

# Synthetic momentum-space lattices: exploring the interplay of topology, frustration, disorder, and interactions

Gadway group

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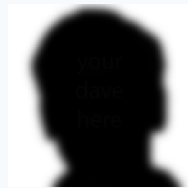
**Synthetic dimensions in quantum engineered systems**

**Pauli Center for Theoretical Studies**

**November 20, 2017**



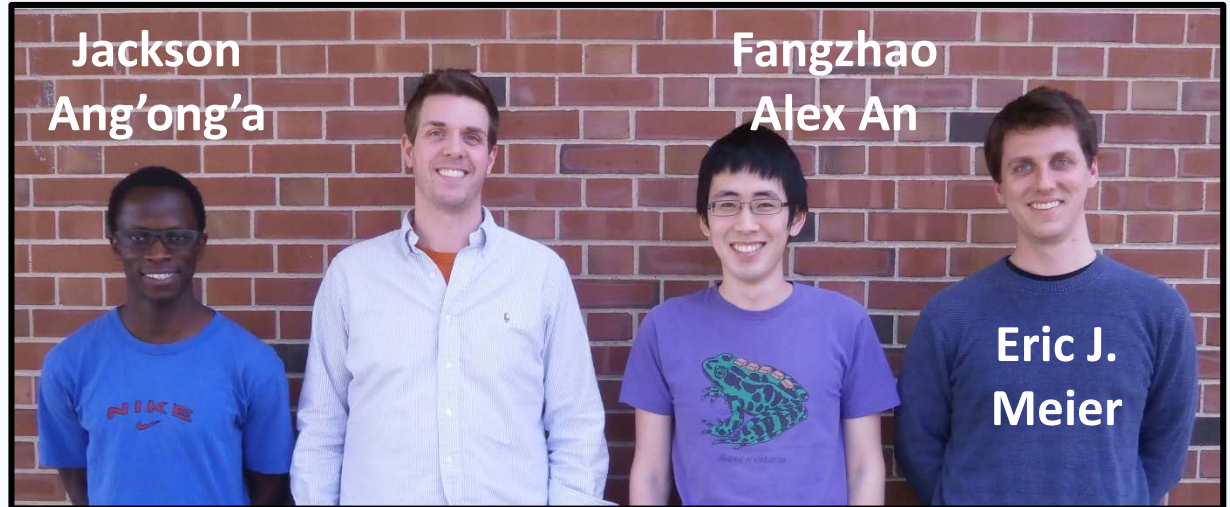
# Acknowledgments



Muyan Du



Michael Highman



Current UG researchers / visitors: Hannah Manetsch, Samantha Lapp, Zejun Liu

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## Theory friends & collaborators:

- Pietro Massignan, Maria Maffei, & Alexandre Dauphin (ICFO)
- Kaden Hazzard (Rice University)
- Taylor Hughes (UIUC)
- Smitha Vishveshwara (UIUC)



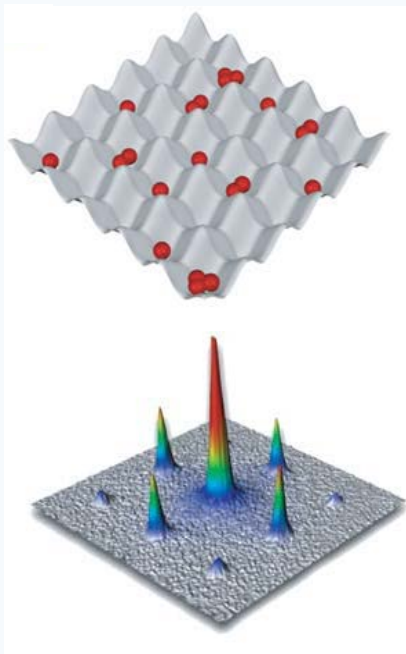
# The promise of synthetic lattices

(as I see it, for cold atoms)

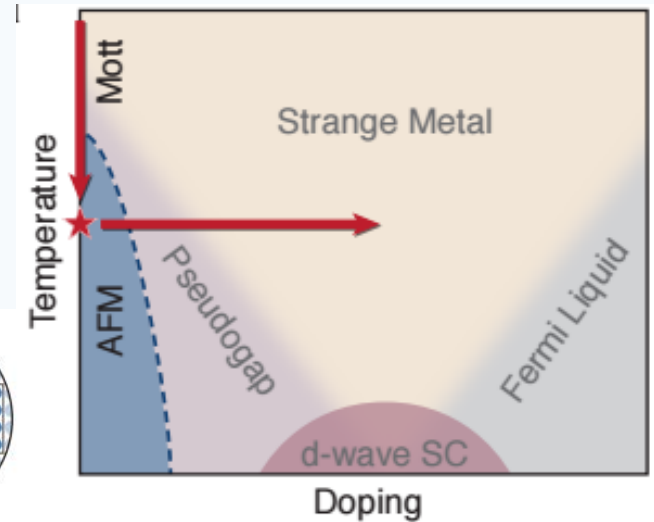
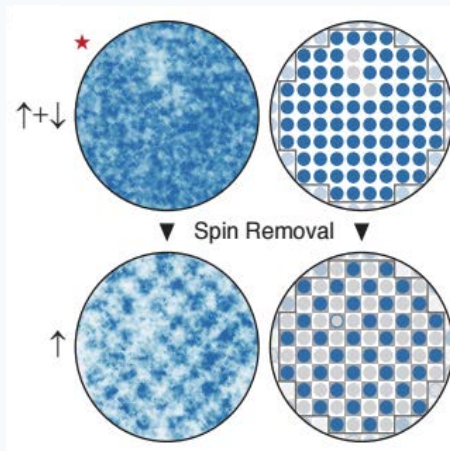
For some problems, it's best to just let atoms act naturally:

cold atoms in optical lattices  $\approx$  ideal lattice Hubbard model

Jaksch, *et al.* PRL (1998)



Greiner, Bloch, Esslinger, Hänsch - Nature (2002)



Mazurenko, *et al.*, Nature (2017)

# The promise of synthetic lattices

(as I see it, for cold atoms)

For other problems, one has to go to heroic efforts to engineer certain effects in real space lattices – often at a price

## Examples:

- Mimicking the coupling of electrons to electromagnetic gauge fields
- Realizing periodic boundary conditions / hard-wall boundary conditions
- Realizing generic types of / forms of disorder
- Realizing higher-dimensional ( $d \geq 4$ ) physics
- Random unitary lattices for boson sampling?

Some of these problems become much easier (even trivial) if one of the “dimensions” to a system is represented by discrete quantum states, such as internal states

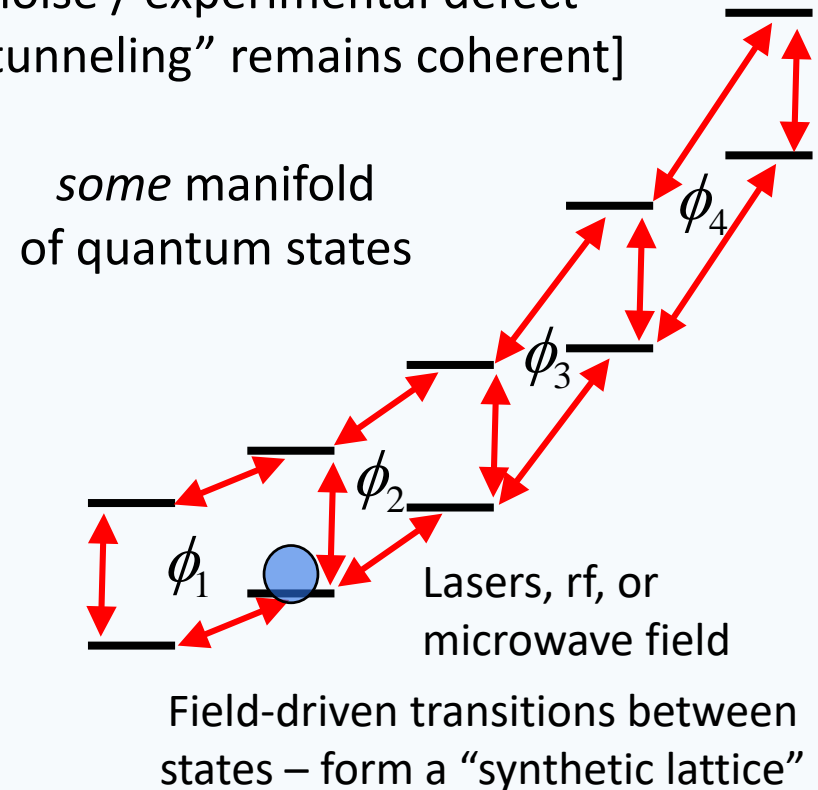
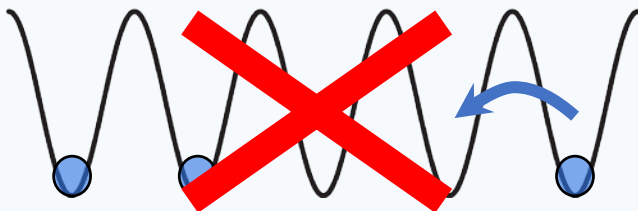
Boada, *et al.* PRL (2012)

Celi, *et al.* PRL (2013)

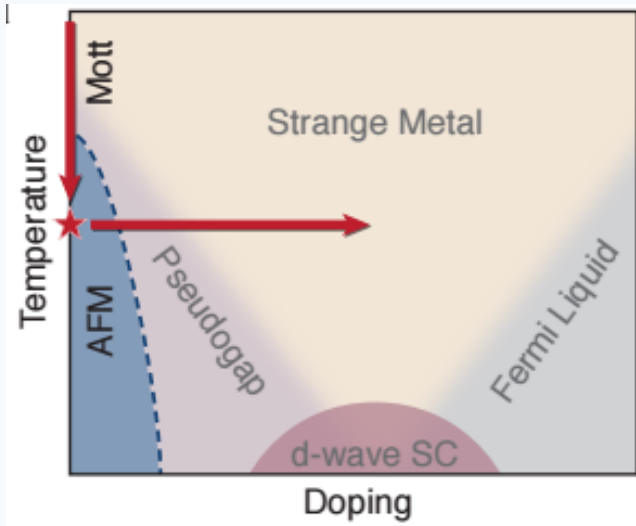
# An ideal picture: a fully synthetic approach

## Synthetic lattice wish list

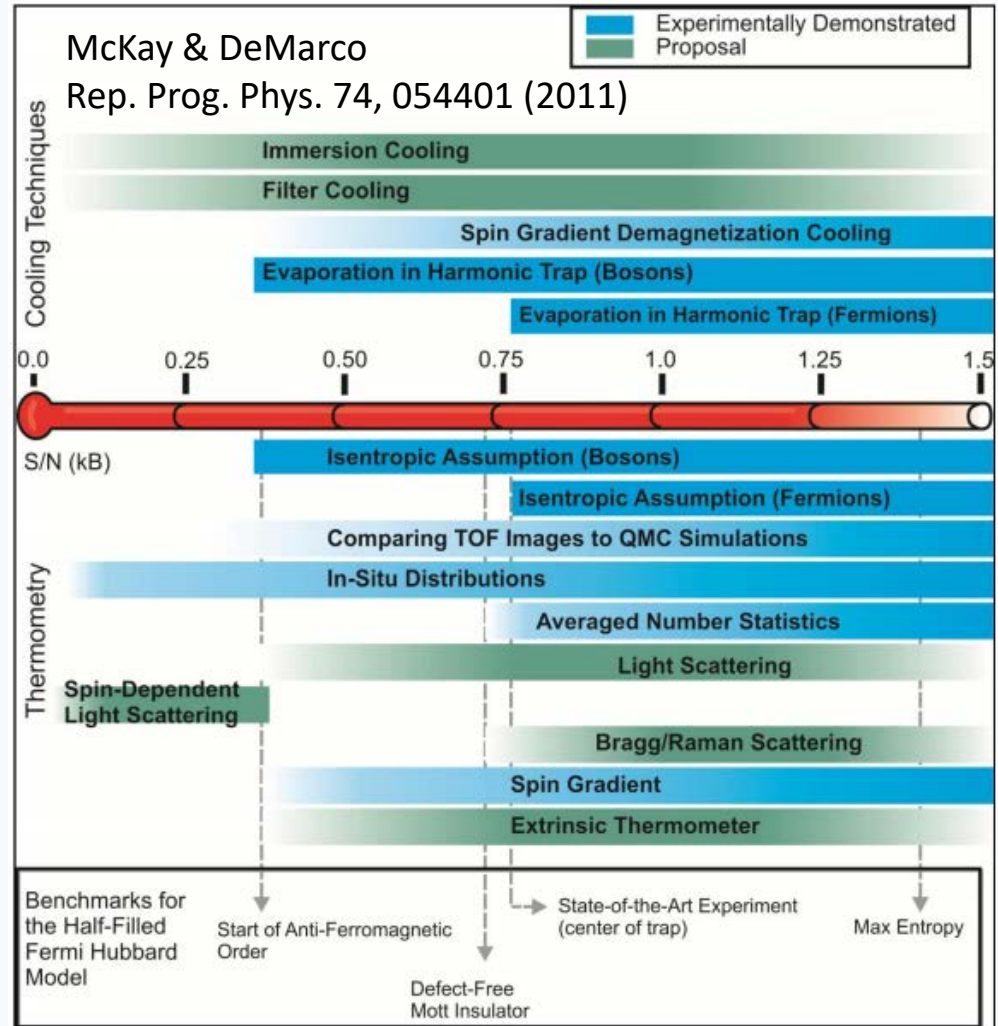
- A large collection of discrete quantum states
- State-to-state energies are unique  
(i.e. we have full spectroscopic engineering of Hamiltonian)
- Transition frequencies are insensitive to noise / experimental defect  
[can make interactions dominant while “tunneling” remains coherent]



# One future promise of fully synthetic lattices



Mazurenko, *et al.*,  
Nature (2017)

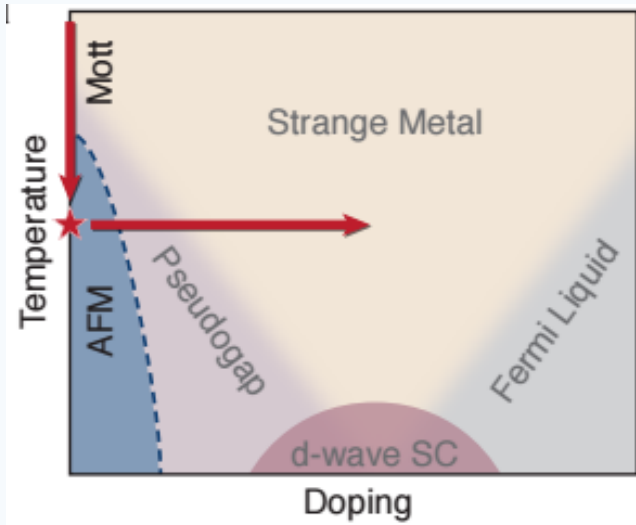


Measuring and lowering the temperature of atoms has been a big challenge for the past decade +

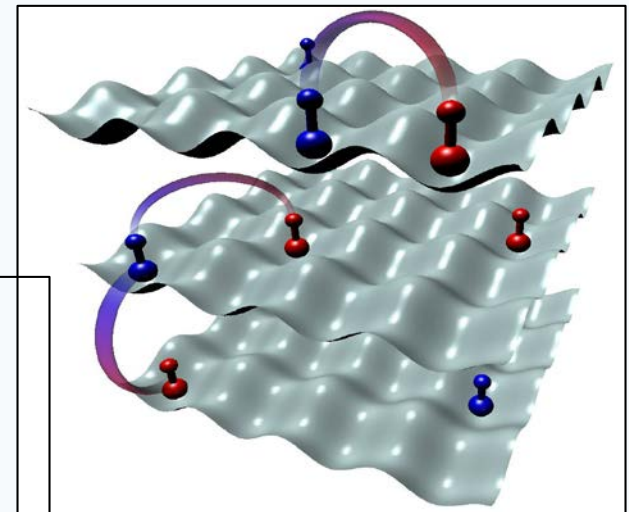
# One future promise of fully synthetic lattices

If real-space motion is **fully** frozen out, then we are freed from coupling to residual motional entropy & heating effects

→ and we're really good at preparing / preserving low-entropy spin states and preserving coherence



Mazurenko, *et al.*,  
Nature (2017)



Yan, *et al.* Nature (2013)  
Hazzard, *et al.* PRL (2014)

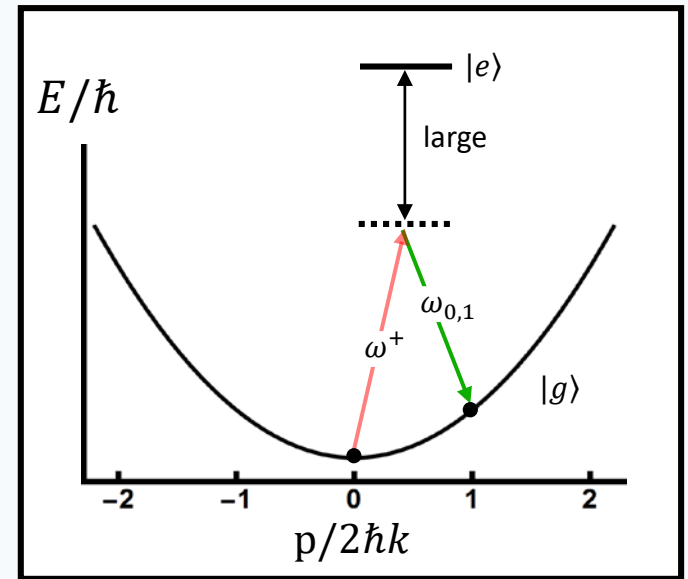
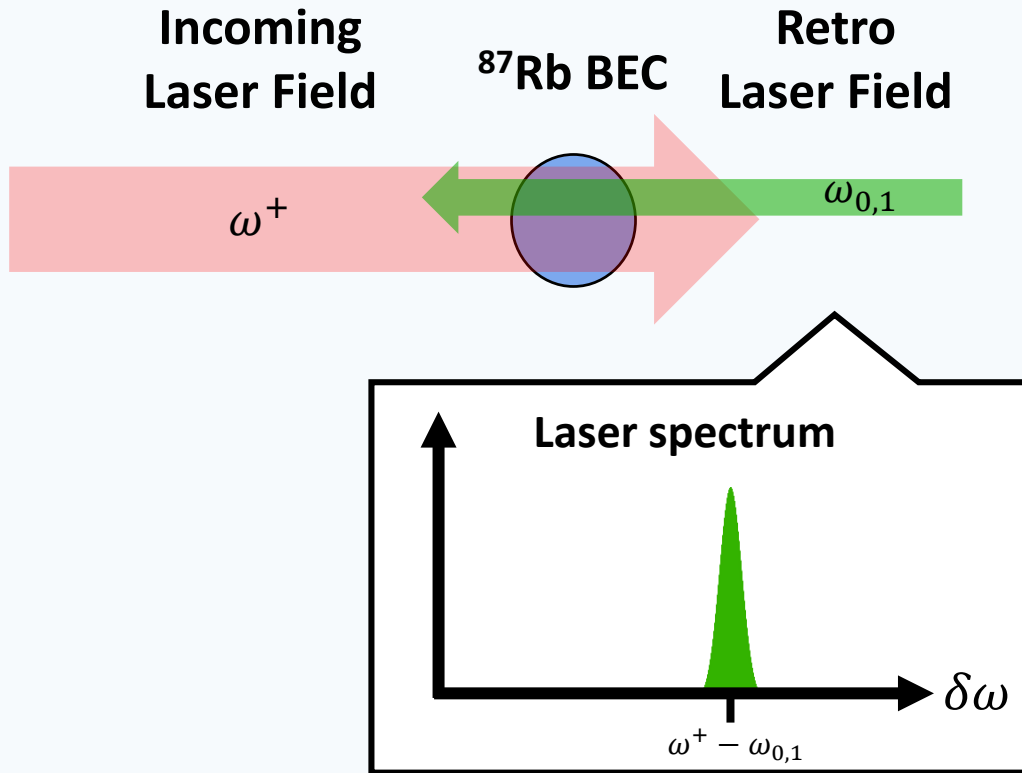
Such an approach would be limited, but well-suited to certain problems

