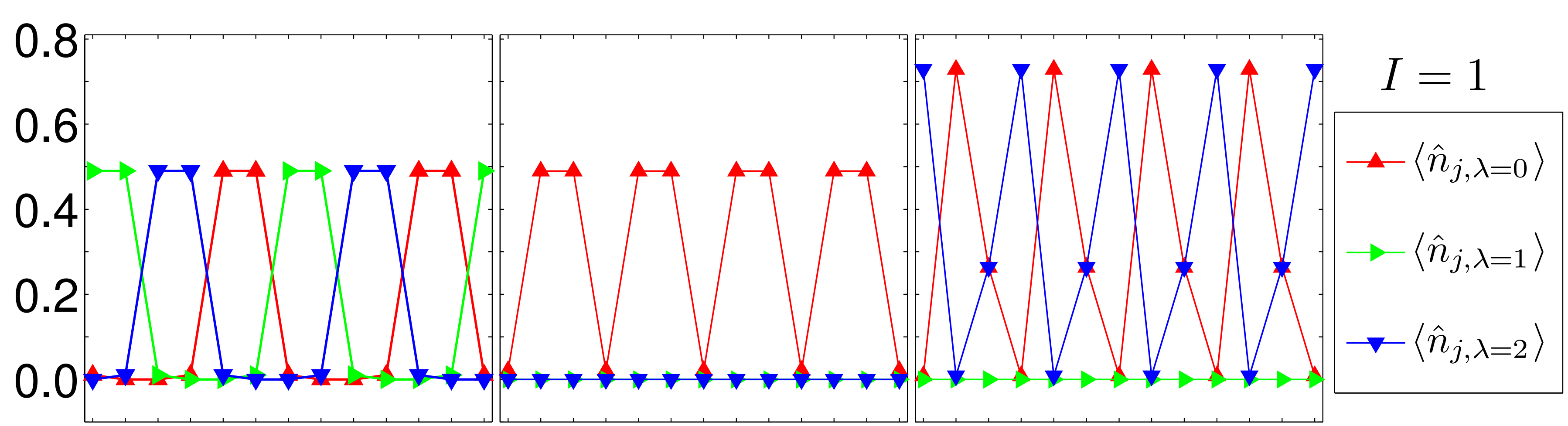
A simple picture emerges in the limit by diagonalising the Hamiltonian at t=U=0

When the space periodicity matches the inter particle distance one expects a crystalline phase.

The spin structure of the lattice is determined in this basis

A small, finite, hopping the system will lower its energy by delocalising the particles. This tendency is however strongly suppressed by the large on-site energy U. The resulting competition leads to the formation of dimers locked together to form a crystal

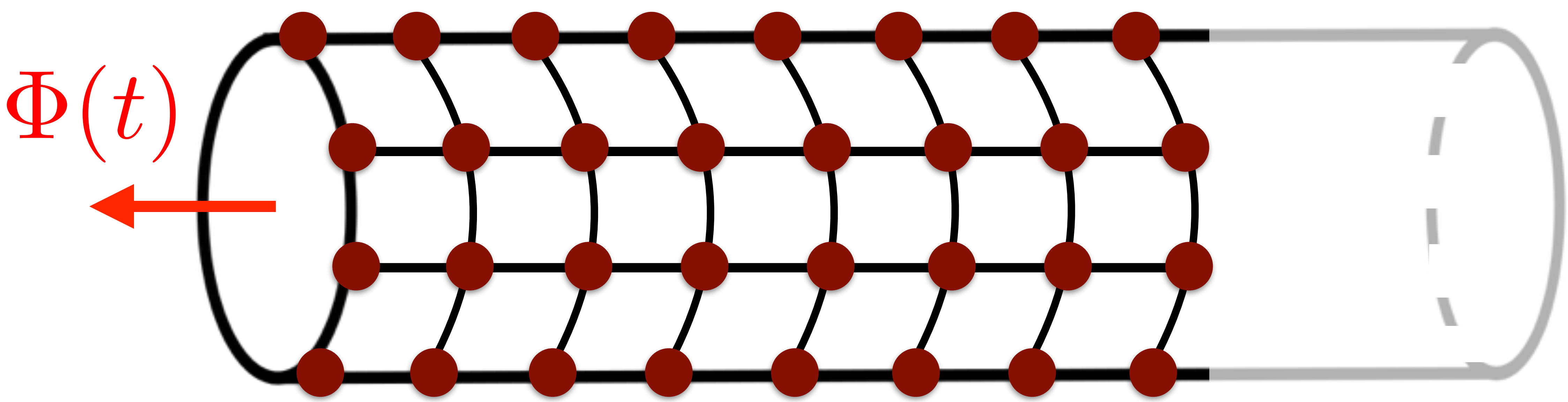




Quantum pumping

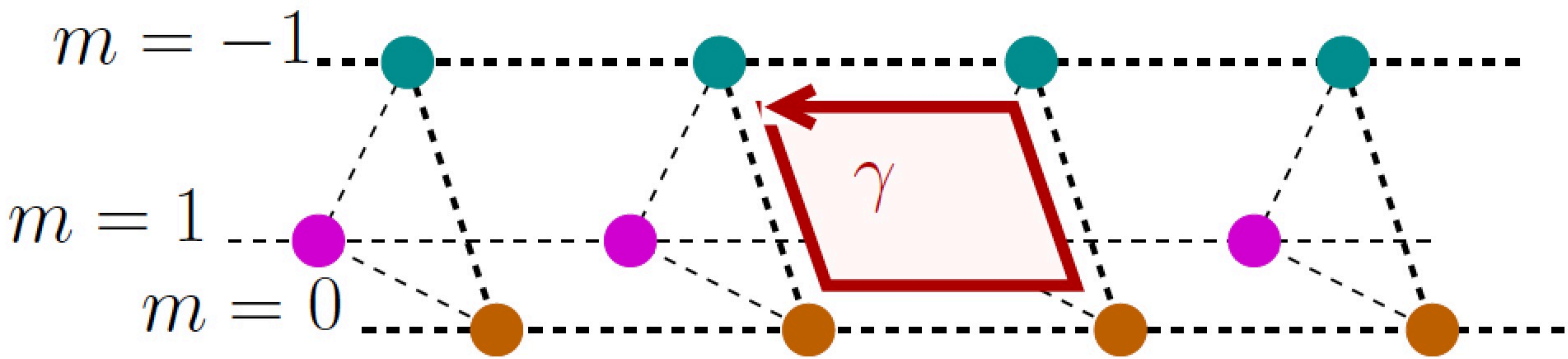
Add a phase factor to the links in the synthetic dimension

$$e^{i2\pi\Phi(t)/N}$$



After a cycle particles will be adiabatically pumped along the real direction

$$Q$$

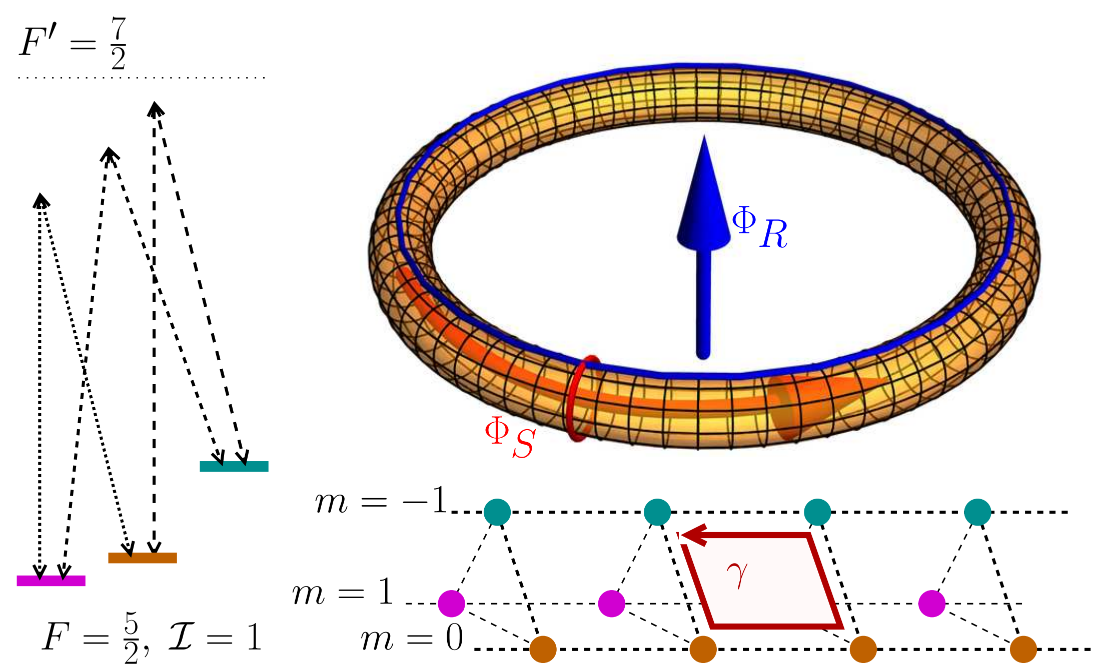

$m = -1$
$m = 1$
$m = 0$
γ

The model

$$\mathcal{H} = \sum_{j,m}\left[-t\hat{c}^{\dagger}_{j,m}\hat{c}_{j+1,m} + \Omega_{j,m}e^{-i\gamma j}\hat{c}^{\dagger}_{j,m}\hat{c}_{j,m+1} + \text{H.c.}\right]$$
$$+\sum_{i,j,m,m'} U^{m,m'}_{i,j}\hat{n}_{i,m}\hat{n}_{j,m'} + w_0\sum_{j,m}(j-j_0)^2\,\hat{n}_{j,m}.$$

$$t \;\rightarrow\; t(\tau) = te^{i\Phi_R(\tau)/L}$$

$$\Omega \;\rightarrow\; \Omega = \Omega e^{i\Phi_S(\tau)/(2\mathcal{I}+1)}$$



rescaled time $s = \tau / T$

Pumped charge/mass over a period

$$Q^{(h)} = 2T \sum_m \int_0^1 ds \Re \langle t \hat{c}^\dagger_{j,m}(s) \hat{c}_{j+1,m}(s) \rangle_h$$

Adiabatic expansion

$$|\Psi^h(s)\rangle \simeq |\Psi^h(s)\rangle_0 + |\Psi^h(s)\rangle_1 + \ldots,$$

$E_n(s)$ $|n^h(s)\rangle$ Instantaneous eigenstates and energies

A system whose ground state is degenerate can pump a charge which is topological, fractional and related to the many-body topological number

$$\langle Q_P \rangle_{\Phi_R} = \frac{1}{q} \int \frac{d^2\Phi}{2\pi} \sum_h [\Omega_{WZ}]^{hh} = \frac{p}{q}$$

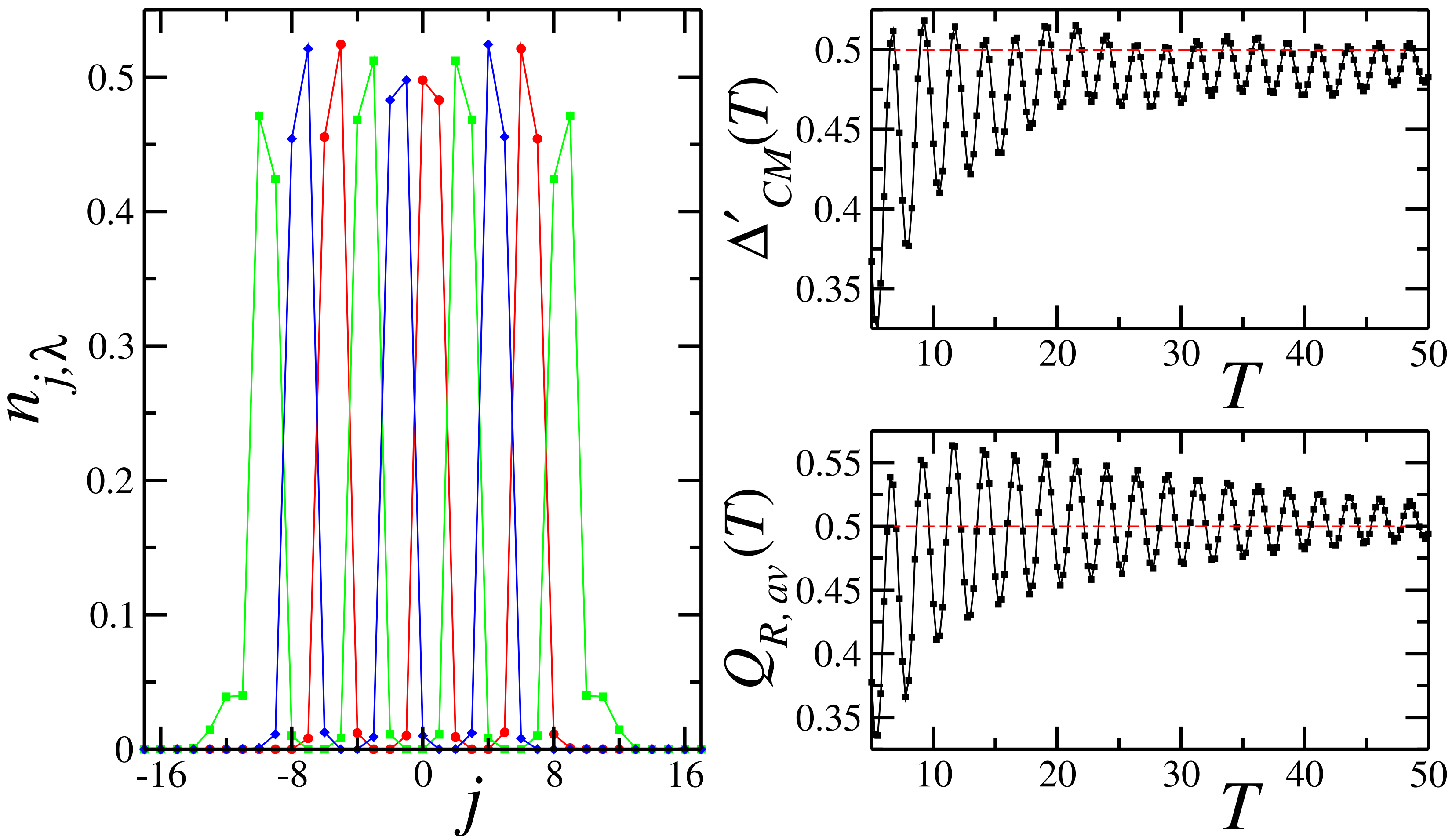
Wilczek-Zee matrix

Non-adiabatic/finite size corrections

$$Q_P = \frac{\mathcal{C}_1}{q} + \mathcal{O}(L^{-1}) + \mathcal{O}\left(\frac{f(\Delta T)}{\Delta T}\right)$$