A testbed for fully synthetic lattices:  
Synthetic momentum-space lattices

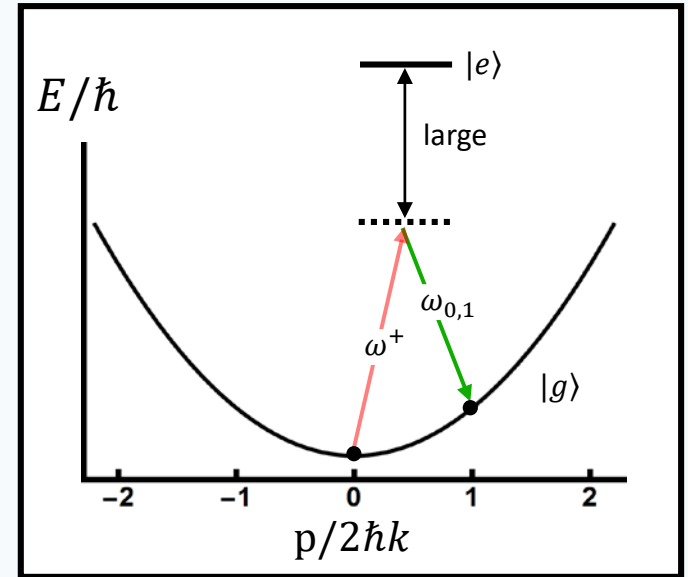
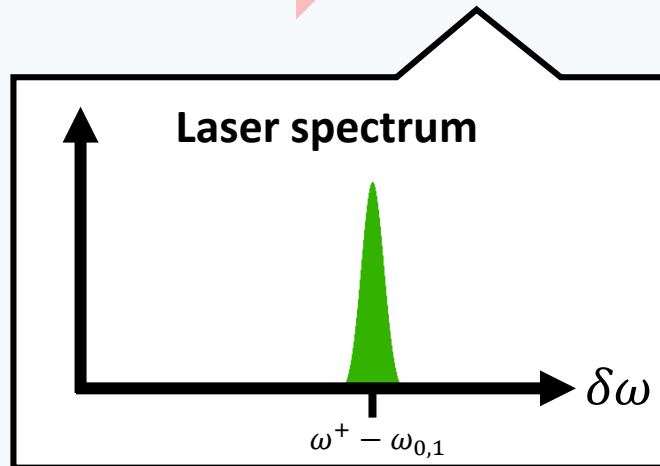
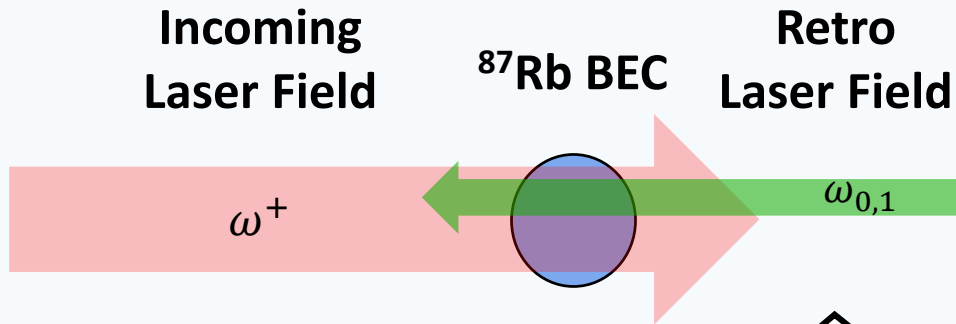
Note: there exists a great body of earlier work on the study of transport phenomena with atoms in momentum space, esp. kicked rotor studies



# Coupling momentum states



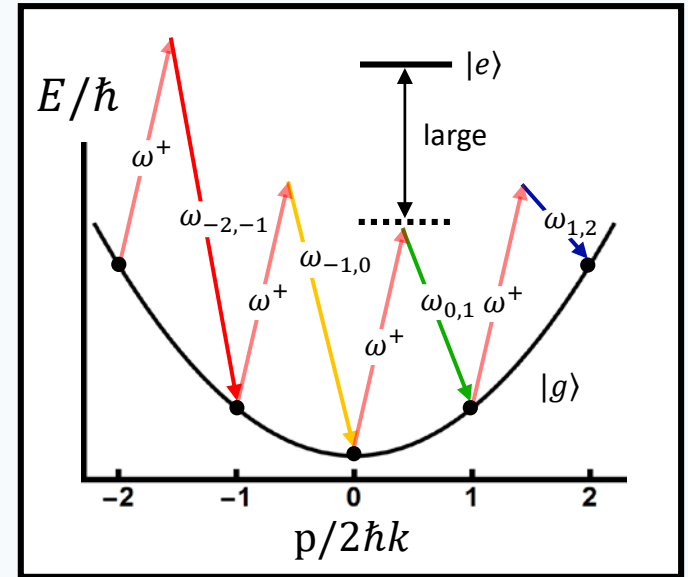
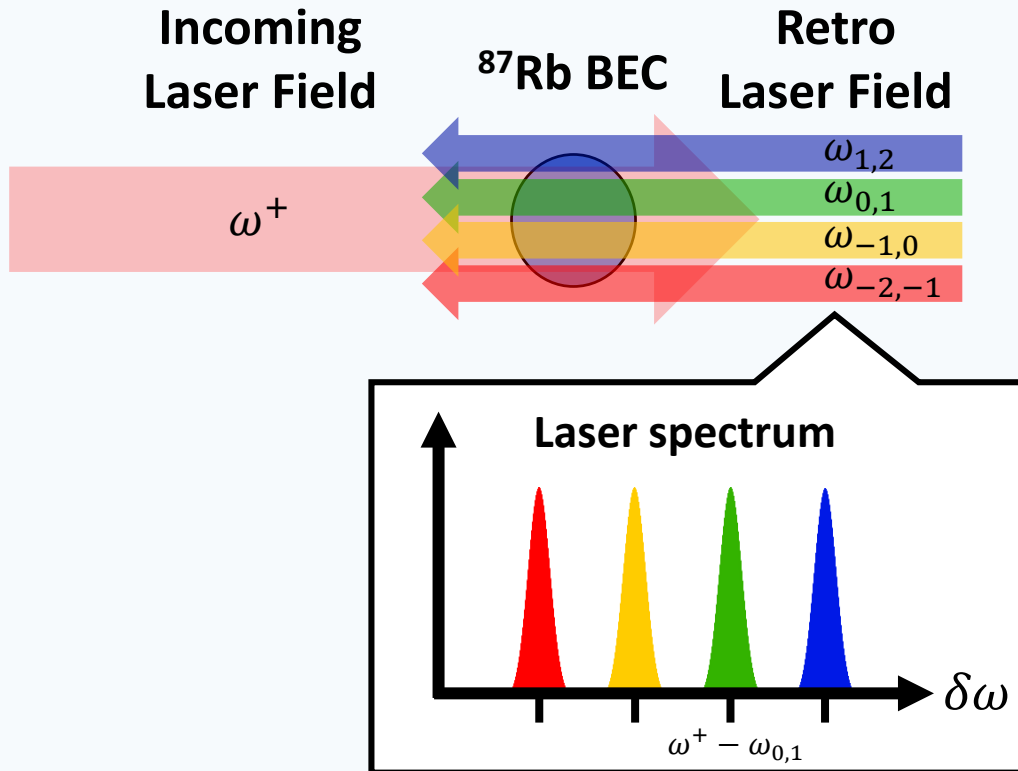
# Coupling momentum states



Two-photon  
Bragg transition



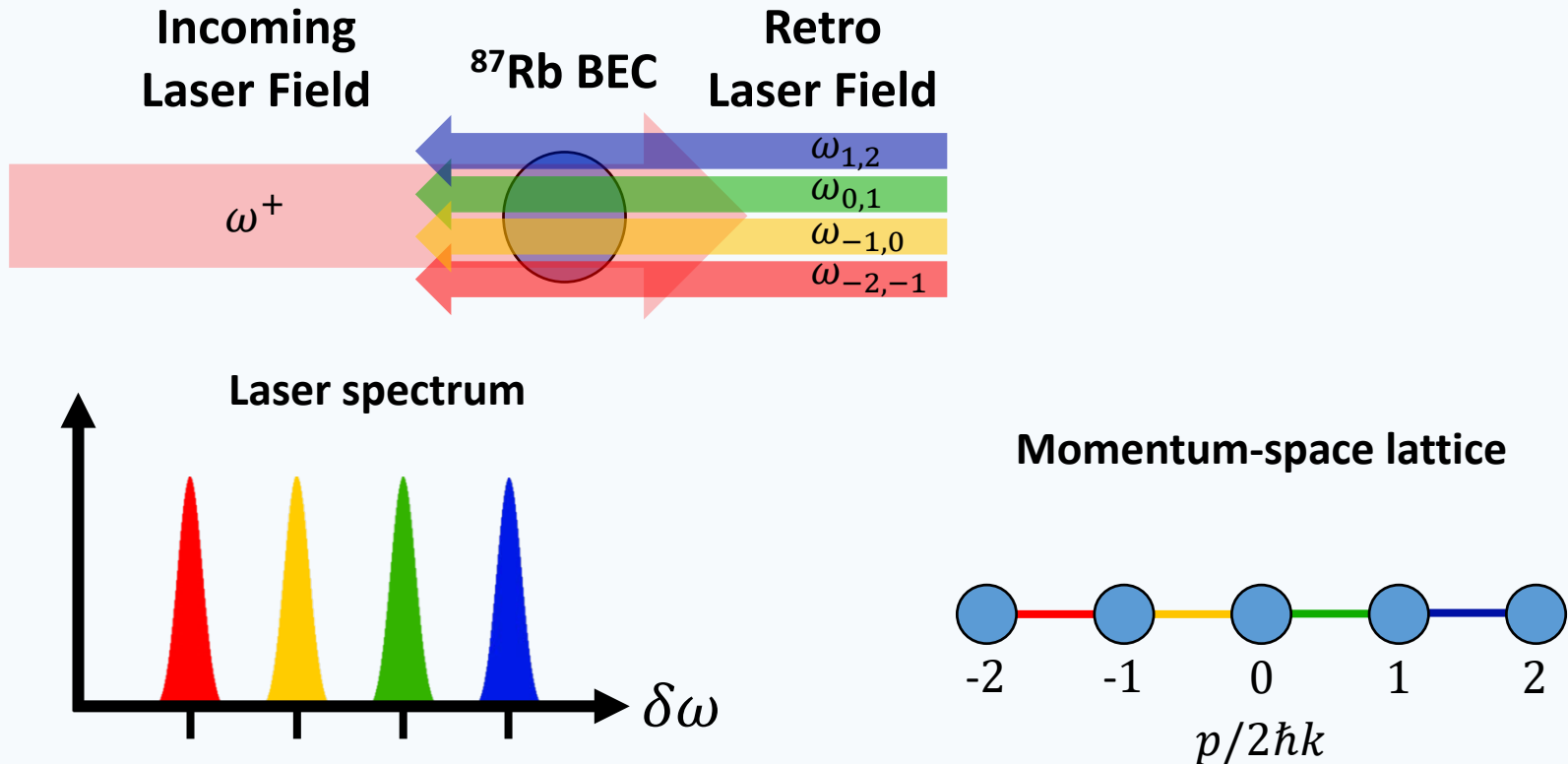
# Synthetic momentum-space lattices



**Two-photon  
Bragg transition**

**Many resonantly-coupled states, with individual addressing of all transitions**

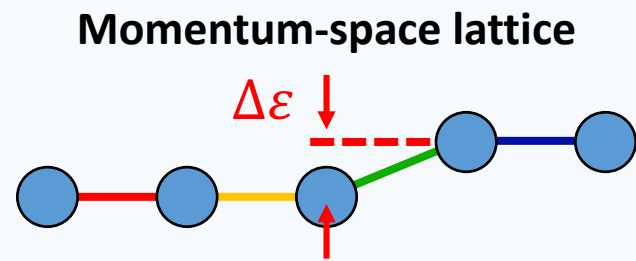
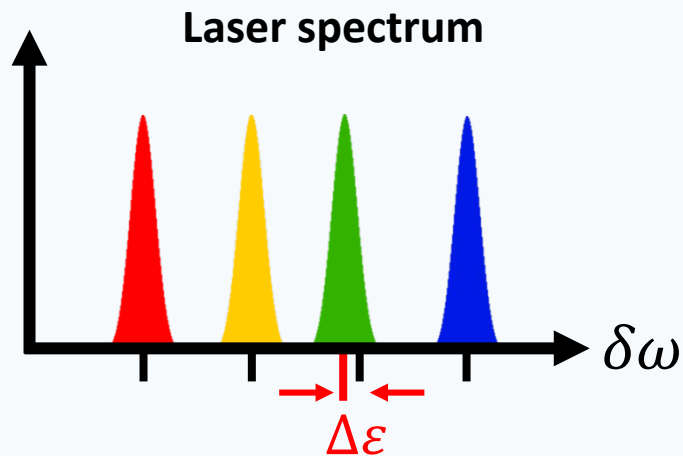
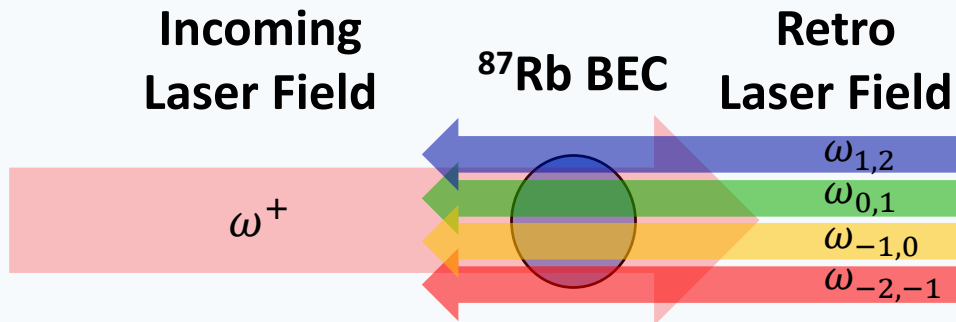
# Spectroscopic Hamiltonian engineering



*potential landscape*      *tunneling amplitudes and phases*

$$H_{s.p.} = \sum_n \varepsilon_n c_n^\dagger c_n - \sum_n t_n \left( e^{i\varphi_n} c_n^\dagger c_{n+1} + \text{h.c.} \right)$$

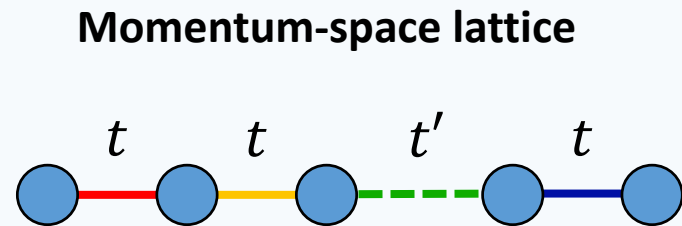
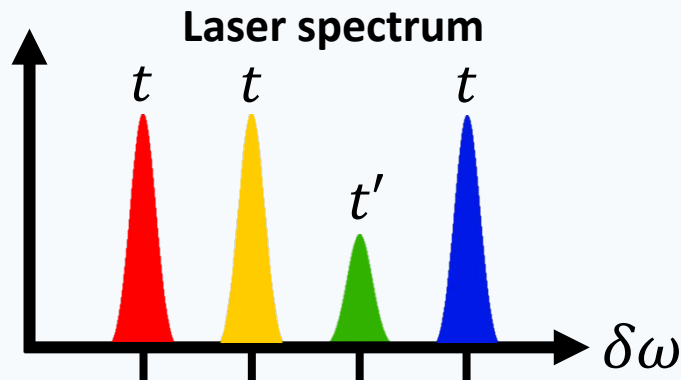
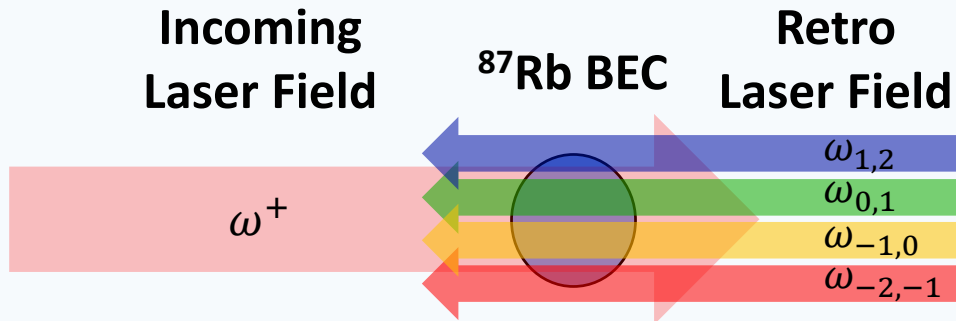
# Spectroscopic Hamiltonian engineering



*potential landscape*

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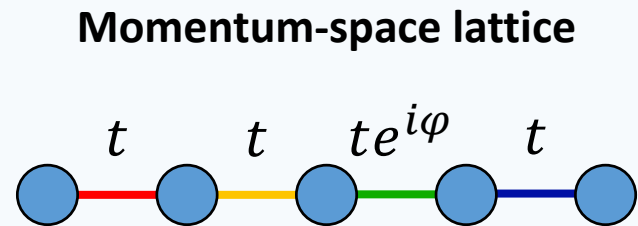
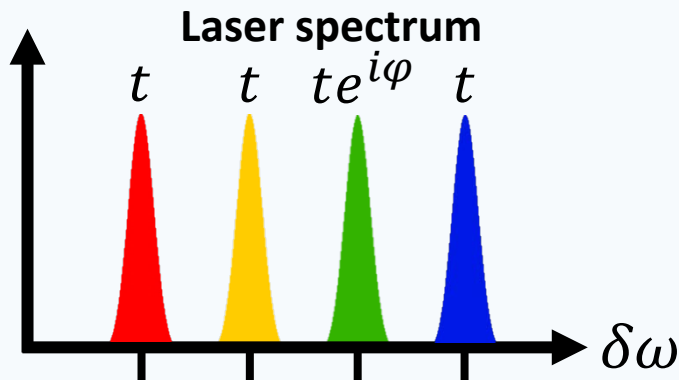
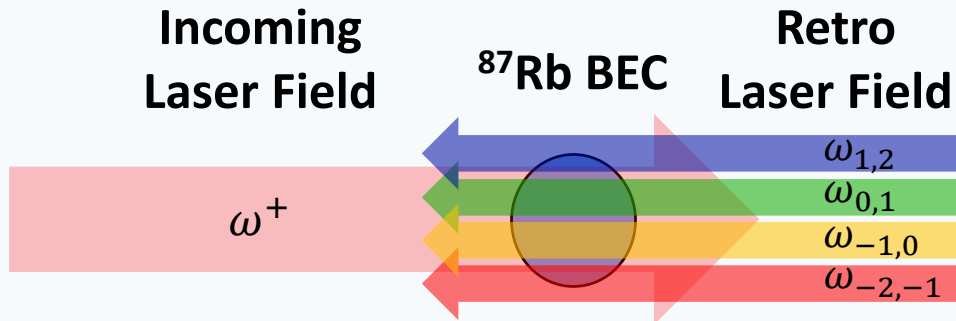
# Spectroscopic Hamiltonian engineering



*tunneling amplitudes*

$$H_{s.p.} = \sum_n \varepsilon_n c_n^\dagger c_n - \sum_n \boxed{t_n} \left( e^{i\varphi_n} c_n^\dagger c_{n+1} + \text{h.c.} \right)$$