DMRG results

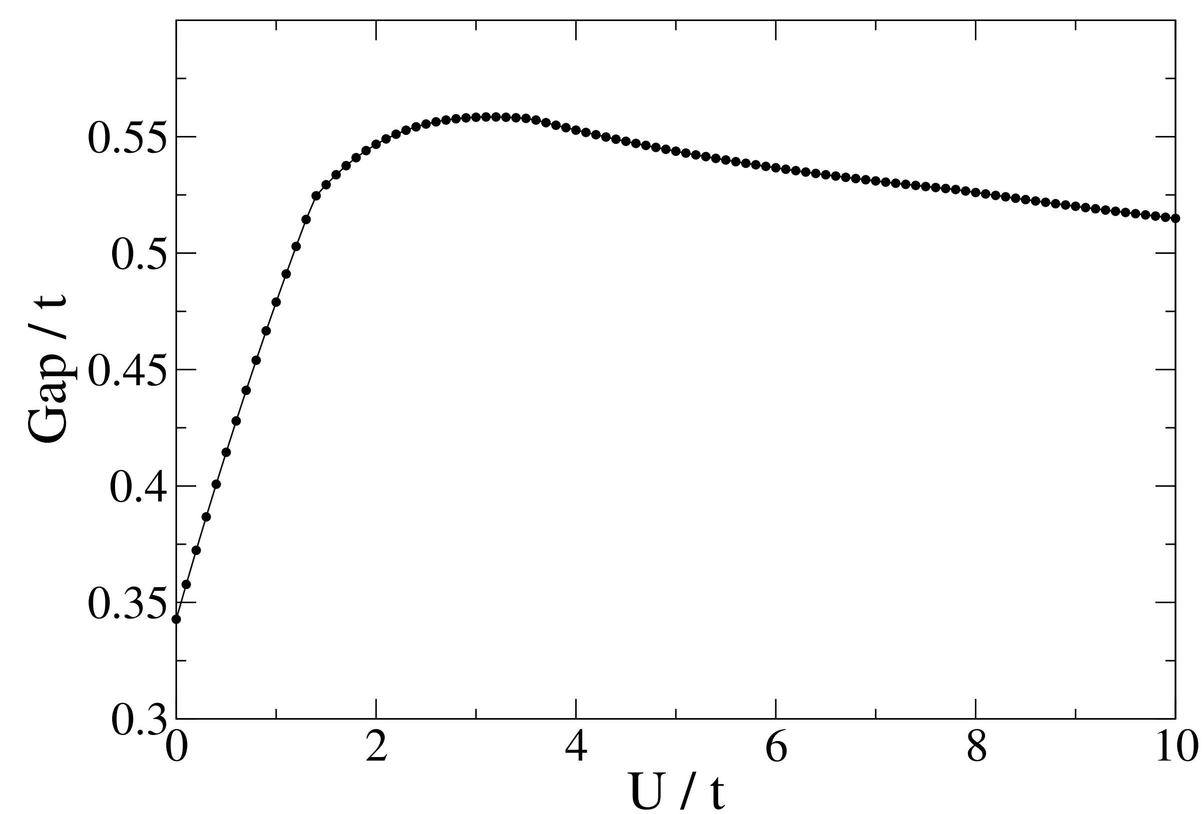
An experimental way to estimate the pumped charge exploits the centre-of-mass displacement of the atomic cloud



H. M. Price and N. R. Cooper, Phys. Rev. A 85, 033620 (2012).
A. Dauphin and N. Goldman, Phys. Rev. Lett. 111,135302 (2013).
H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, and N. Goldman, Phys. Rev. B 93, 245113 (2016).

Imperfect quantisation

- the existence of small metallic wings at the edges of the crystal
- the presence of multiple copies of the system.
- finite temperatures



Conclusions

- Alkaline-earth(-like) atoms, trapped in optical lattices and in the presence of an external gauge field, can form insulating states at given fractional fillings

- By exploiting these properties, it is possible to realise a topological fractional pump.

Funding: EU-QUIC

Searching for Majoranas in a cold atomic setting

In collaboration with

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Peter Zoller

IQI

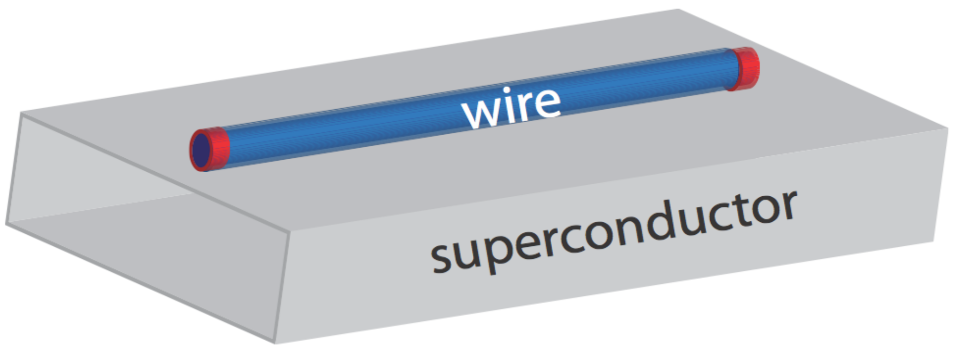
Leonardo Fallani

UNIVERSITÀ DEGLI STUDI FIRENZE

F. Iemini, L. Mazza, L. Fallani, P. Zoller, R. Fazio, and M. Dalmonte, Phys. Rev. Lett. **118**, 200404 (2017)

Realisation of the Kitaev Model

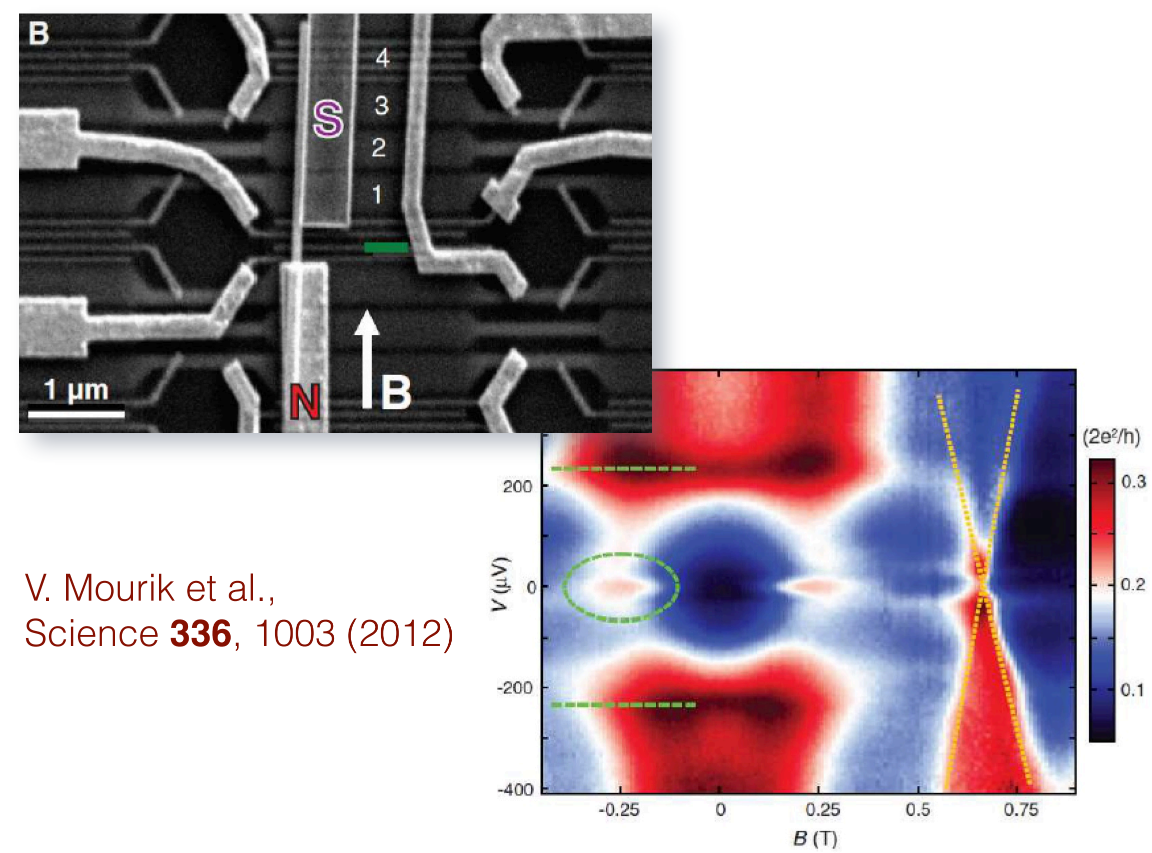
R.M. Lutchyn, J.D. Sau, and S. Das Sarma, Phys. Rev. Lett. **105**, 077001 (2010).
Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).