# Spectroscopic Hamiltonian engineering



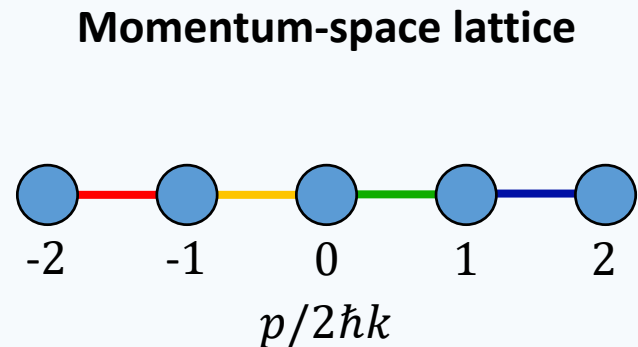
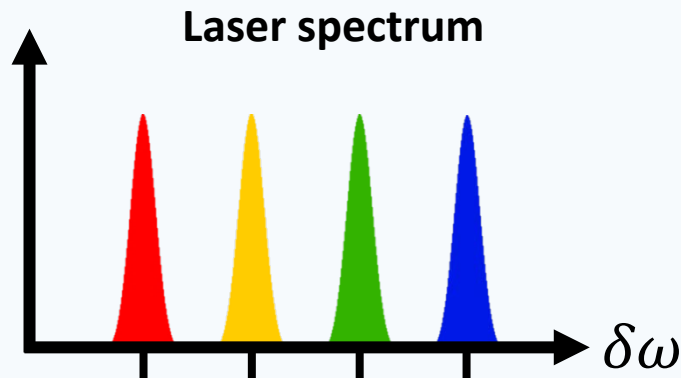
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*tunneling phases*

# Spectroscopic Hamiltonian engineering

*disorder, topology, hard-wall boundaries, ...*

- local, time-dependent control [*interesting for Floquet physics*]
- higher-dimensions, long-range tunneling [*interesting for synthetic gauge fields*]
- multiple internal states [*interesting for local loss, classical  $U(2)$  gauge fields, etc.*]
- **interactions** are naturally present and can be dominant over  $t$ 's &  $\varepsilon$ 's

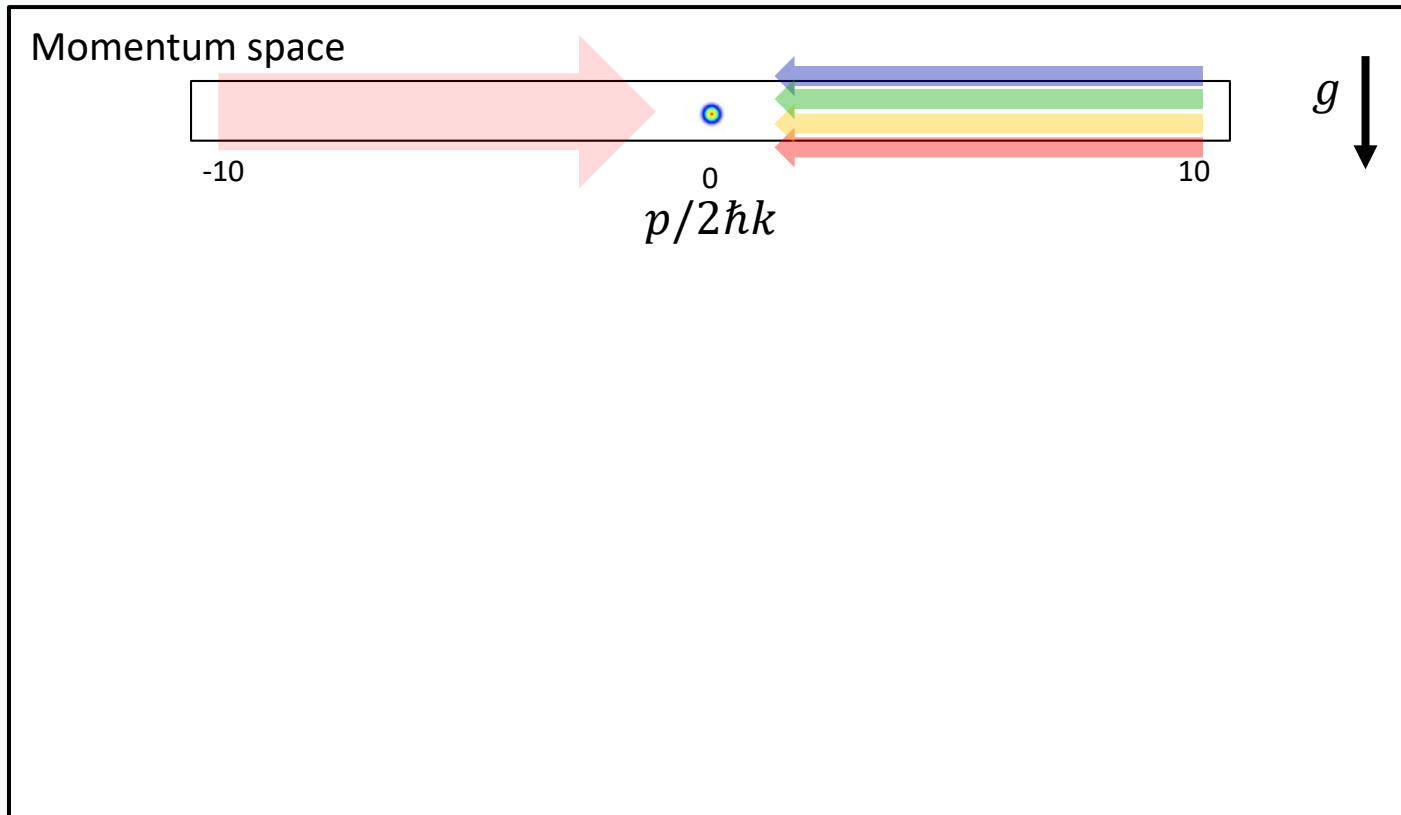


$$H_{s.p.} = \sum_n \varepsilon_n c_n^\dagger c_n - \sum_n t_n \left( e^{i\varphi_n} c_n^\dagger c_{n+1} + \text{h.c.} \right)$$

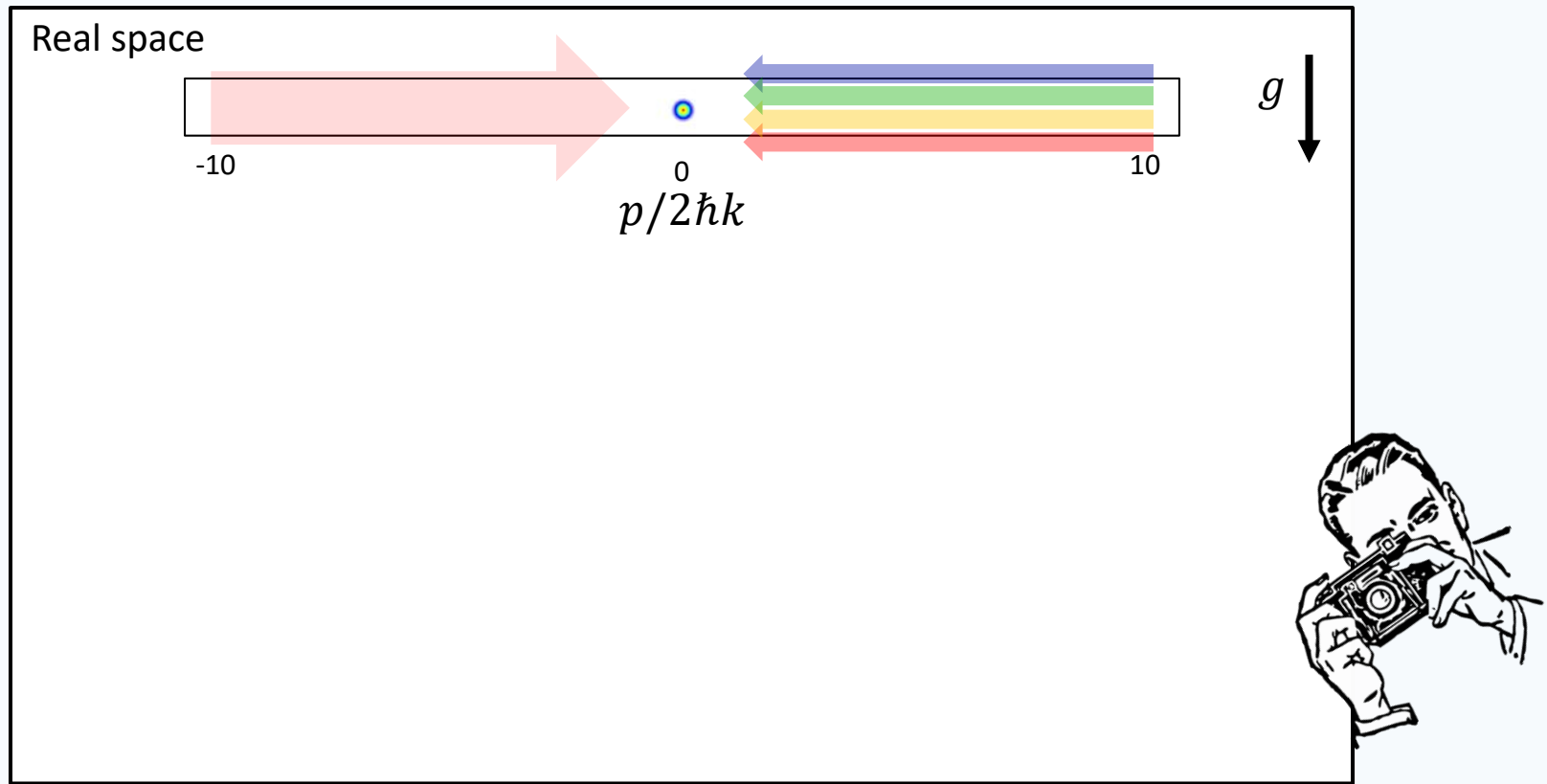
*potential landscape*      *tunneling amplitudes and phases*



# How we measure the atomic distributions

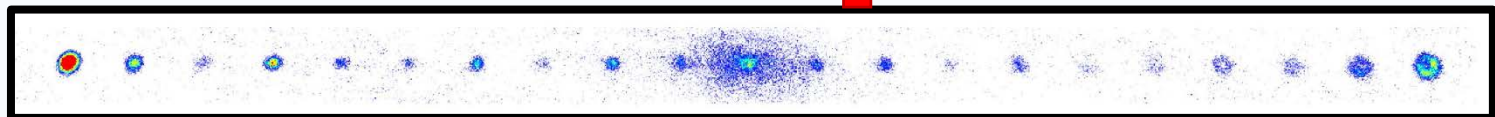
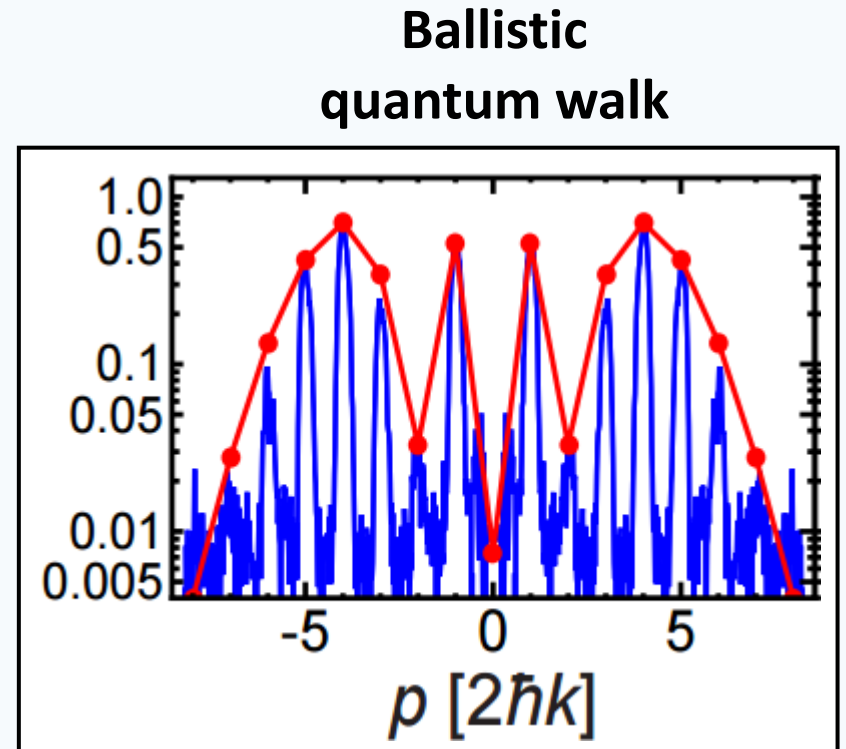
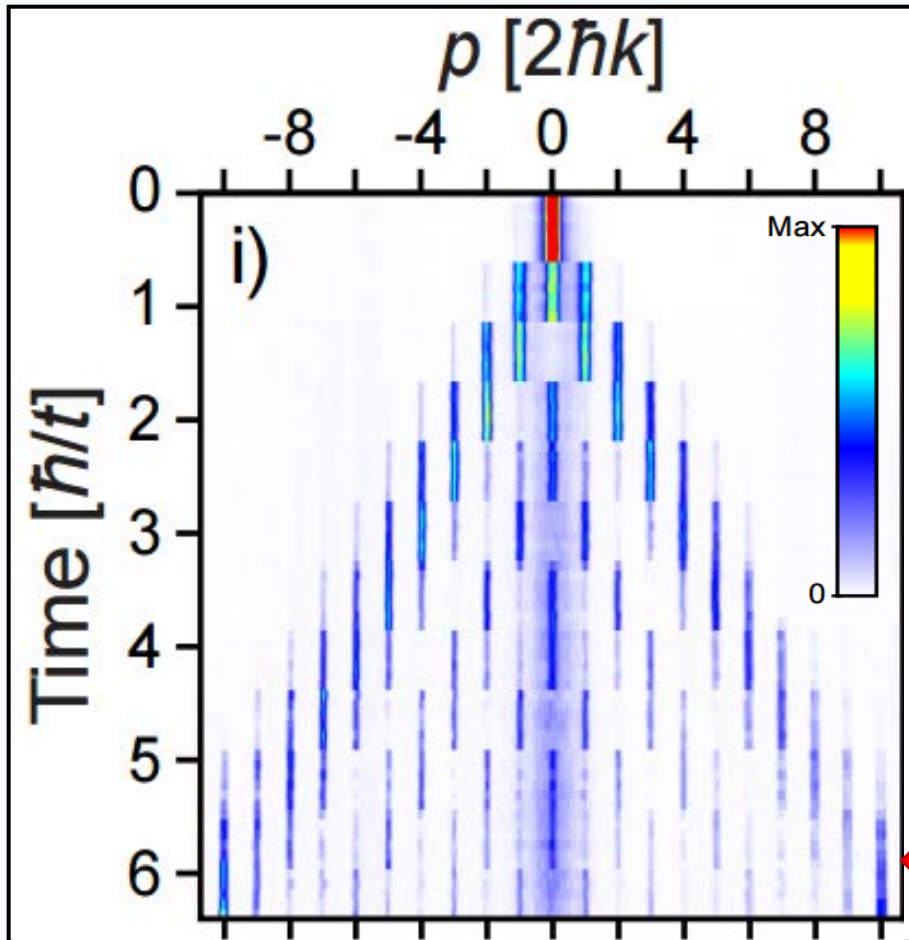


# How we measure the atomic distributions

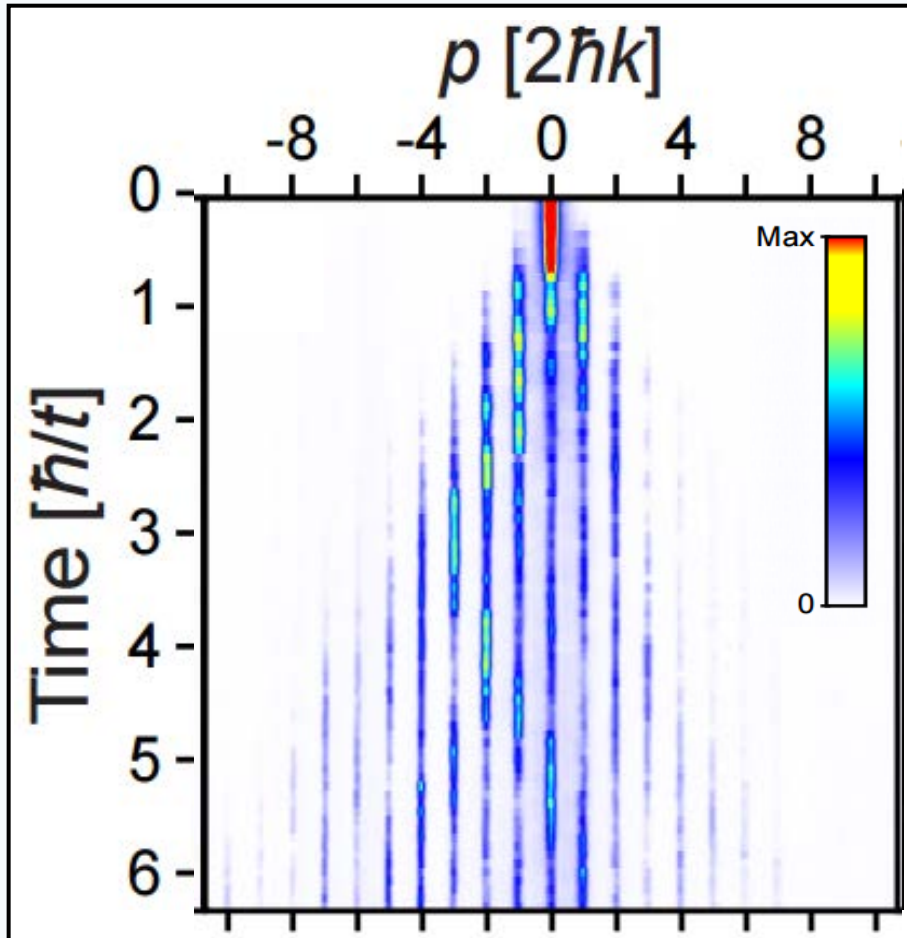


site-resolved measurement comes for free

# Simplest type of measurement: quench dynamics

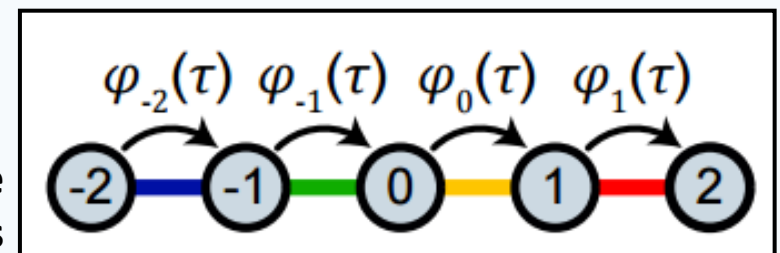
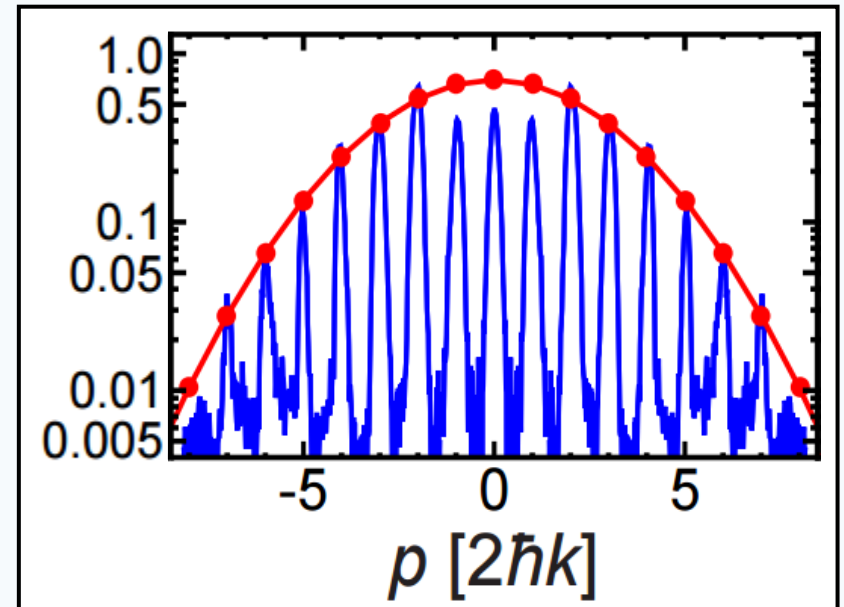


# Quench dynamics in dynamical disorder



Random, **dynamical** disorder in the tunneling phases

Dynamical phase disorder  
→ “classical” random walk

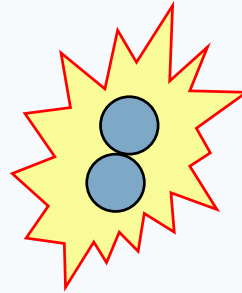


# Interactions in momentum-space lattices

# Momentum-space interactions

## real space

$$V_{ij} \approx g \sum_{i,j} \delta(\vec{r}_i - \vec{r}_j)$$



only short-range  
s-wave collisions  
 $T \ll 100 \mu\text{K}$

real-space interactions are nearly zero range (contact) at low energy

Should relate to **infinite-ranged** interactions in momentum space ...right?