First implementation is solid-state:

A semiconducting wire, in the presence of

i) Spin-orbit interaction,
ii) Zeeman fields,
iii) Superconducting order parameter induced by an s-wave superconductor located in proximity of the nanowire



V. Mourik et al.,
Science **336**, 1003 (2012)

Cold Atom setting

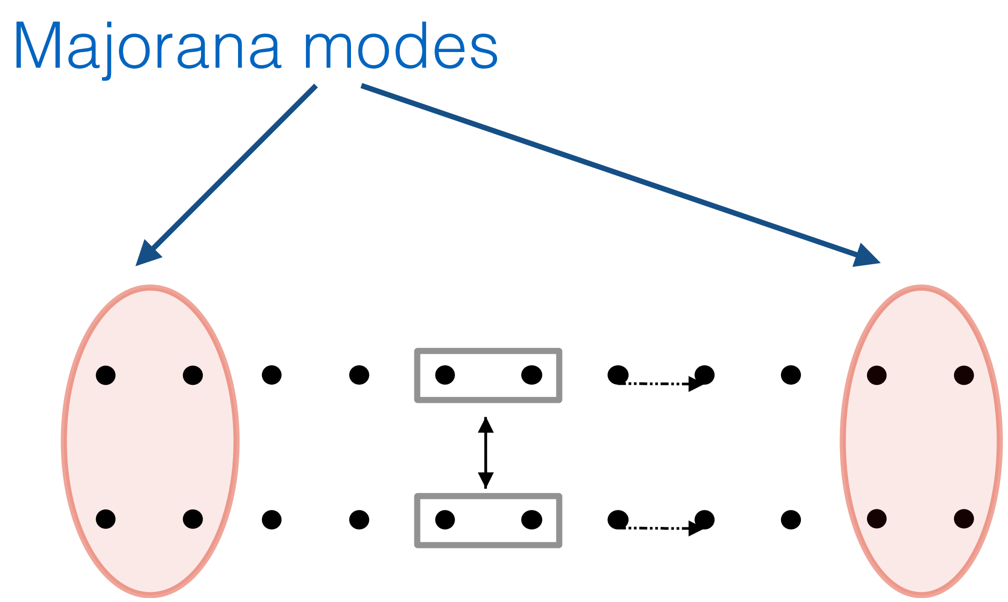
The same ingredients (inducing an effective spin-orbit interaction and magnetic field) are found in atoms with two internal states of the atoms couple via an optical Raman transition. Molecular BEC cloud generates a s-wave pairing.

L. Jiang et al, Phys. Rev. Lett. **106**, 220402 (2011)

Majorana modes in a number conserving setting

Besides the interest in itself, the study of topological superconductivity at fixed N can be relevant for some experimental platforms.

e.g. C. V. Kraus, *et al*, Phys. Rev. Lett. **111**, 173004 (2013).



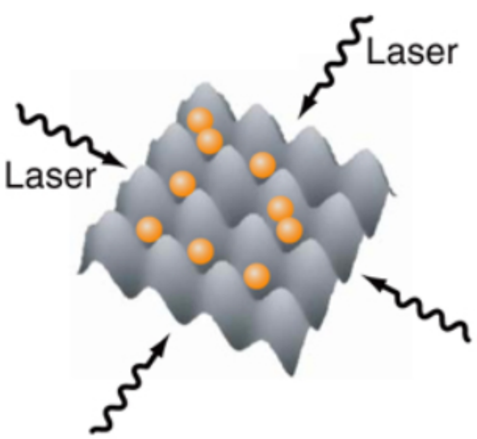
With number conservation:

- *From one wire to two wires*
- *Interaction is important* (complex many-body problem)

Our work

Question: is there a naturally occurring model where a parity symmetry / pairing correlations are realised in a number-conserving fashion?

Our work: Fermion atoms in optical lattices with local orbital + spin degrees of freedom