## momentum space

all-to-all:  $V_{ij} = V_{kl} \quad \forall i, j, k, l$

**NO!**

(although this simple argument holds for distinguishable particles)

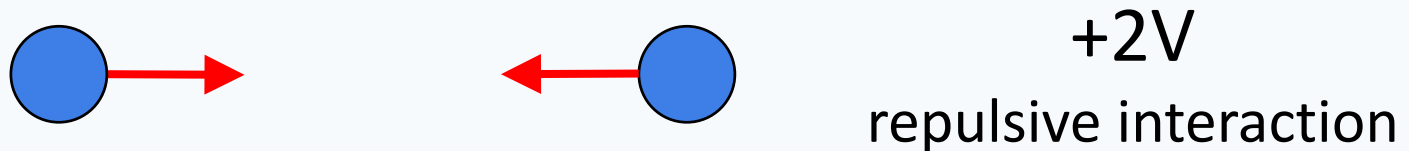
# Direct + exchange interactions

Consider two identical bosons at rest in their center of mass frame, i.e. with the same momentum



For repulsive interactions, let's assume two atoms experience a positive energy shift  $V$

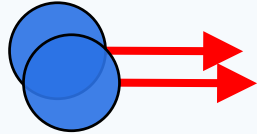
Now consider two identical bosons colliding with some relative momentum (let's assume mode-preserving interactions in 1D)



**Total pair wave function needs to be symmetric**

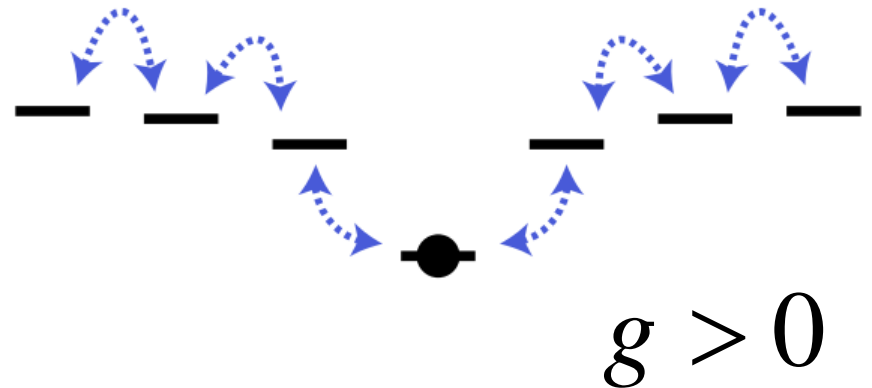
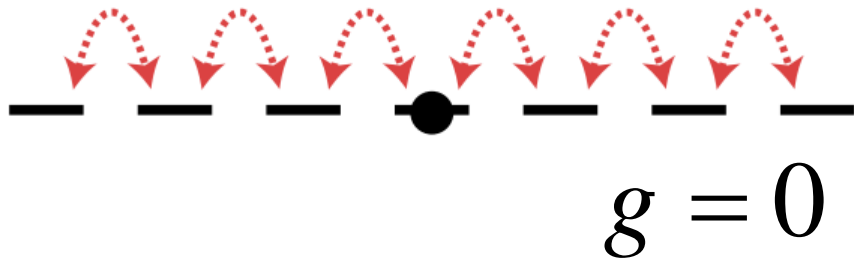
**→ factor of 2 enhancement of collisional energy shift  
(added “exchange energy”)**

# Effective momentum-space attraction



weaker repulsive interaction

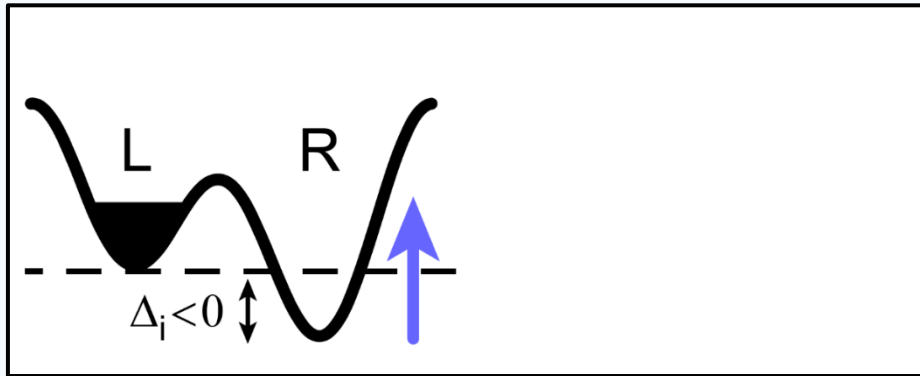
Looks like an effective **local attraction** for particles in same momentum state (site)!



The “attractive” interaction is mostly local (on-site)  
[some off-site component due to screening effects, which vanish for  $k\xi \gg 1$ ]

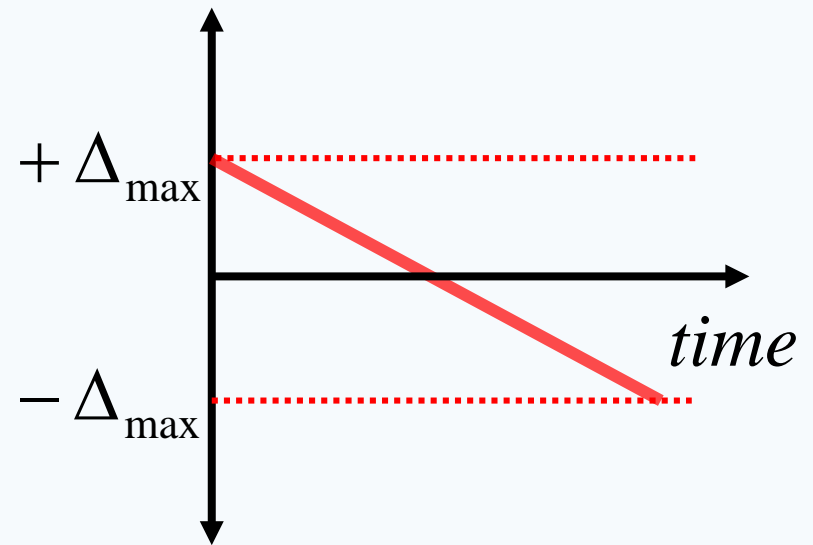
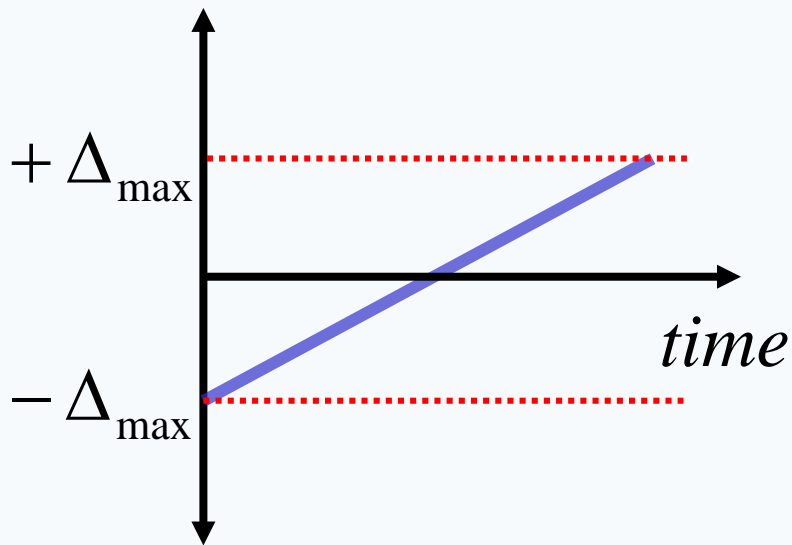


# Interactions in a double-well system

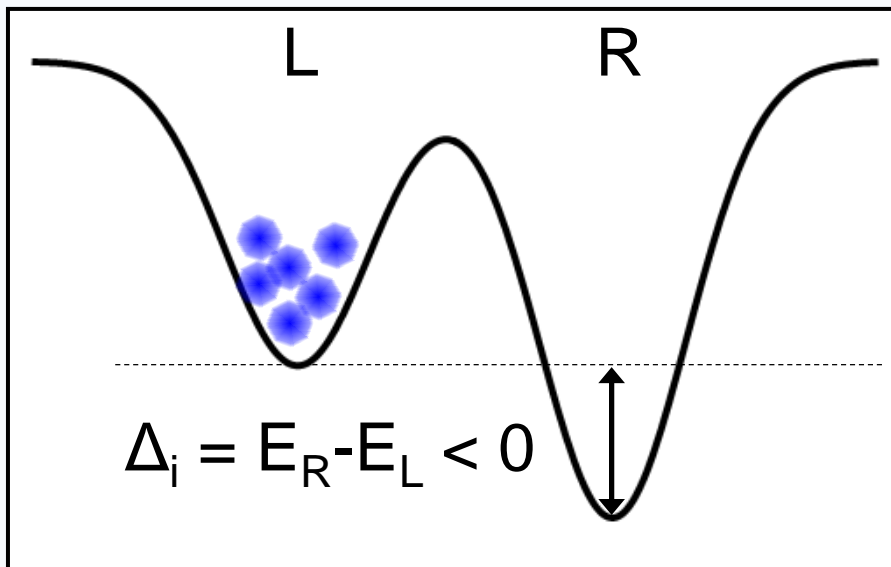


**$U = 0$  prediction**

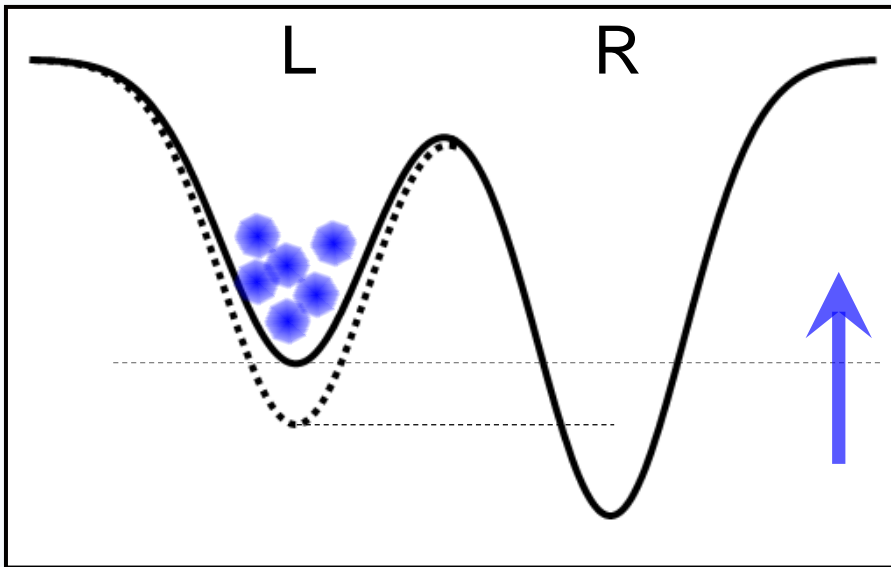
Details of population transfer should be *independent* of the sweep direction



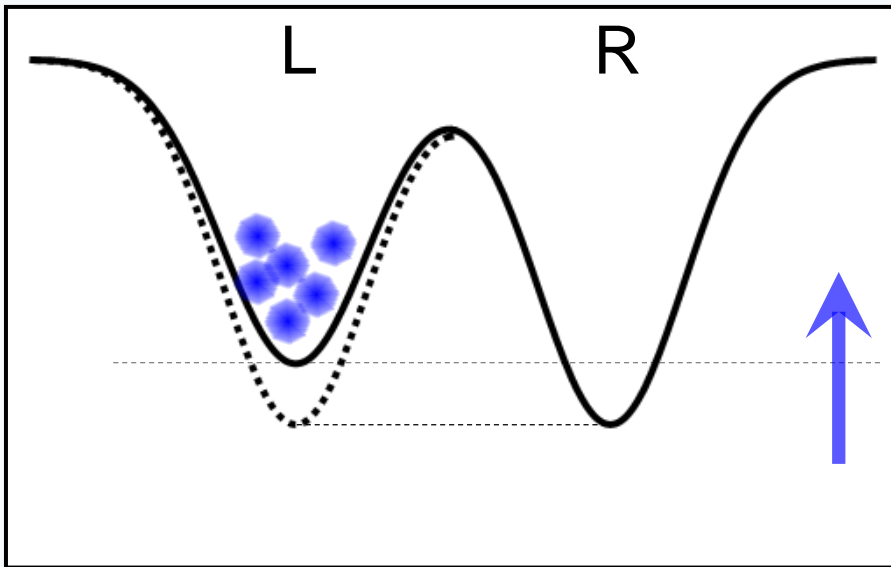
# Interactions in a double-well system



# Interactions in a double-well system

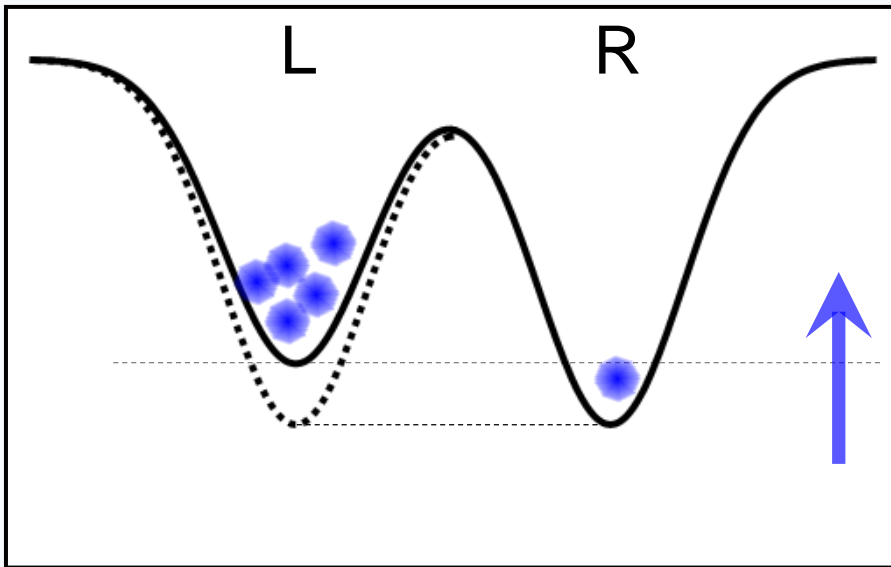


# Interactions in a double-well system



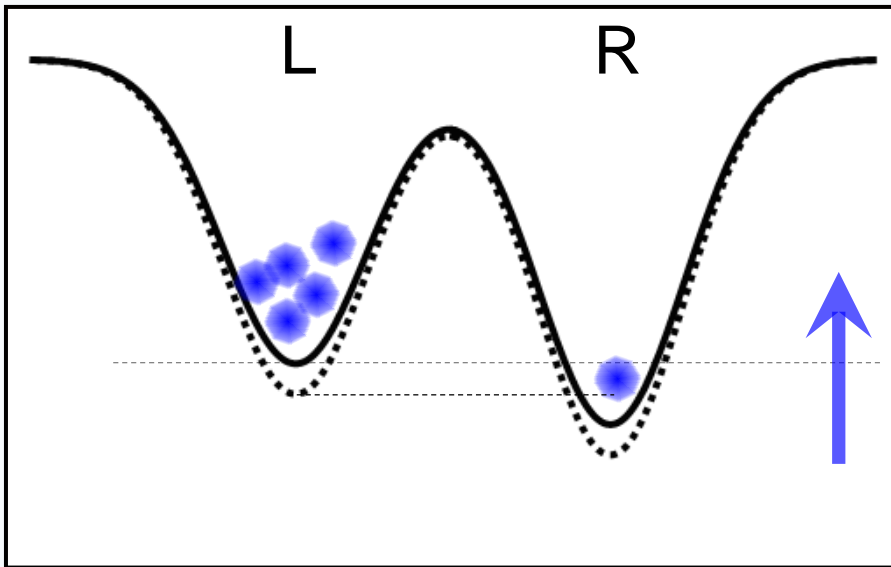
- appears to transfer “early”

# Interactions in a double-well system



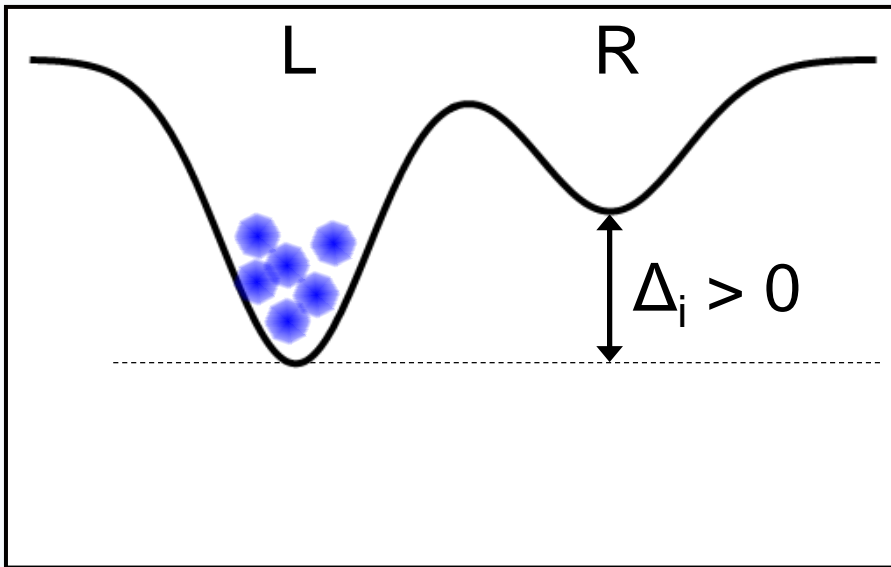
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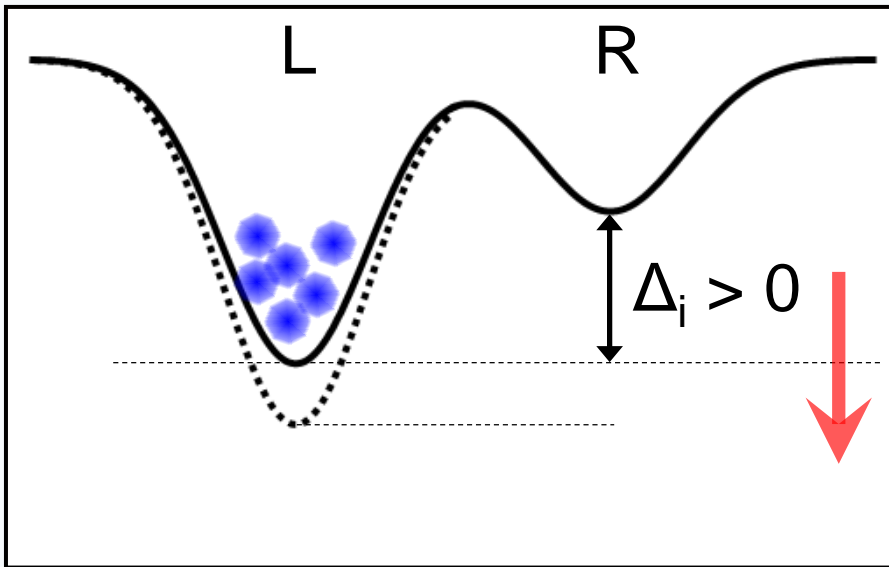


- appears to transfer “early”
- ramp direction allows for continued transfer

# Interactions in a double-well system

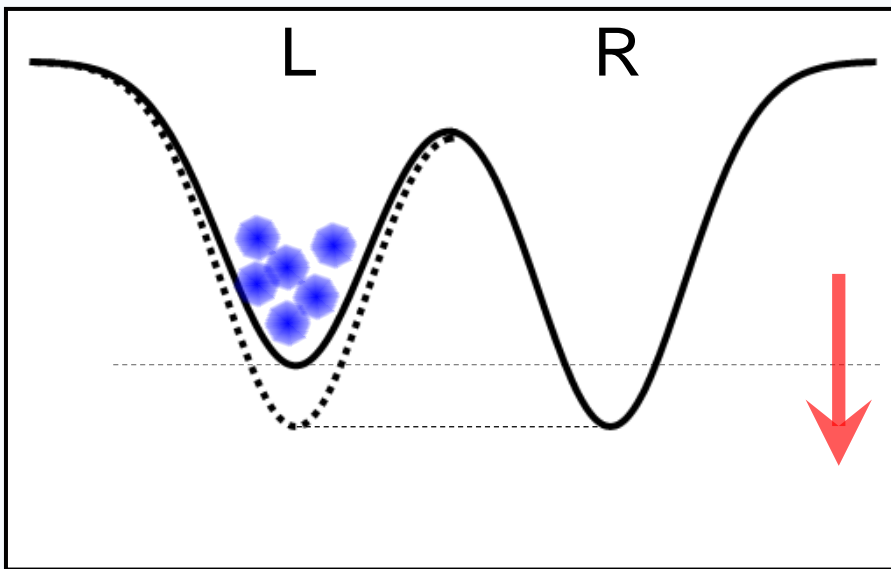


# Interactions in a double-well system



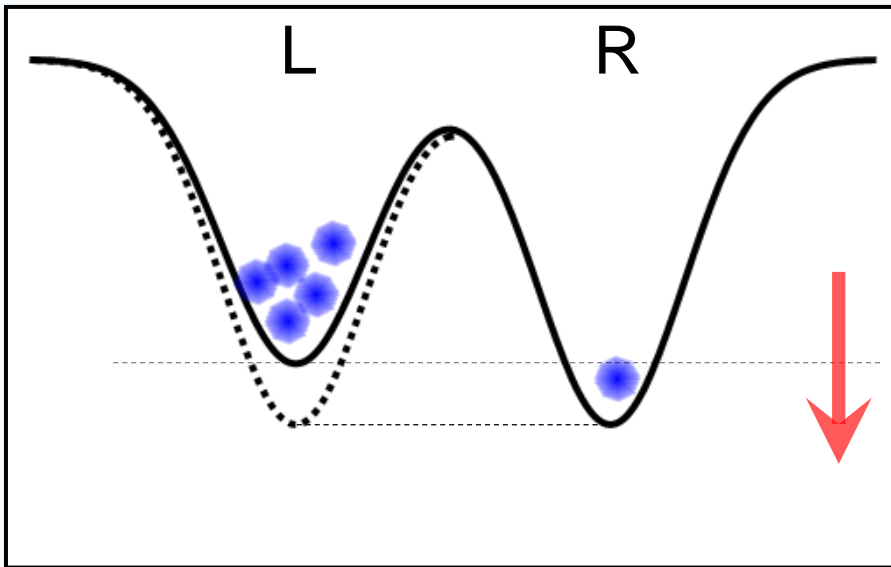


# Interactions in a double-well system



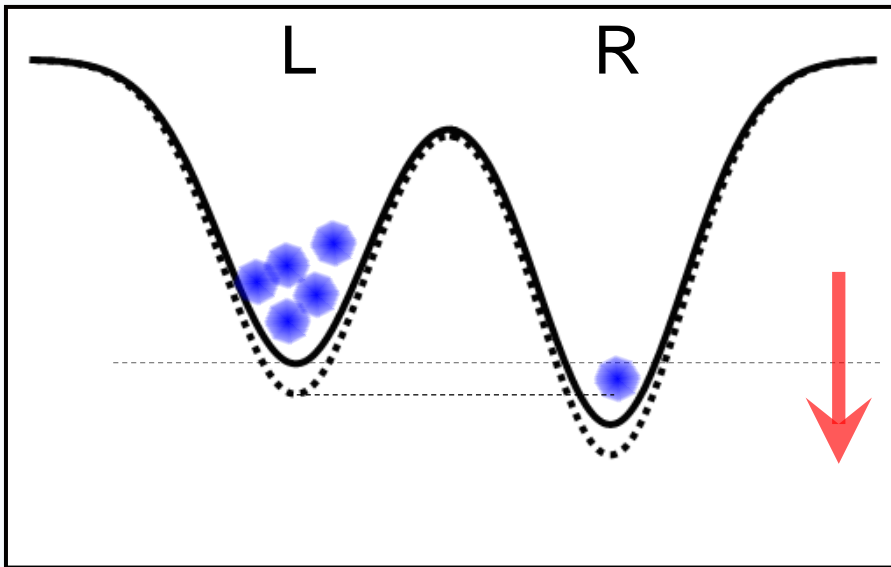
- appears to transfer "late"

# Interactions in a double-well system



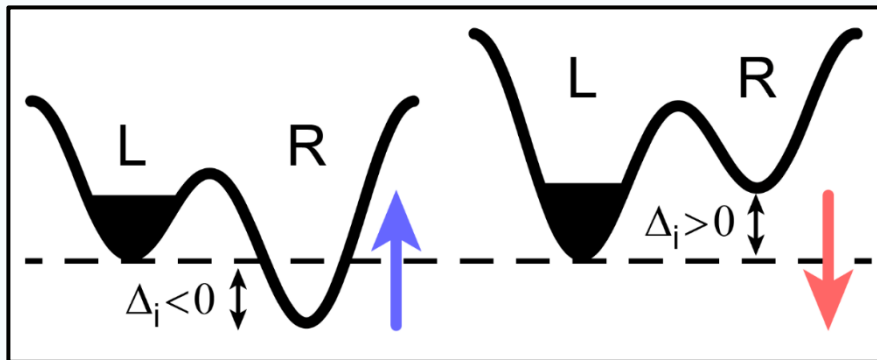
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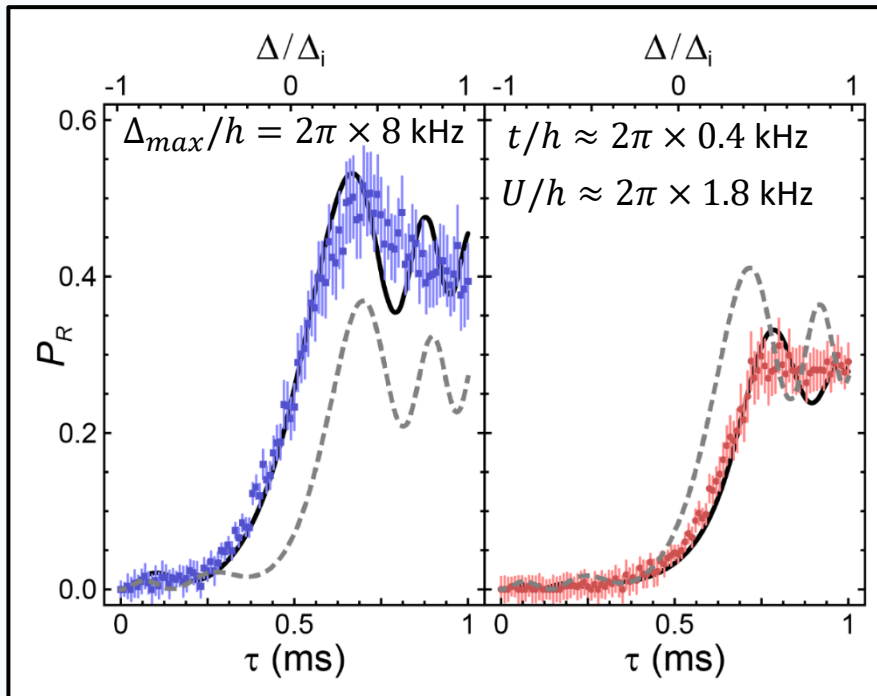


- appears to transfer “late”
- ramp direction runs past resonance condition

# Interactions in a double-well system



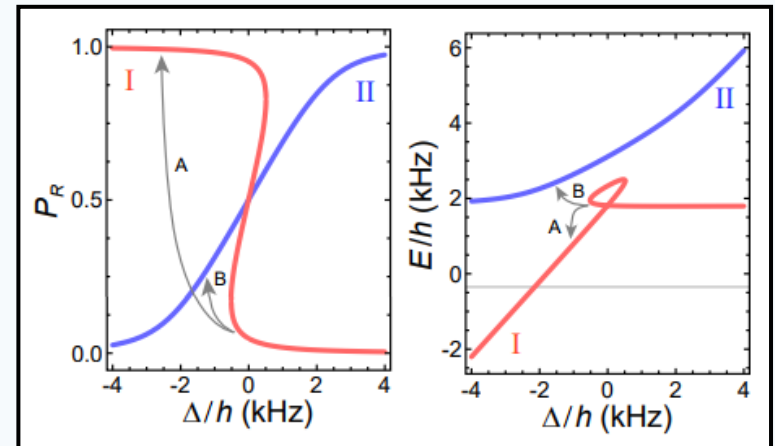
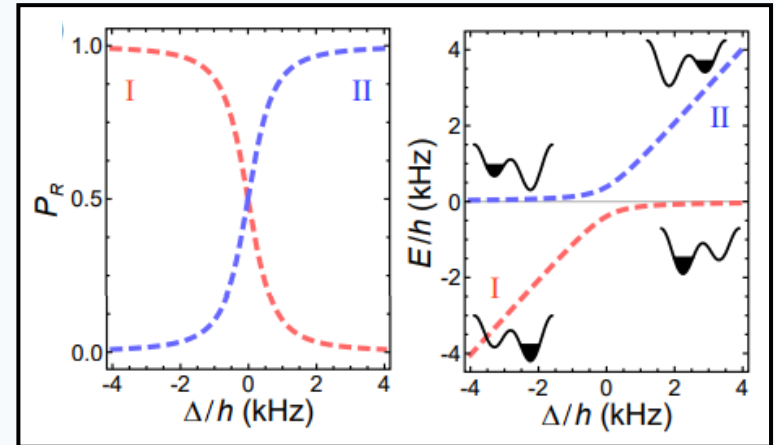
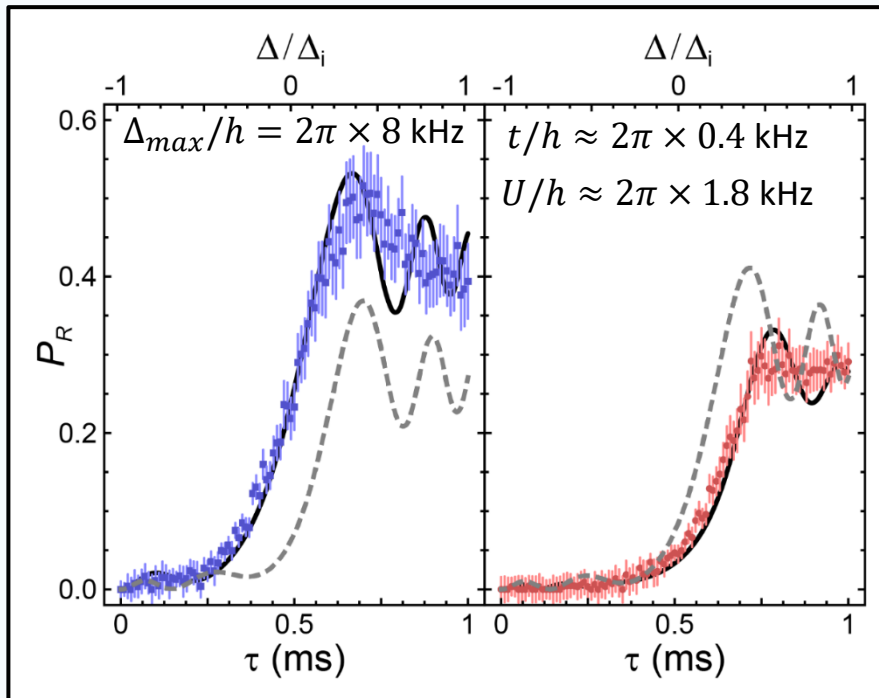
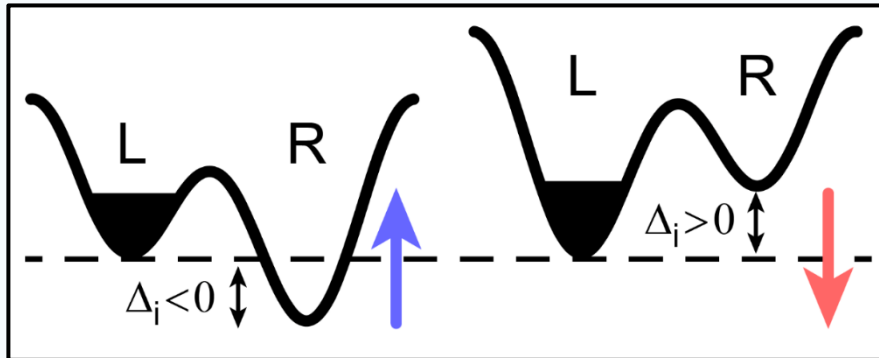
**What do we observe?**



onset of nonlinear self-trapping  
due to interactions

- positive ramp transfers early
- negative ramp transfers late
- enhanced/suppressed transfer depending on sweep direction

# Interactions in a double-well system

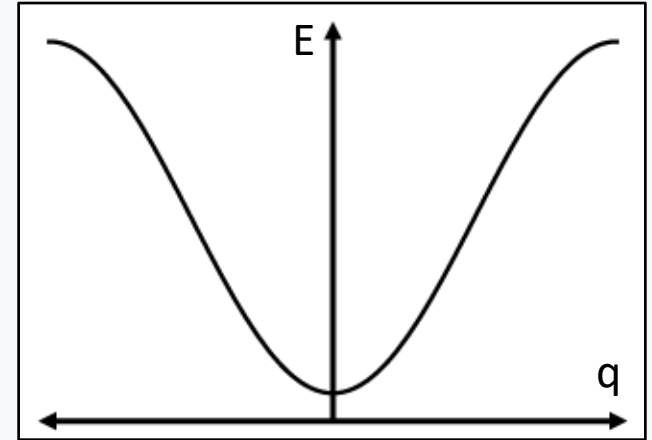
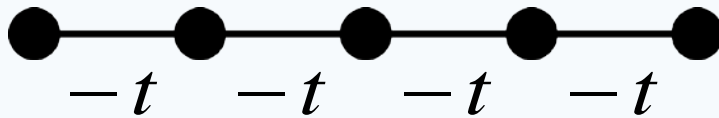


- same physics underlying bosonic Josephson effects, hysteresis in interacting spin systems
- can give rise to k-space “squeezing”

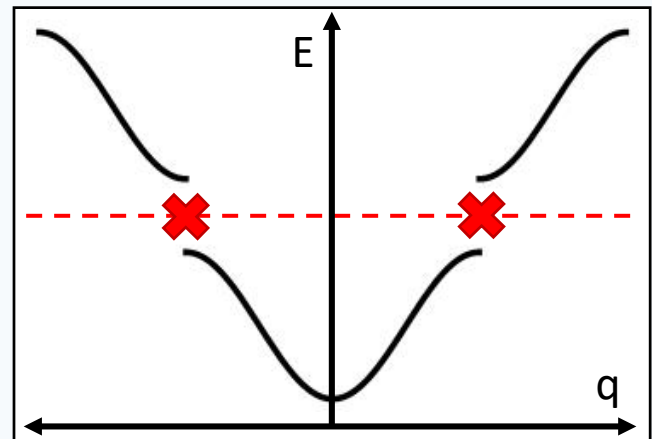
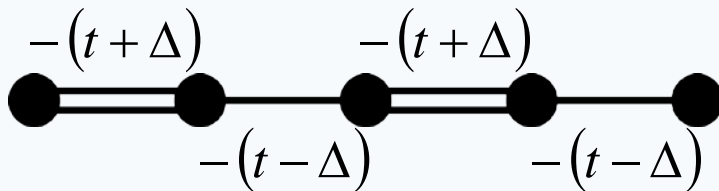
Exploring the  
interplay of topology & disorder

# 1D topological wires: Realizing the Su-Schrieffer-Heeger model

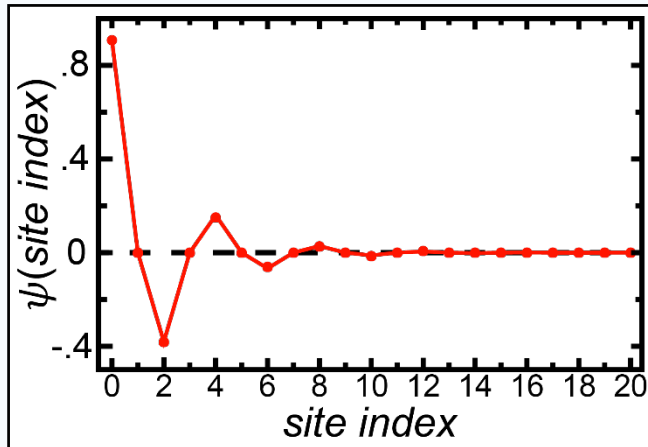
Simple cosine-like band structure



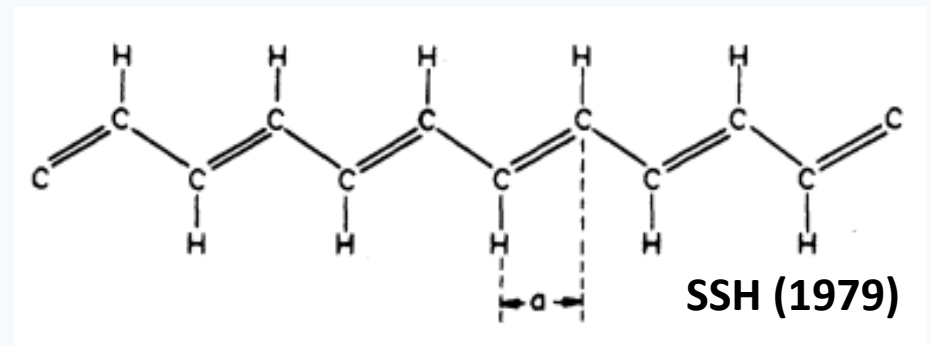
1D topological wires (Su-Schrieffer-Heeger model)  
“textbook” topological free-fermion model



# Probing the boundary modes



Bound state at defects / interface

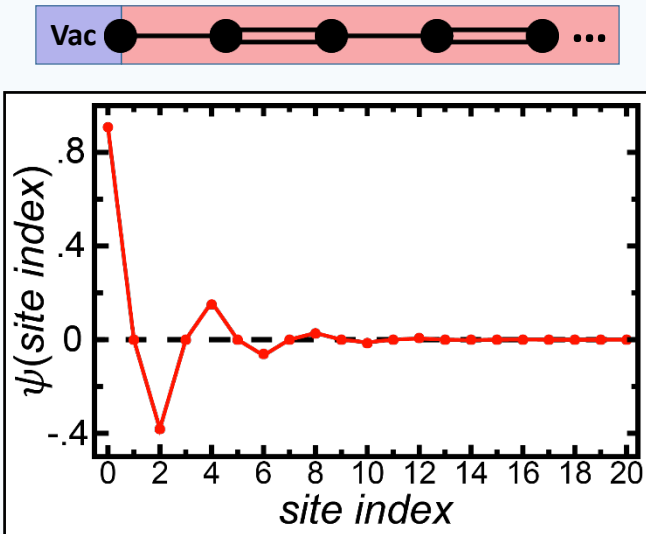


$10^9$  x increase in conductivity of doped polyacetylene



# Probing the boundary modes

Three methods to probe our midgap state:



Bound state at defects / interface

## 1. Nonadiabatic projection

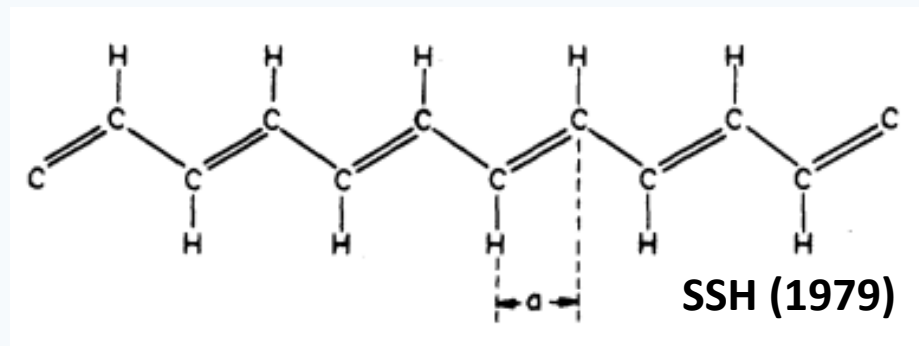
Inject population directly at system boundary.