Exps

Spin-exchange

G. Cappellini et al., Phys. Rev. Lett. **113**, 120402 (2014)

F. Scazza et al., Nat. Phys. **10**, 779 (2014

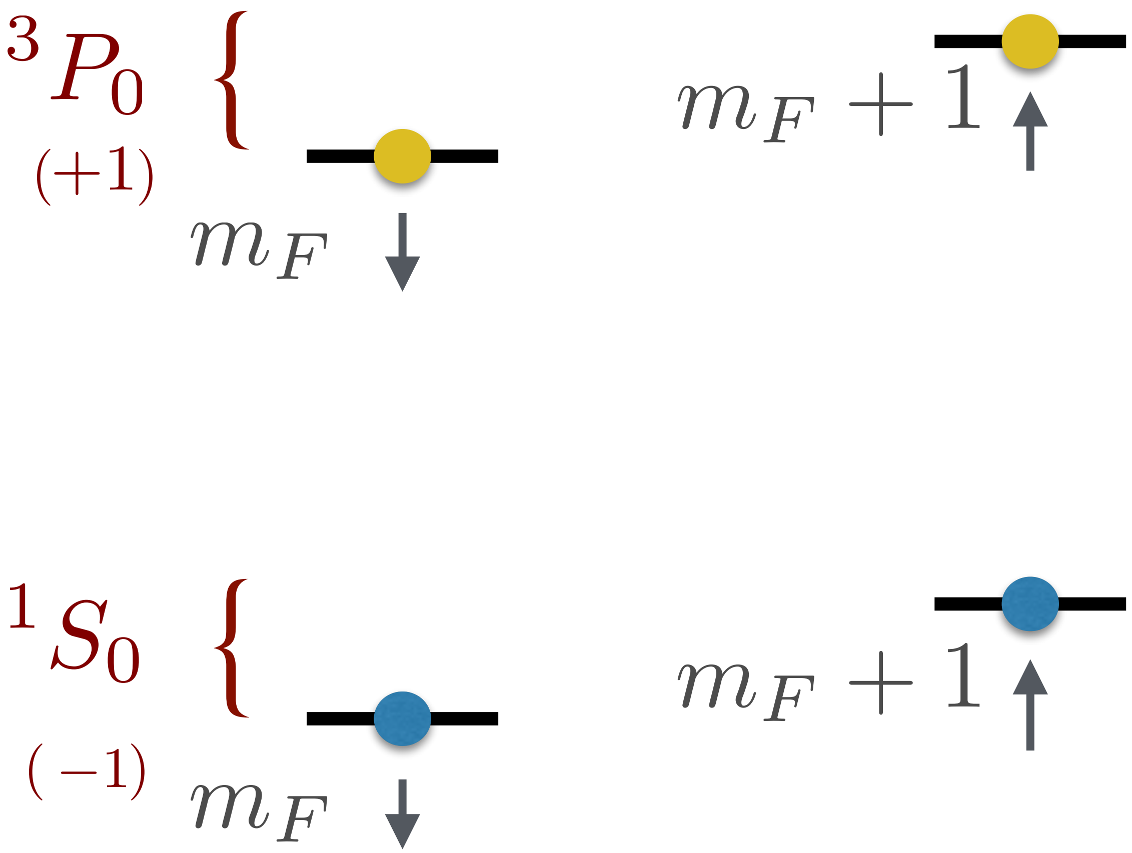
Spin-orbit

M. L. Wall et al., Phys. Rev. Lett. **116**, 035301 (2016)

L. F. Livi et al., Phys. Rev. Lett. **117**, 220401 (2016)

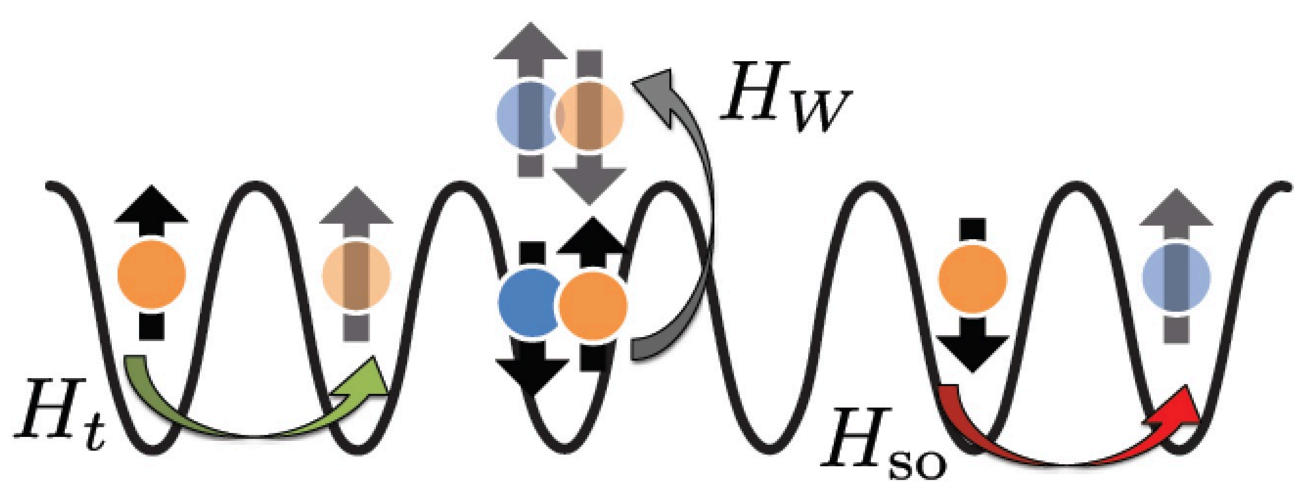
S. Kolkowitz et al., Nature **542**, 66 (2017)

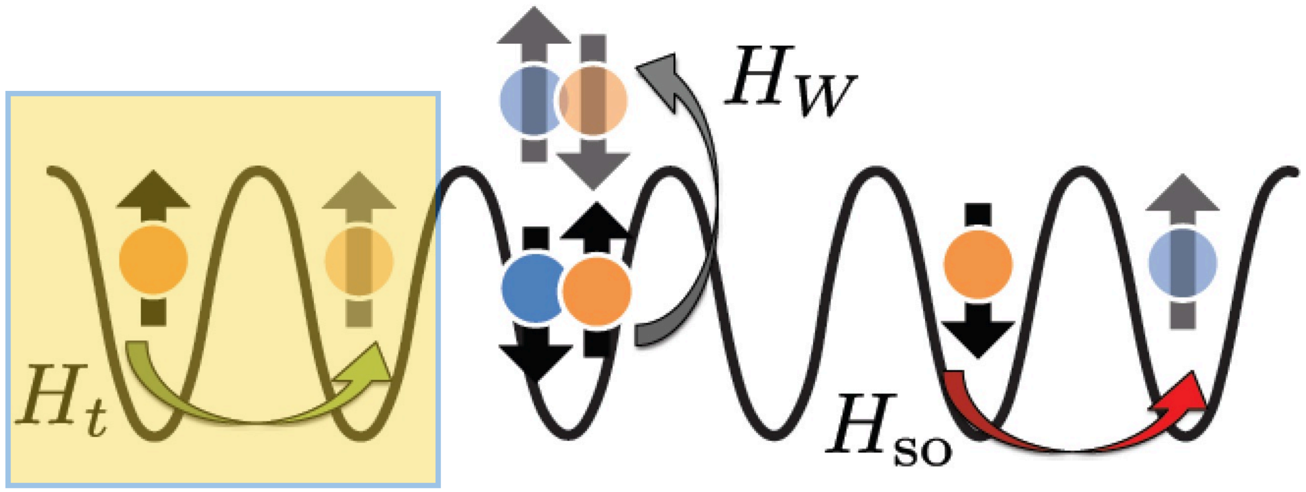
- Local parity symmetry occurs naturally in the presence of spin-orbit coupling + spin-exchange interactions;
- Symmetry is protected by Angular Momentum Conservation
- Spin-exchange acts effectively as“pairing correlations”



- Nuclear spin (m_F, m_F+1)
- Orbital state $(+1, -1)$

$$H = \sum_j (H_{t,j} + H_{U,j} + H_{W,j} + H_{\mathrm{so},j})$$



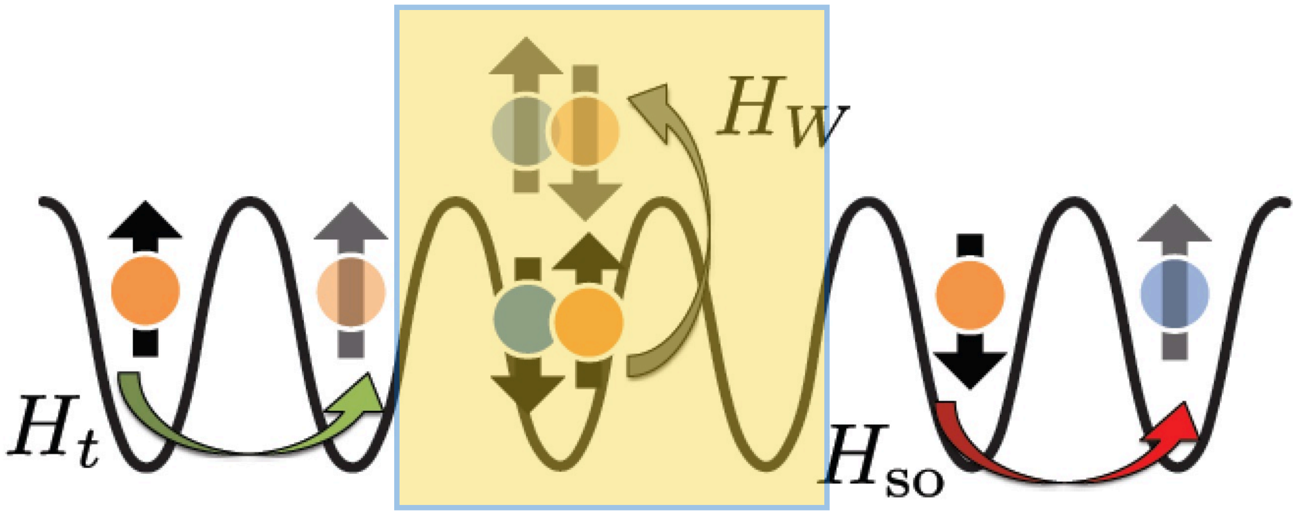


Tunneling

$$H_{t,j} = \sum_{\alpha,p} t(c^\dagger_{j,\alpha,p} c_{j+1,\alpha,p} + \text{h.c.})$$

Spin-orbit interaction

$$H_{\rm so,j} = \sum_p \left\{ (\alpha_R + b) c^\dagger_{j,\uparrow,p} c_{j+1,\downarrow,-p} + (b - \alpha_R) c^\dagger_{j+1,\uparrow,p} c_{j,\downarrow,-p} + \text{h.c.} \right\}$$



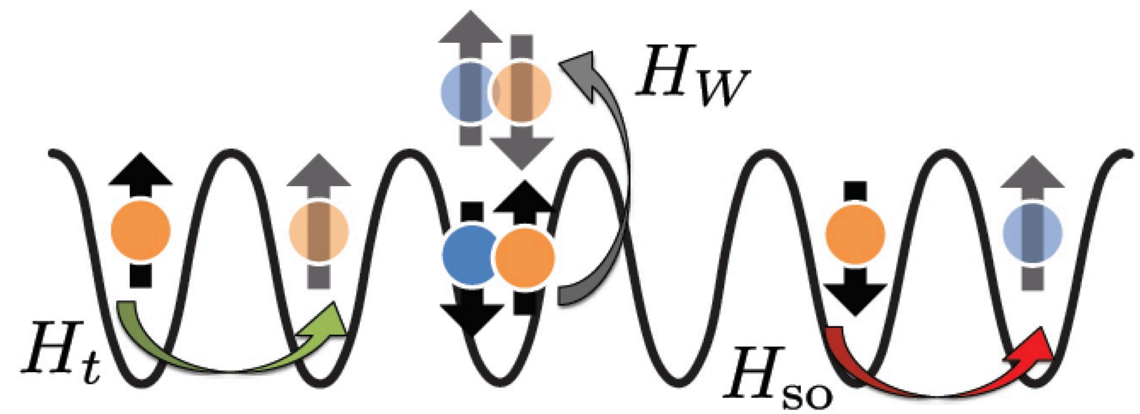
Hubbard interaction

$$H_{U,j} = \sum_p U_p n_{j,\uparrow,p} n_{j,\downarrow,p} + U \sum_{\alpha,\beta} n_{j,\alpha,-1} n_{j,\beta,1}$$

Exchange term

$$H_{W,j} = W(c^\dagger_{j,\uparrow,-1} c^\dagger_{j,\downarrow,1} c_{j,\downarrow,-1} c_{j,\uparrow,1} + \mathrm{h.c.})$$

Spin-orbit interaction



$$H_{\mathrm{so,j}} = \sum_p \left\{ (\alpha_R + b) c^\dagger_{j,\uparrow,p} c_{j+1,\downarrow,-p} + (b - \alpha_R) c^\dagger_{j+1,\uparrow,p} c_{j,\downarrow,-p} + \mathrm{h.c.} \right\}$$

The presence of spin-orbit coupling reduces the global spin symmetry to an angular momentum parity symmetry.

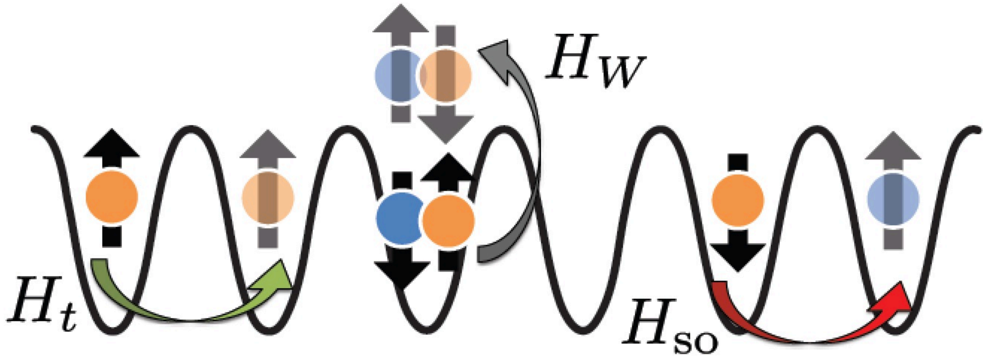
$$\sim S^x_j L^x_j$$

The number of fermions in each pair of states, [(↑,1),(↓,-1)] and [(↑,-1),(↓,1)], coupled by spin-orbit coupling is conserved mod(2)

$$P = (-1)^{\sum [n_{j,\uparrow,1} + n_{j,\downarrow,-1} - 1]}$$

The system described by the model

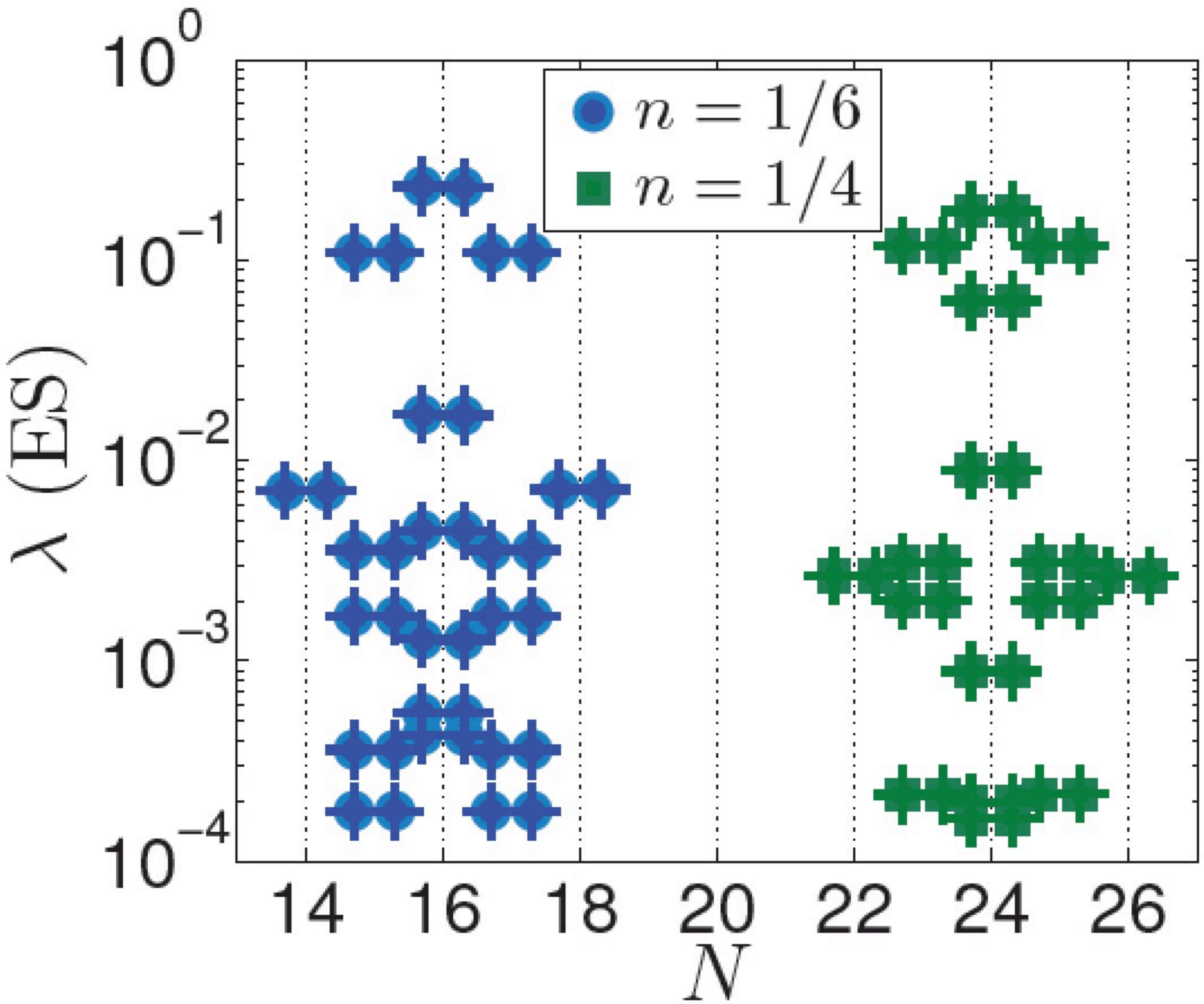
$$H = \sum_j (H_{t,j} + H_{U,j} + H_{W,j} + H_{\mathrm{so},j})$$