## 2. Phase-sensitive projection

Initialize atoms in a state that matches amplitude and phase of midgap state.

## 3. Adiabatic preparation

Begin in ground state of  $H_{\text{initial}}$ , slowly evolve towards  $H_{\text{final}}$



$10^9$  x increase in conductivity of doped polyacetylene

# Probing the boundary modes

## 1. Nonadiabatic projection

Inject population directly at system boundary.

## 2. Phase-sensitive projection

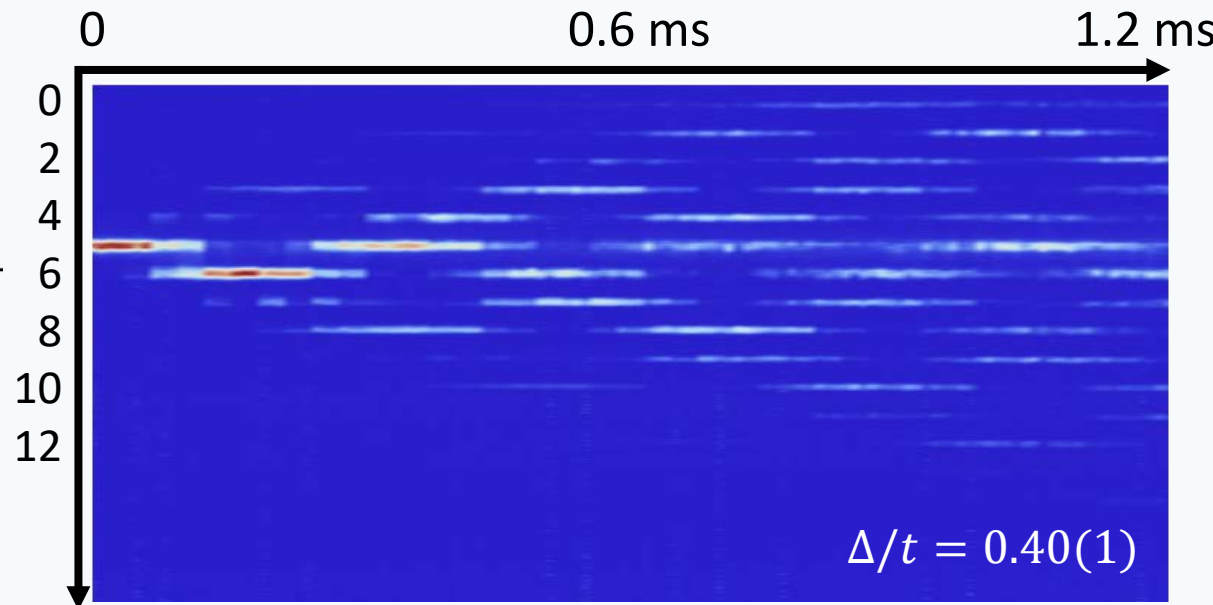
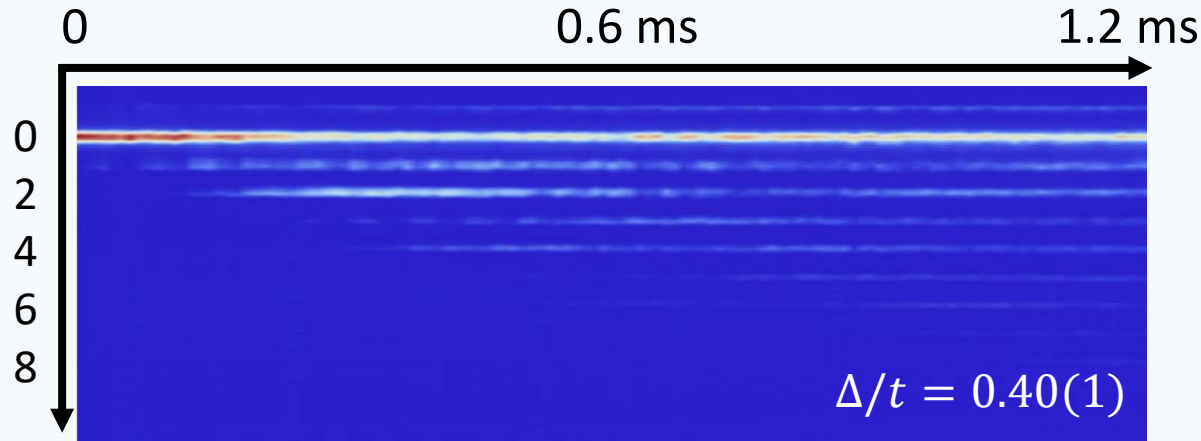
Initialize atoms in a state that matches amplitude and phase of midgap state.

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Begin in ground state of fully dimerized lattice; adiabatically ramp up weak links.

$$\frac{p}{2\hbar k}$$

$$\frac{p}{2\hbar k}$$



# Probing the boundary modes

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Inject population directly at system boundary.

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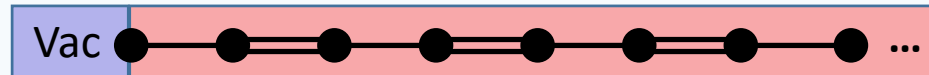
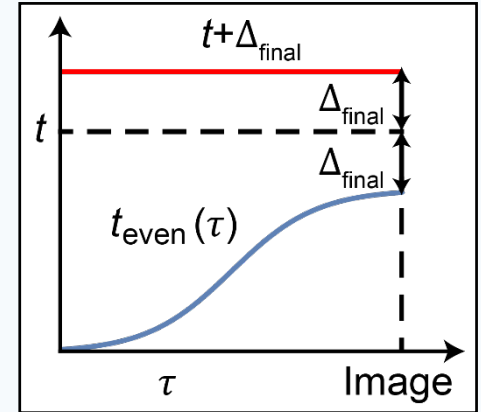
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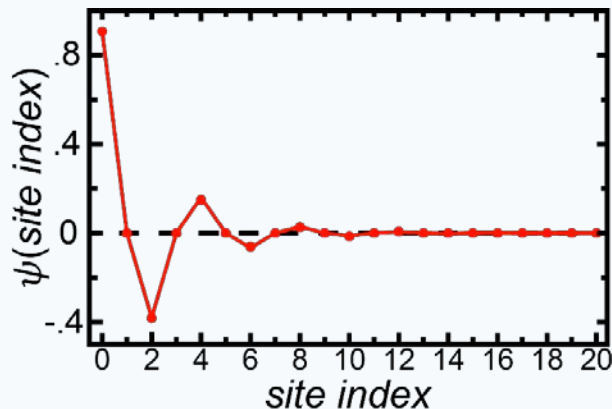
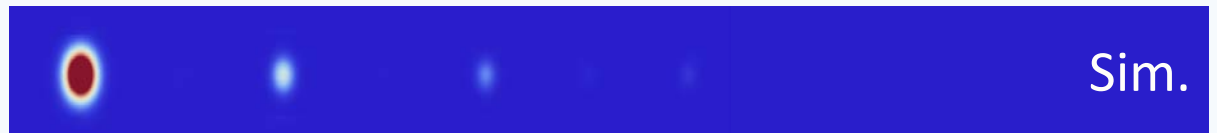
Begin in ground state of fully dimerized lattice; adiabatically ramp up weak links.

## Adiabatically prepare ground state of interest

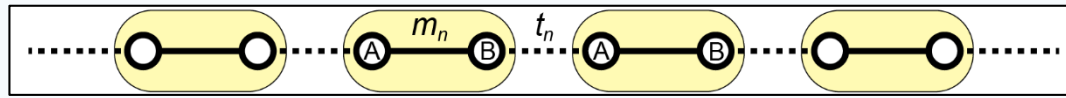
- Start with strong links at  $t+\Delta_{final}$  and weak links at zero.
- Ramp up weak links to a final value of  $t-\Delta_{final}$  while holding strong links constant.



$$\Delta/t = 0.38(1)$$



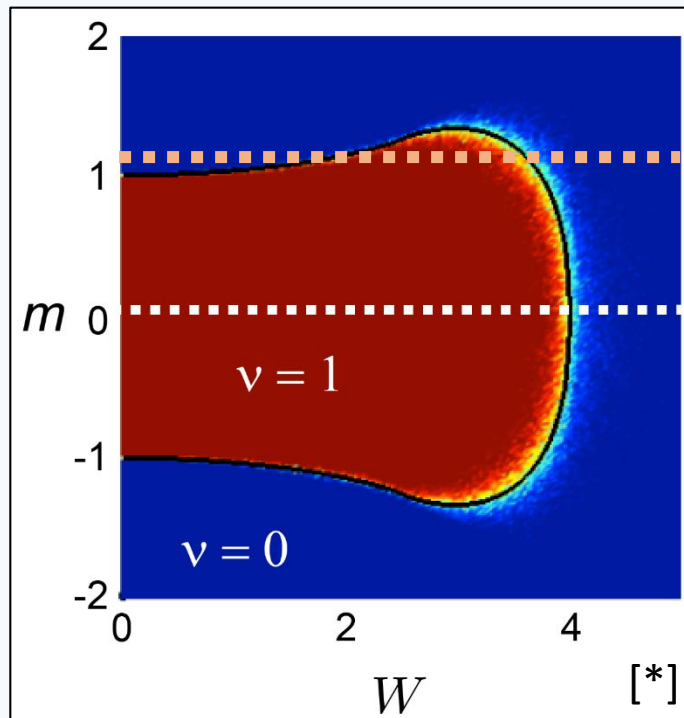
# Probing the influence of disorder



$$\mathcal{C} = 2 \langle n \otimes \sigma_z \rangle$$

$$t_n = t \left( 1 + \frac{W}{2} \omega_n \right)$$

$$m_n = t \left( m + W \omega'_n \right)$$

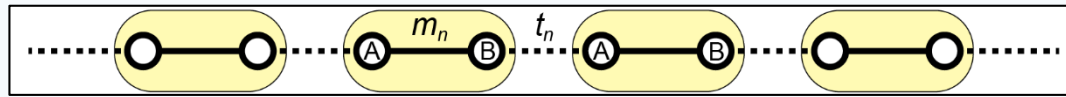


\* I. Mondragon-Shem, T. L. Hughes, J. Song, and E. Prodan, Phys. Rev. Lett. 113, 046802 (2014).

F. Cardano, et al. Nat. Comms. 8, 15516 (2017)

M. Maffei, A. Dauphin, F. Cardano, M. Lewenstein, and P. Massignan, arXiv:1708.02778

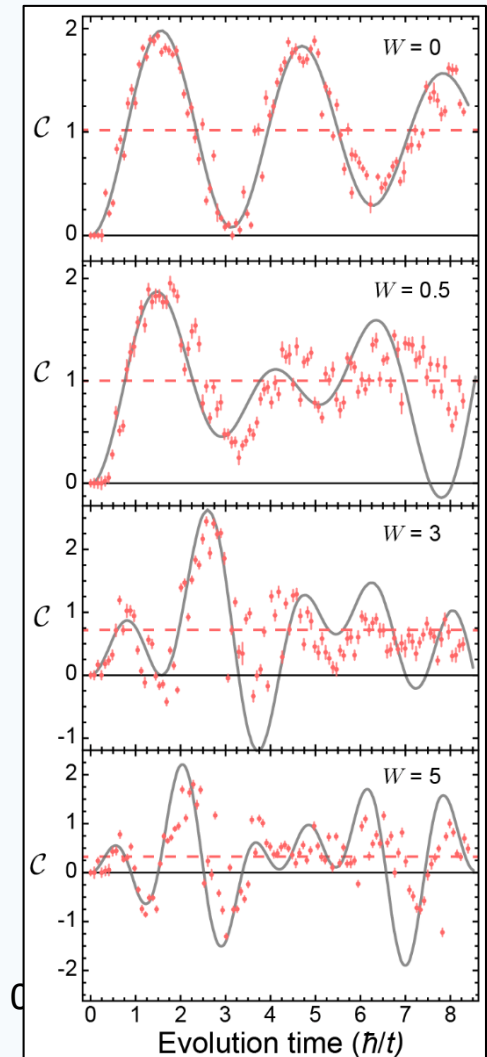
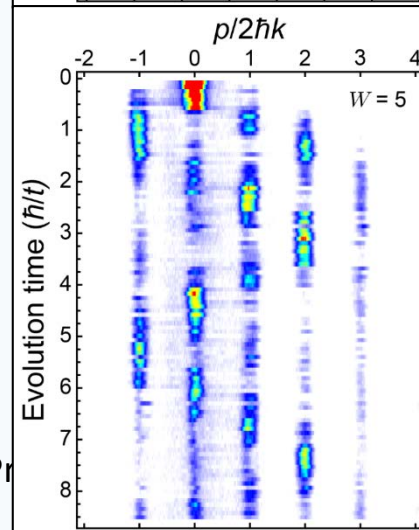
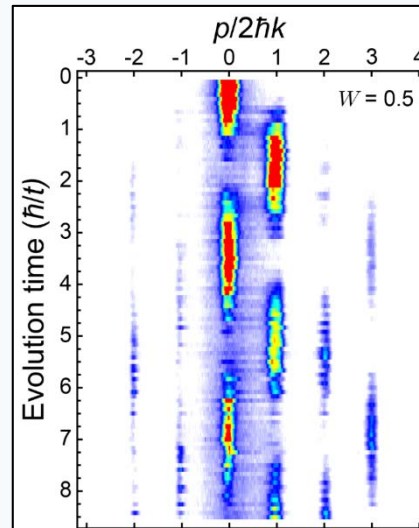
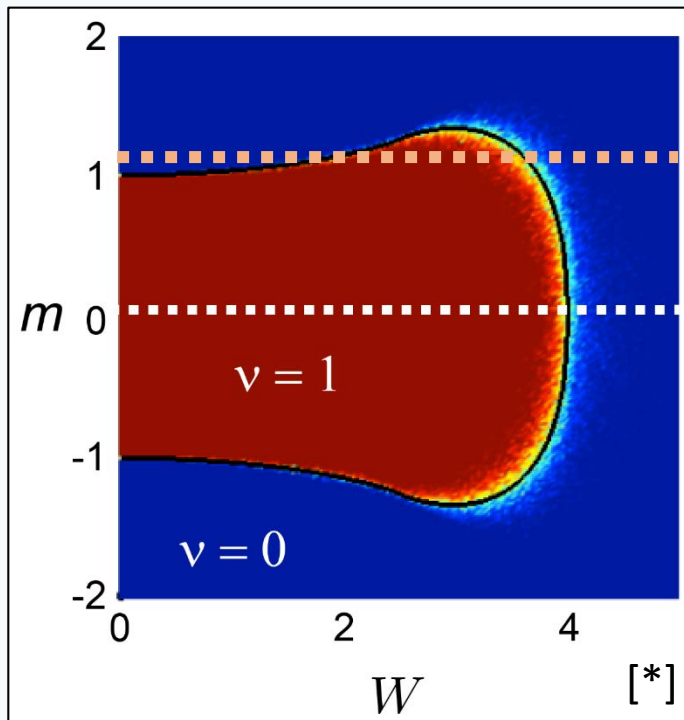
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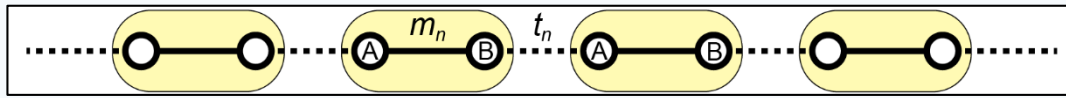


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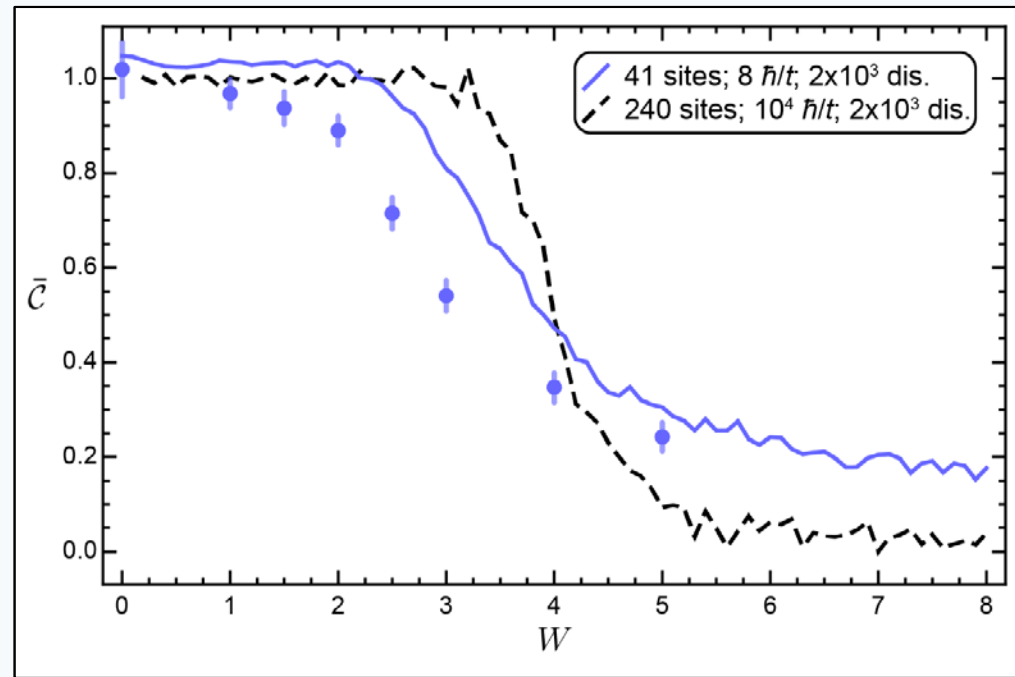
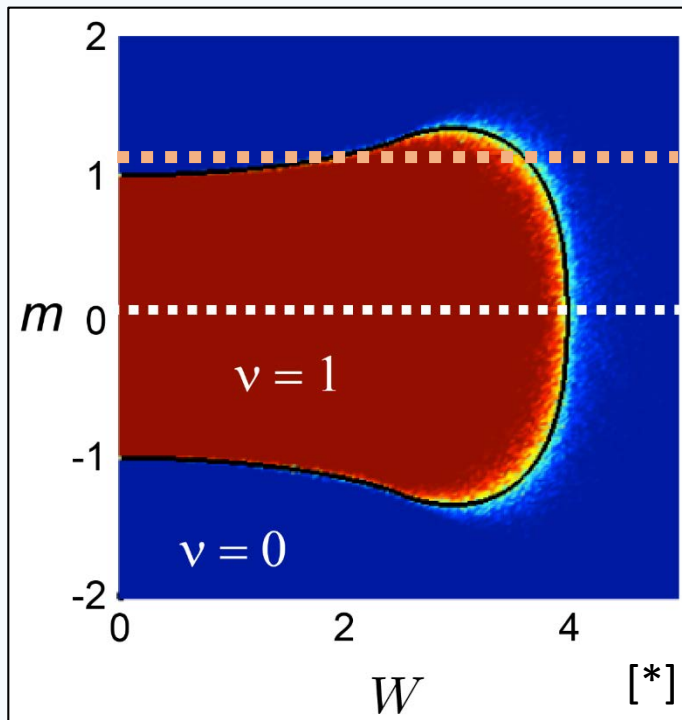
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evidence for a topological-to-trivial  
crossover driven by disorder

now investigating predicted “topological Anderson insulator”