supports Majorana edge modes

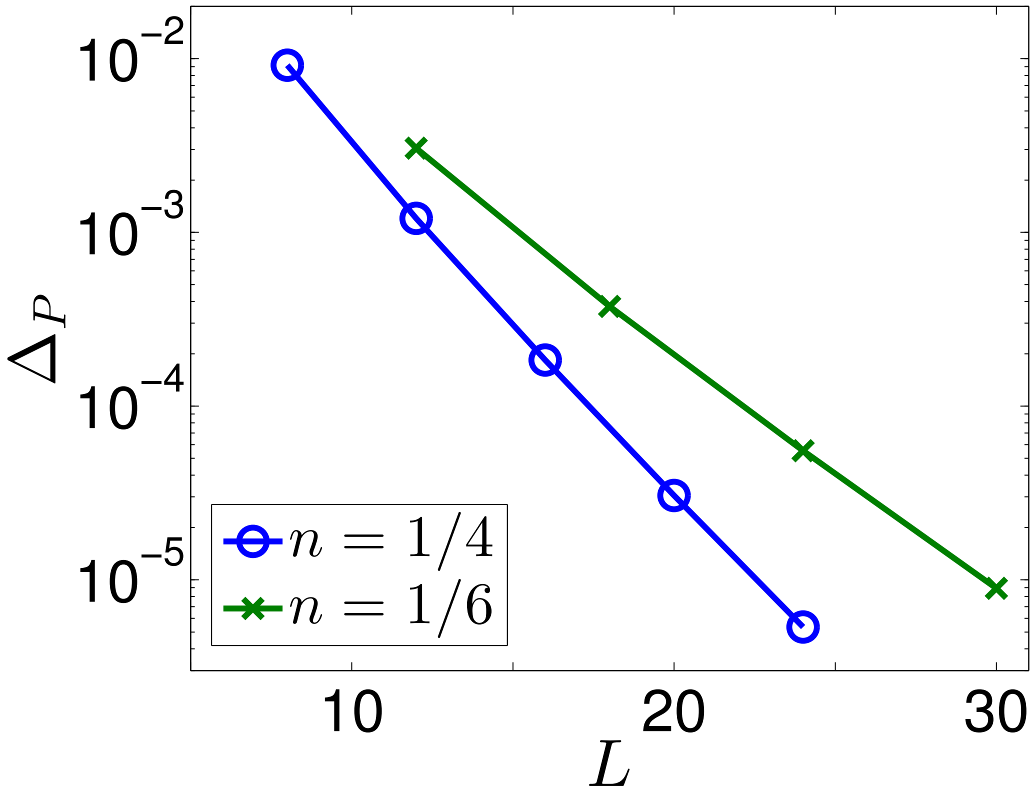
DMRG - Entanglement spectrum

Given the reduced density matrix with respect of a bipartition of the system, the entanglement spectrum is the collection of its eigenvalues



DMRG - Energy gaps

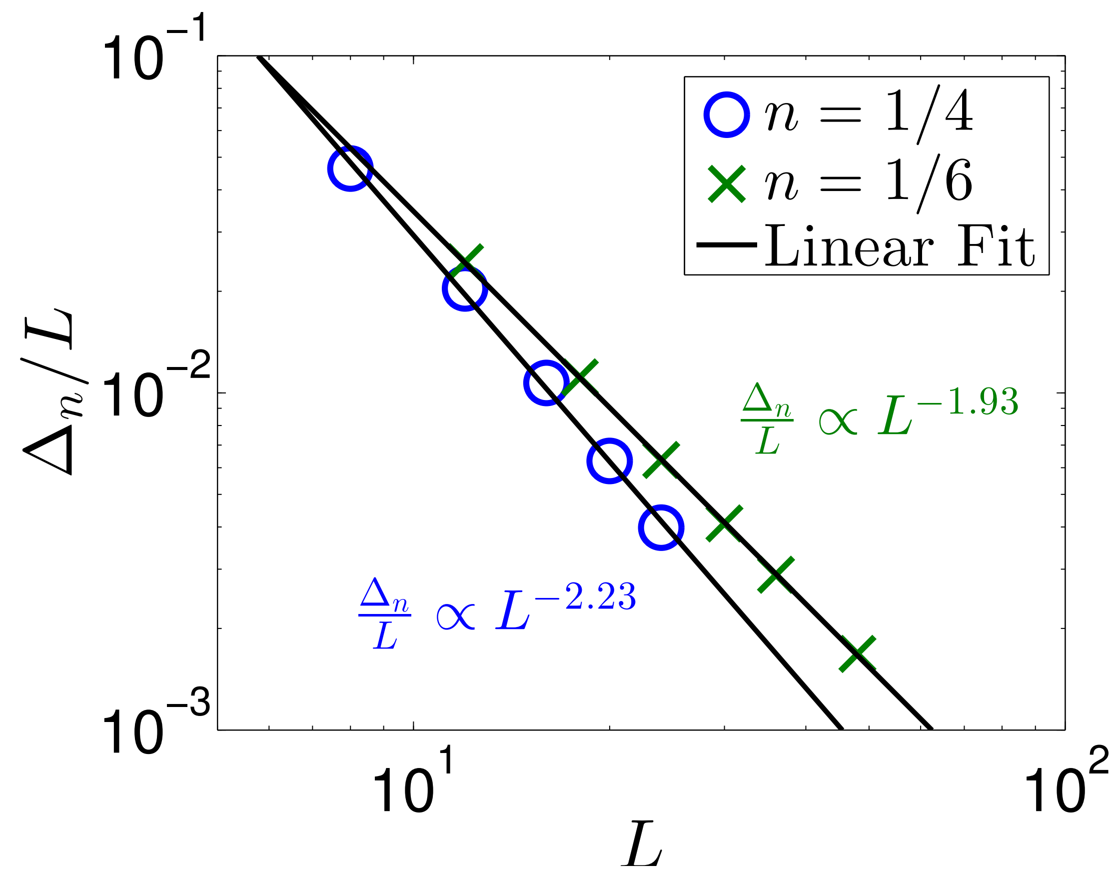
$$\Delta_P = E^0_L[N,-1] - E^0_L[N,1]$$



The parity gap is sensitive exclusively to spin excitations and it closes exponentially with the system size

$$\Delta_n = E^1_L[N,P] - E^0_L[N,P]$$

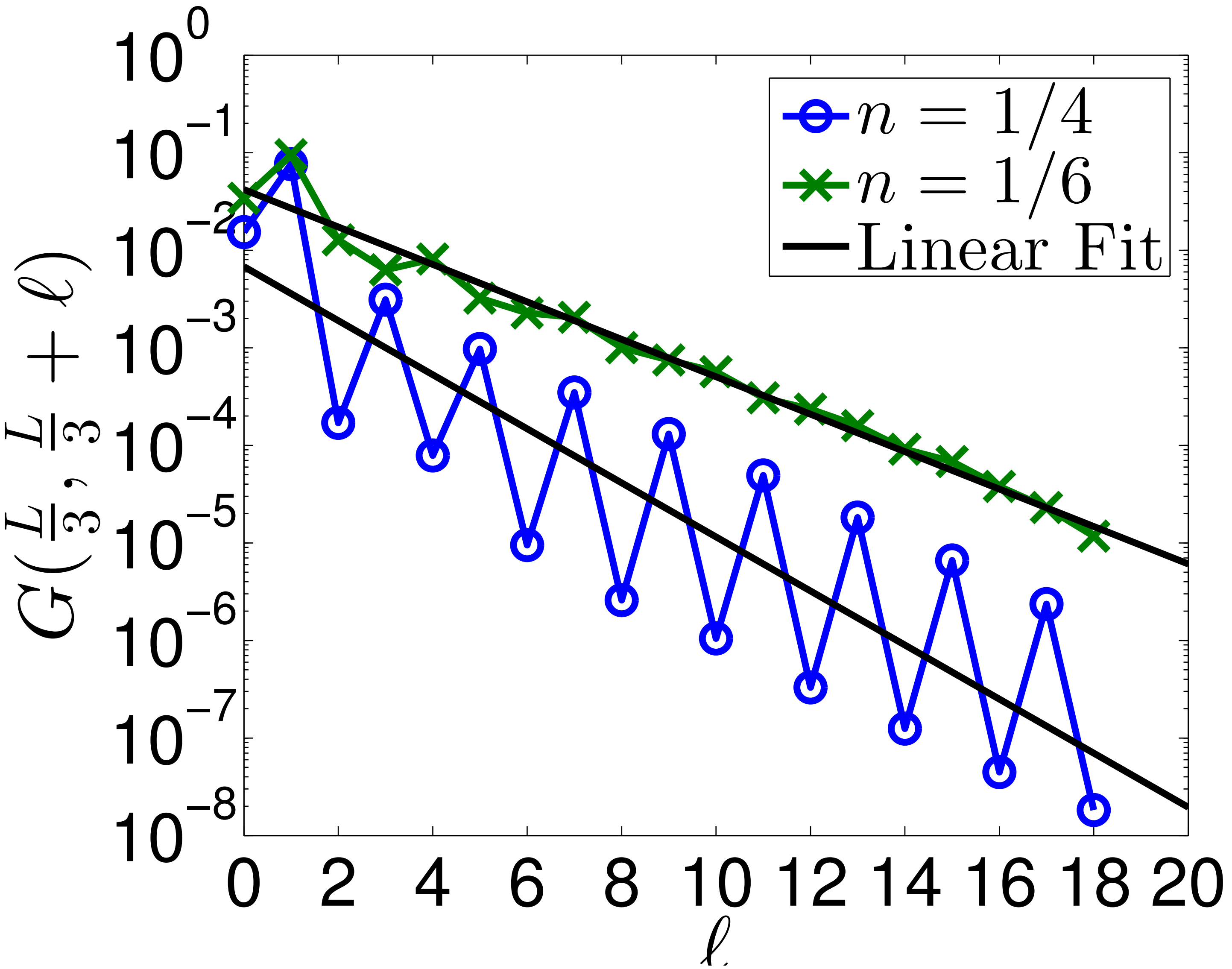
In the topological phase, this gap decays algebraically due to the presence of a gapless charge excitation



DMRG - Correlation functions

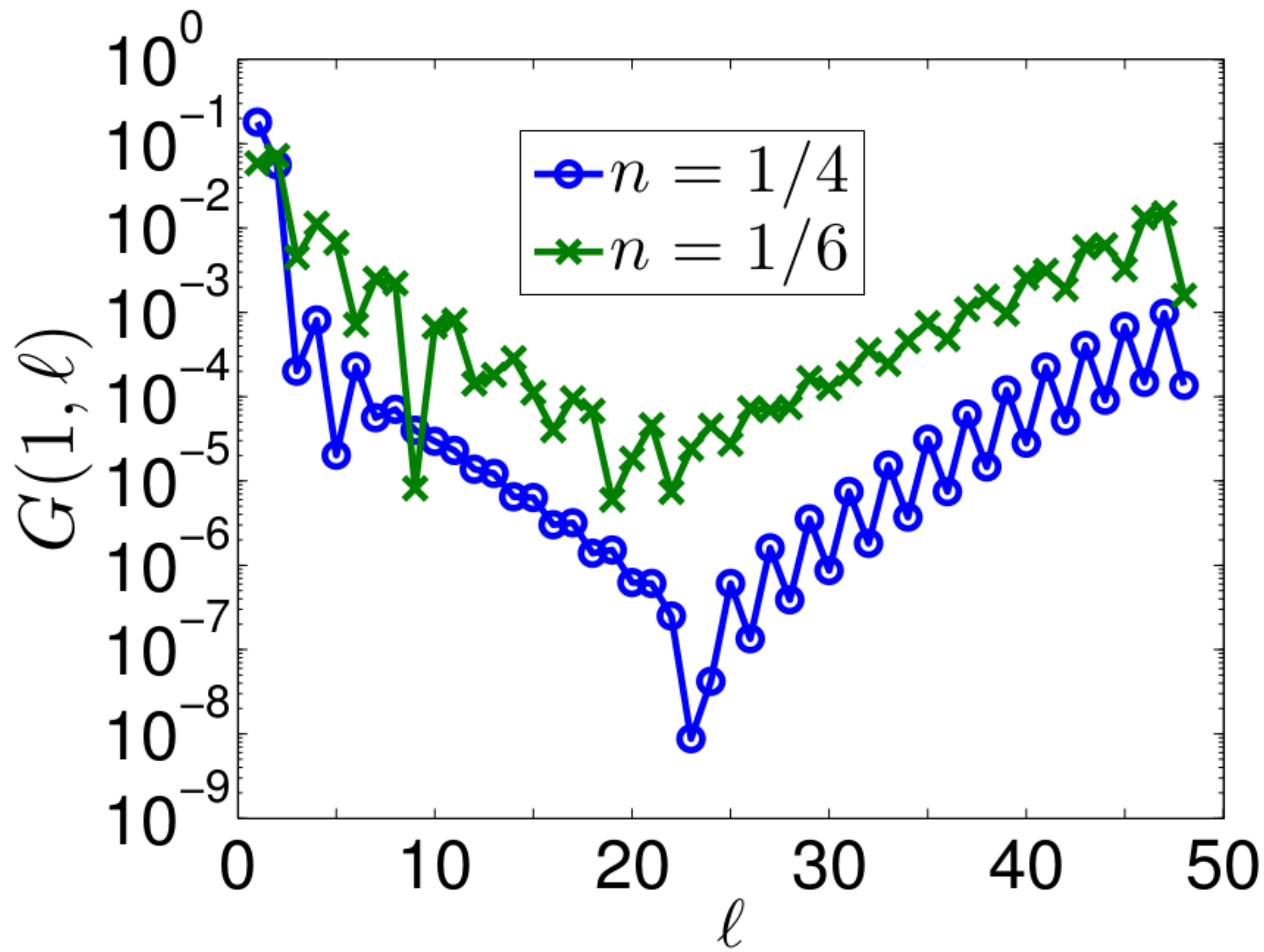
A finite bulk gap in the spin sector is signalled by an exponential decay of the Green functions

$$G(j,\ell) = \langle c^\dagger_{j,\uparrow,1} c_{\ell,\uparrow,1} \rangle$$



DMRG - edge correlations

$$G(j,\ell) = \langle c^\dagger_{j,\uparrow,1} c_{\ell,\uparrow,1} \rangle$$



The Green functions are sensitive to the presence of edge modes,

Here the correlation of one boundary site with the rest of the chain

The correlation rapidly decays in the bulk due to the presence of a spin gap, there is a strong revival close to the edge of the system, signalling the presence of MQP edge modes

Conclusions

- Majorana quasi-particles can emerge as edge modes of in optical lattices in the presence of spin-orbit couplings + spin-exchange interactions

- Hopefully such results could help to the observation of Majorana edge modes in such canonical settings, where all basic ingredients for our recipe have been experimentally realised