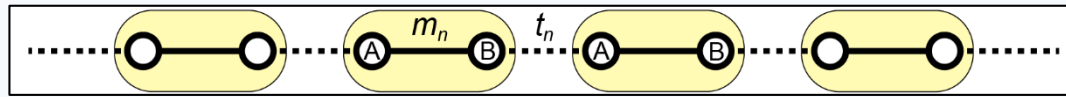
E.J. Meier, *et al.* (in prep)

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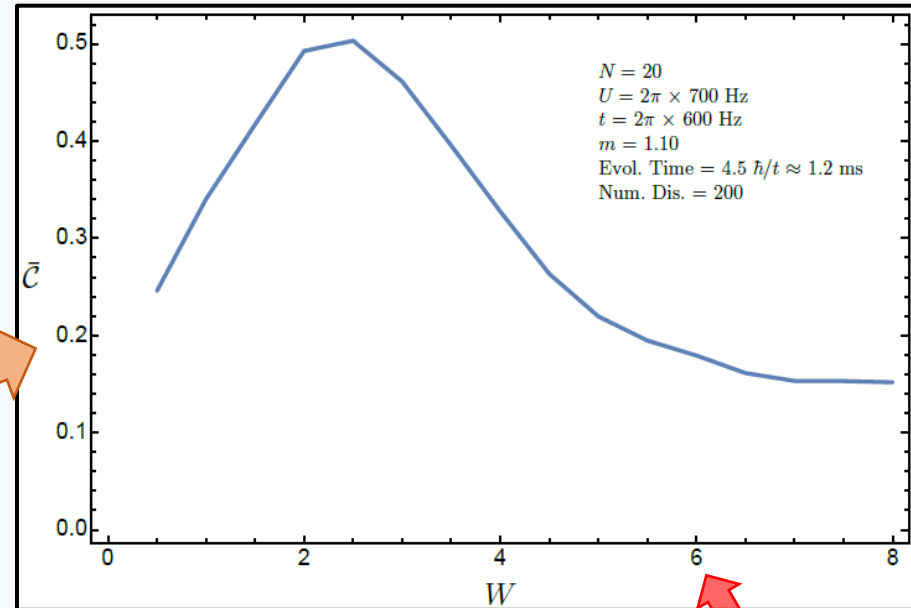
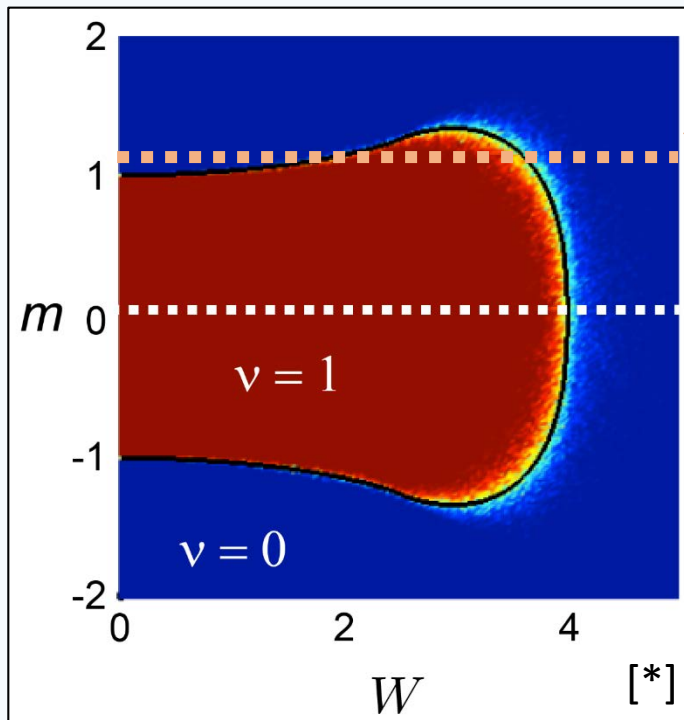
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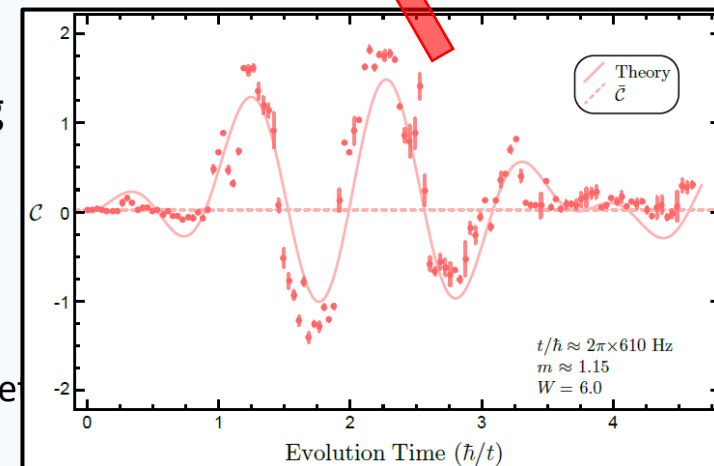
E.J. Meier, *et al.* (in prep)

$$t_n = t \left( 1 + \frac{W}{2} \omega_n \right)$$

$$m_n = t (m + W \omega'_n)$$



Looks promising  
so far, even for  
large  $W$



\* I. Mondragon-Shem, T. L. Hughes, J. Song, and E. Prodan, Phys. Rev. Lett. 117, 085701 (2016)  
 F. Cardano, et al. Nat. Comms. 8, 15516 (2017)  
 M. Maffei, A. Dauphin, F. Cardano, M. Lewenstein, and P. Massignan, arXiv:1708.02778

# Exploring *single-particle* & *many-body* mobility edges in (pseudo)disordered 1D lattices

(interplay of frustration, disorder & interactions)

Previous work on single-particle mobility edges:

**in 3D (~random disorder):**

- Fattori/Inguscio/Modugno group, LENS [Semeghini, *et al.* Nat. Phys. (2012)]
- DeMarco group, UIUC [Kondov, *et al.*, Science (2011)]

**recently, in 1D (correlated disorder):**

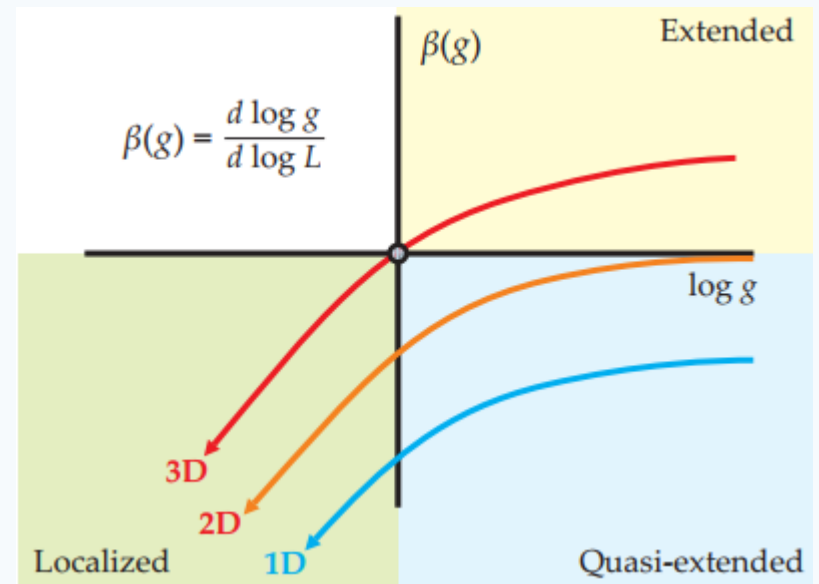
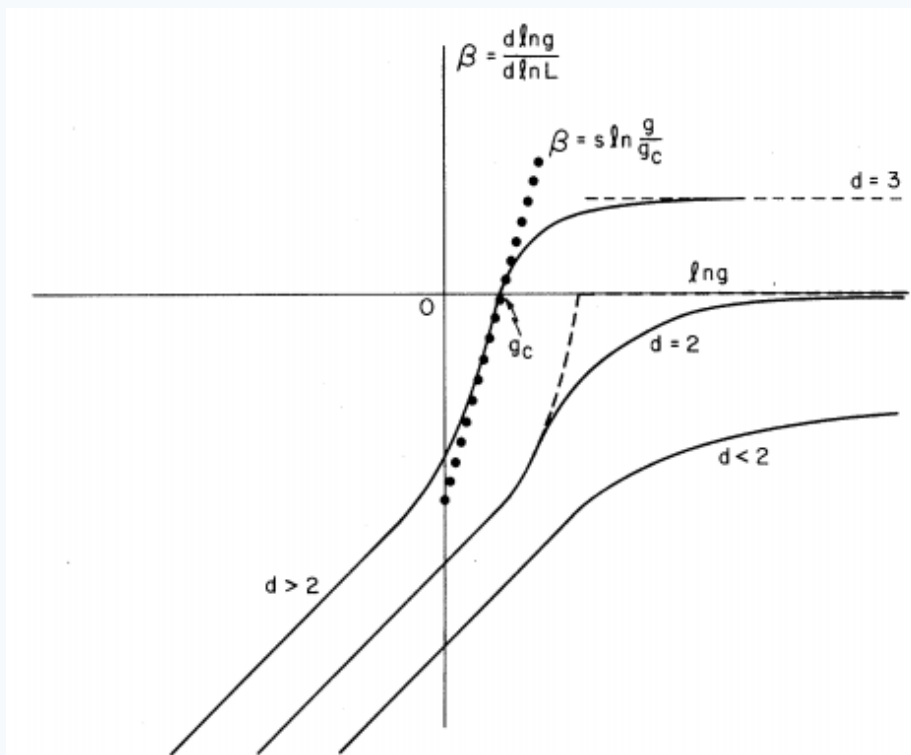
- Bloch group, Munich/Garching [Lüschen, *et al.* arXiv:1709.03478]



# Mobility edges in disordered systems

For free electrons + **random** site-energy disorder: localization in 1D and 2D

## Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions



Legendijk, van Tiggelen, and Wiersma  
(Physics Today, 2009)

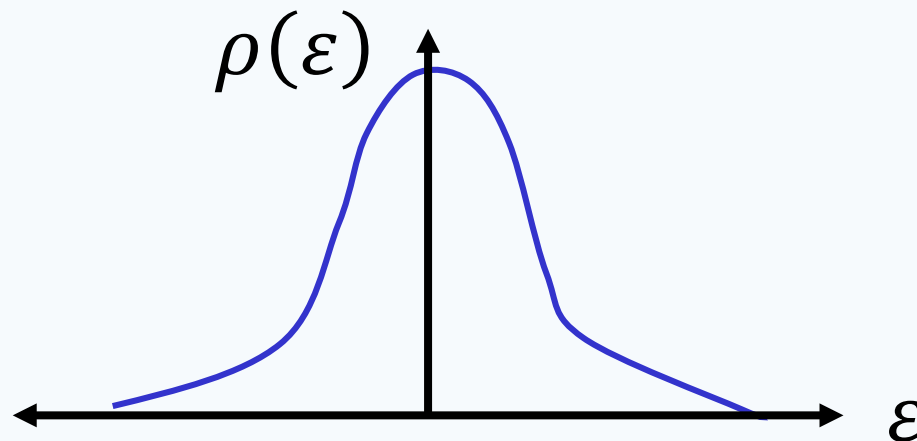
Abrahams, Anderson, Licciardello, and  
Ramakrishnan (PRL, 1979)

# Mobility edges in disordered systems

For free electrons + **random** site-energy disorder: localization in 1D and 2D

In 3D systems with random disorder – mobility edge

→ *Localization depends on the energy of a state, or more specifically the density of states at a particular energy*

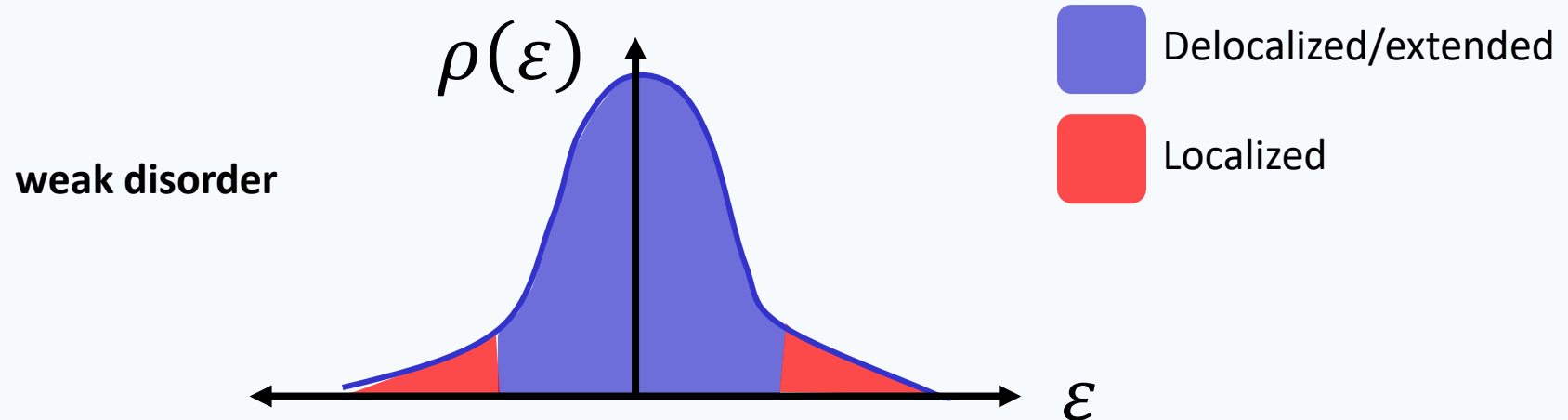


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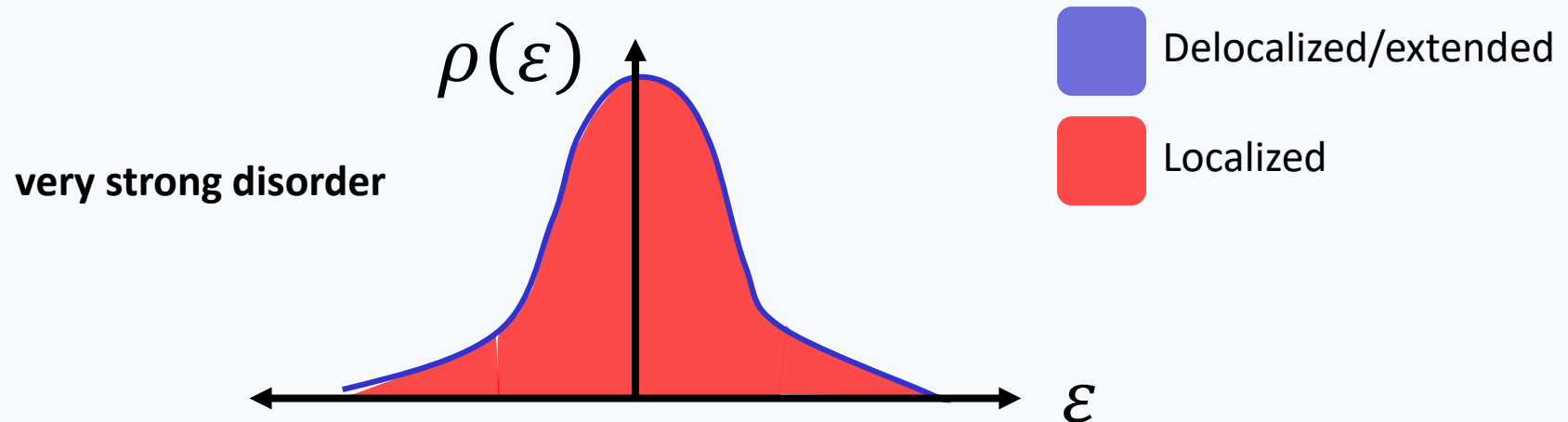


# Mobility edges in disordered systems

For free electrons + **random** site-energy disorder: localization in 1D and 2D

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# The Aubry-André model

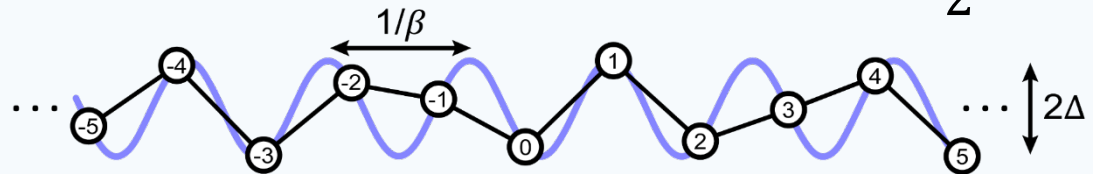
## 1D pseudodisordered model with a localization-delocalization transition

$$H = \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n - t \sum_n (\hat{c}_{n+1}^\dagger \hat{c}_n + \hat{c}_n^\dagger \hat{c}_{n+1})$$

$$\beta = \frac{\sqrt{5} - 1}{2}$$

$$\varepsilon_n = \Delta \cos(2\pi\beta n + \varphi)$$

$\beta$  irrational



### Some funny properties:

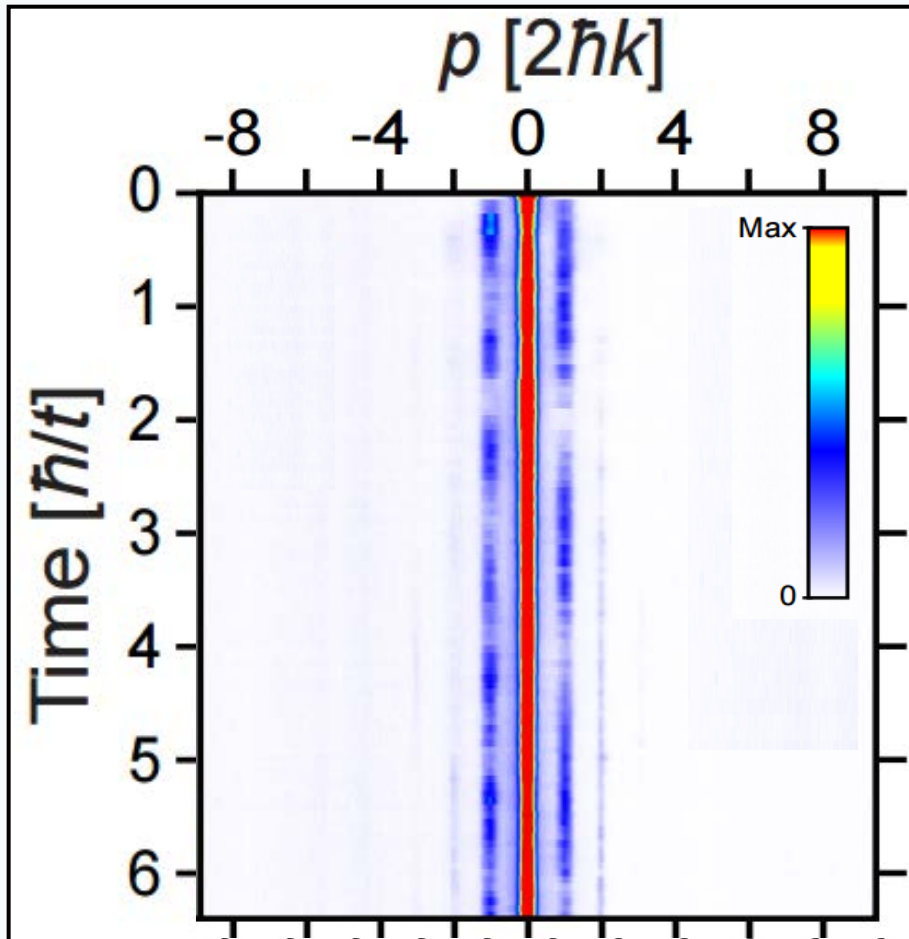
- kinetic energy and site energy terms have matching cosine distributions
- duality between tunneling terms / potential terms
- consequence: **no single-particle mobility edge** -- ALL eigenstates undergo insulator-to-metal transition at same point,  $\Delta = 2t$  !

S. Aubry and G. André, Ann. Israel Phys. Soc. 3, 133 (1980)

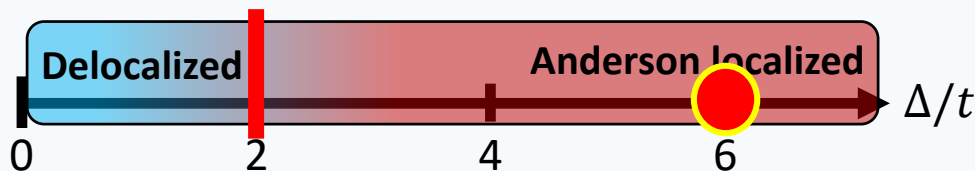
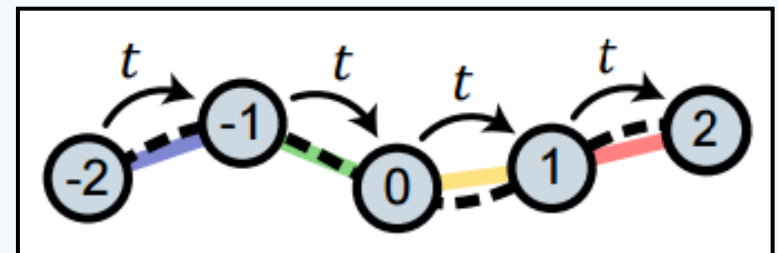
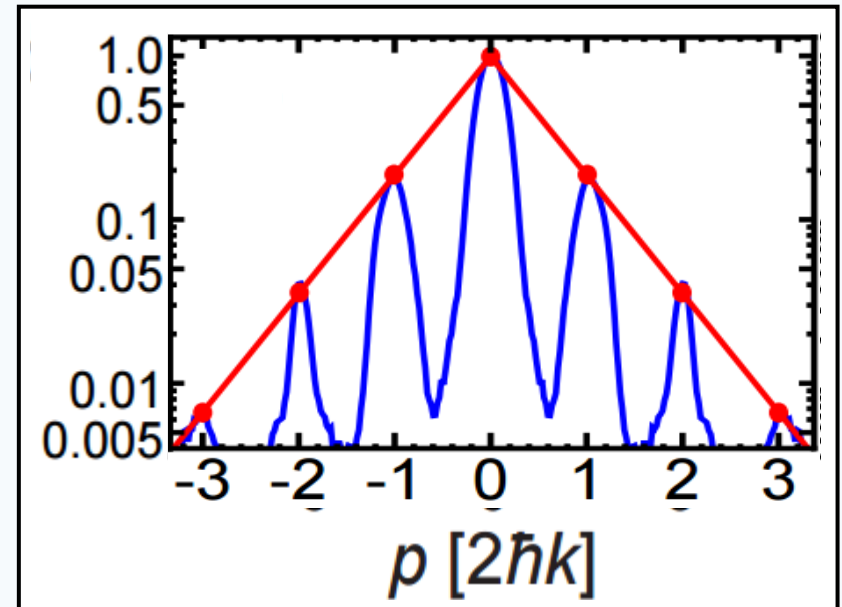
D. R. Hofstadter, Phys. Rev. 14, 2239 (1976)

D. J. Thouless, Phys. Rev. B 28, 4272 (1983)

# Quench in the Aubry-André

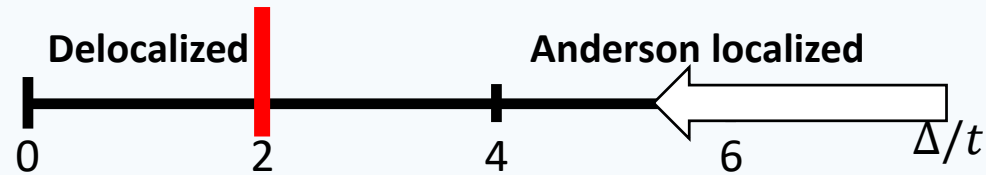
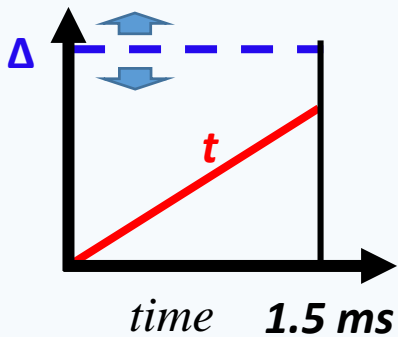


Disordered quantum walk  
→ exponential localization



# Adiabatic probing for localization

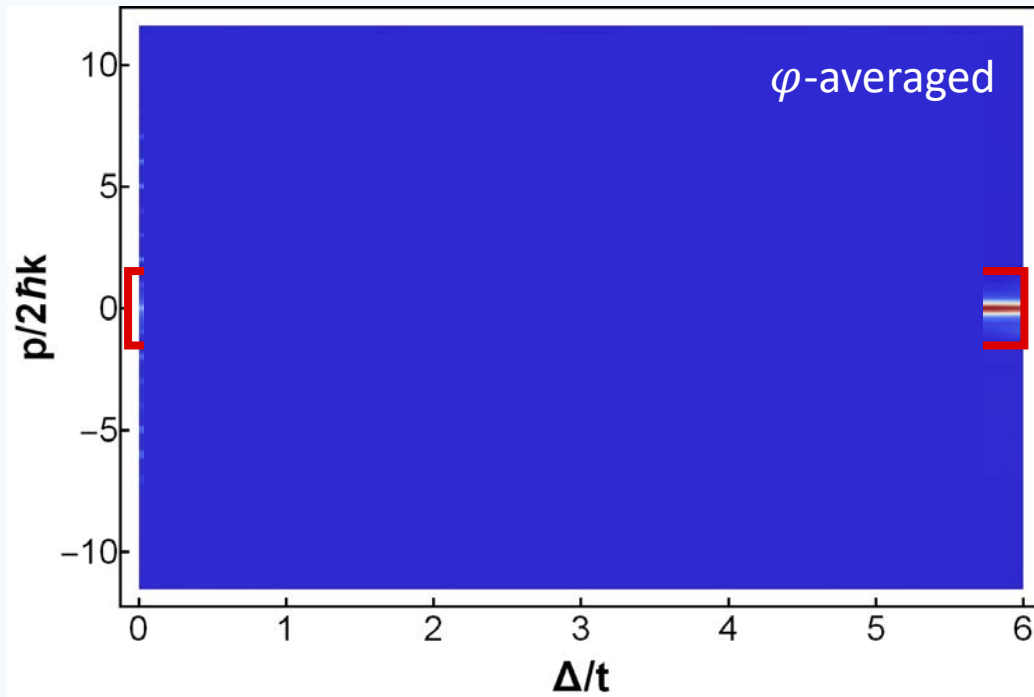
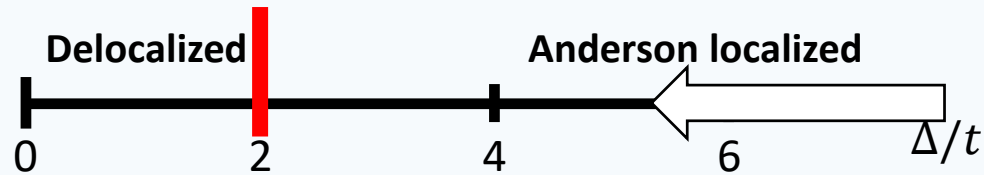
## Aubry-André model



All eigenstates (i.e. at all energies)  
undergo localization transition at same  $\Delta/t$  ratio

# Adiabatic probing for localization

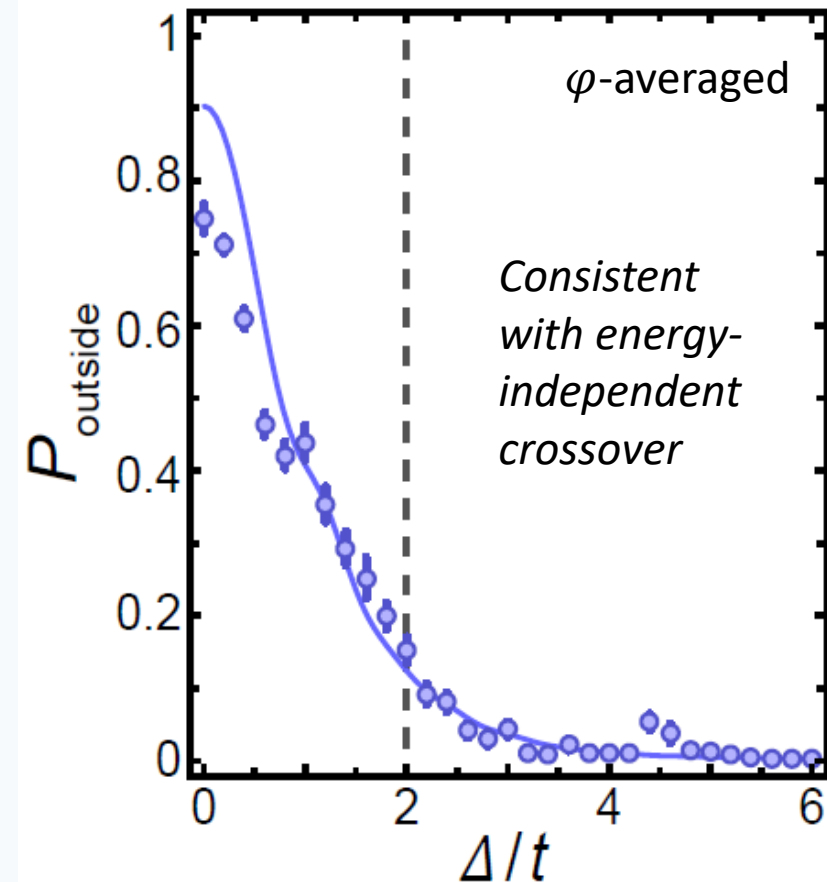
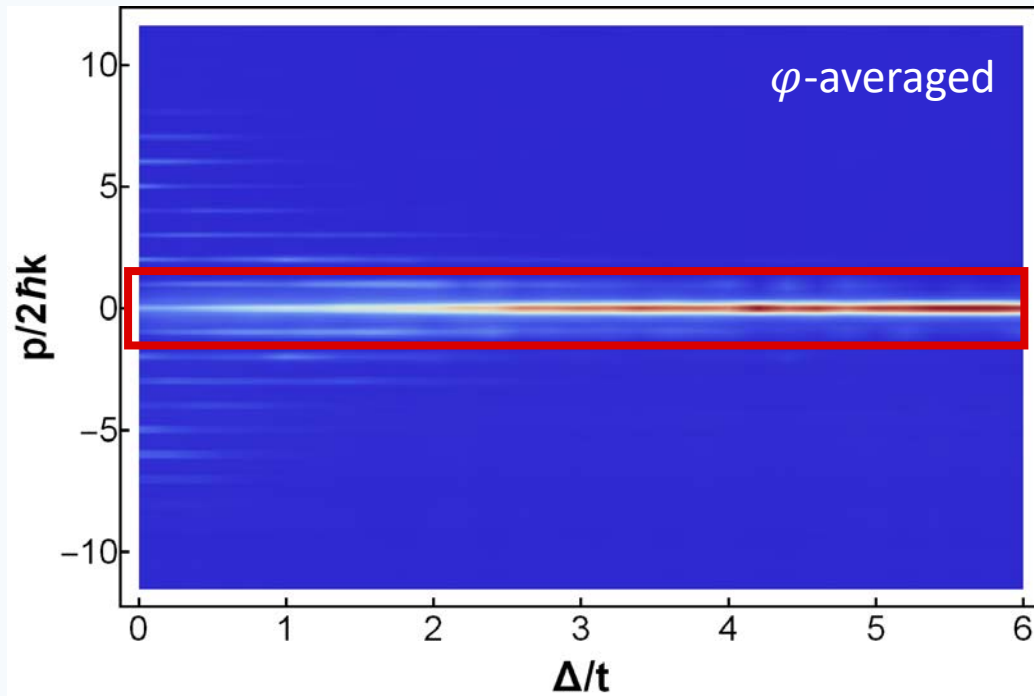
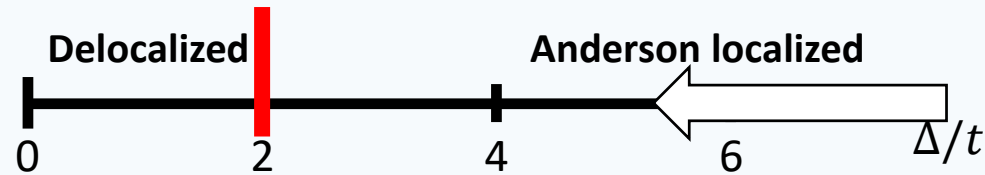
## Aubry-André model



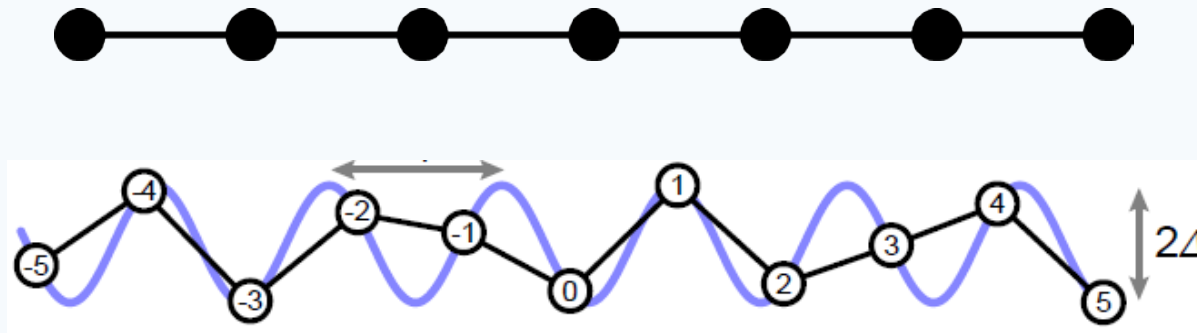


# Adiabatic probing for localization

## Aubry-André model



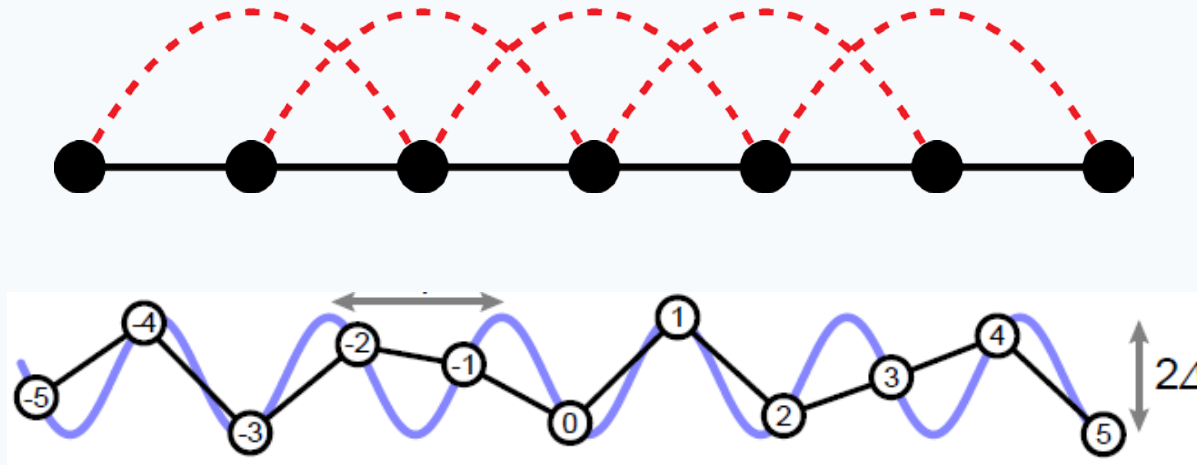
# Engineering a single-particle mobility edge



**The Aubry-André model is, in some sense, fine-tuned.**

- modifying either the dispersion or the disorder potential should introduce a single-particle mobility edge (SPME)

# Engineering a single-particle mobility edge

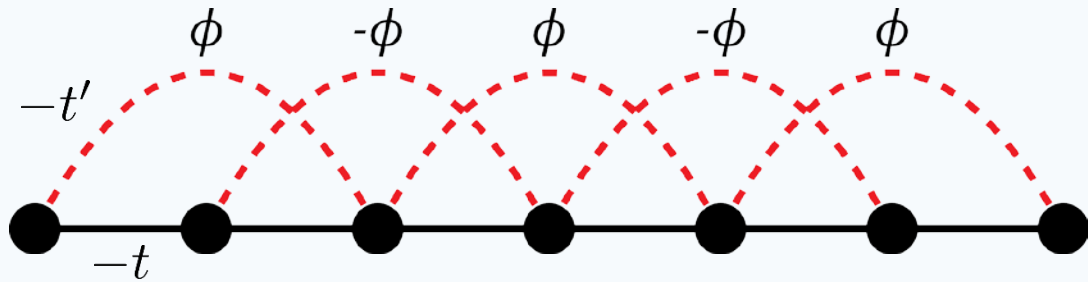


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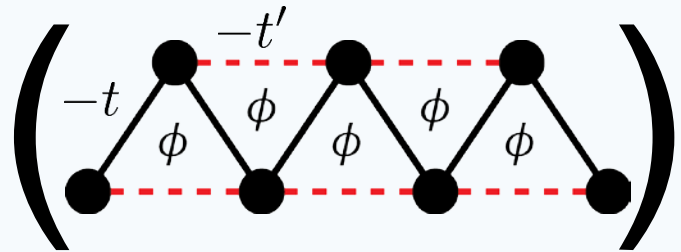
- modifying either the *dispersion* or the disorder potential should introduce a single-particle mobility edge (SPME)

add NNN tunneling

# Creating zig-zag ladders



Nearest and next-nearest neighbor tunneling



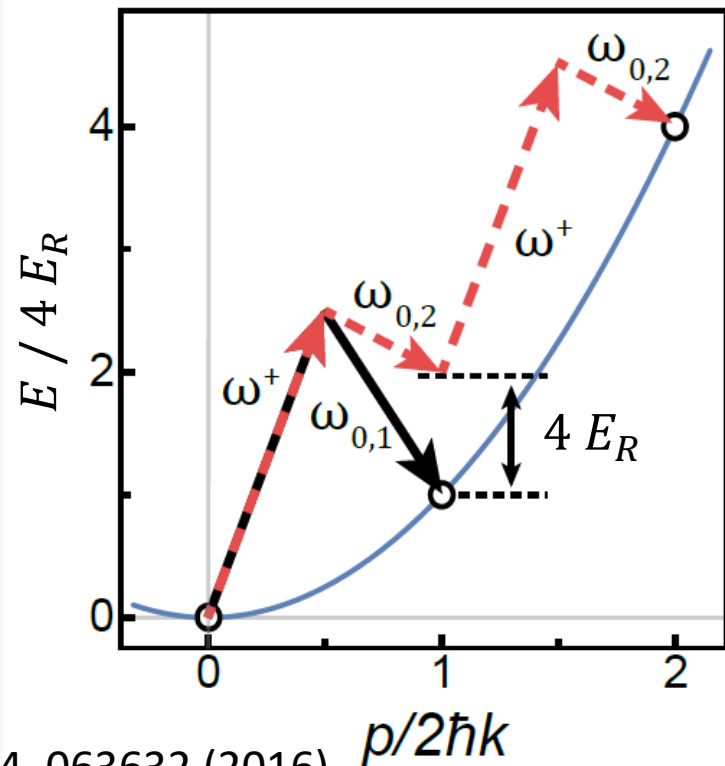
Zig-zag flux ladder

**Nontrivial flux loops!**

add NNN tunneling  
 $\rightarrow$  second-order Bragg transitions

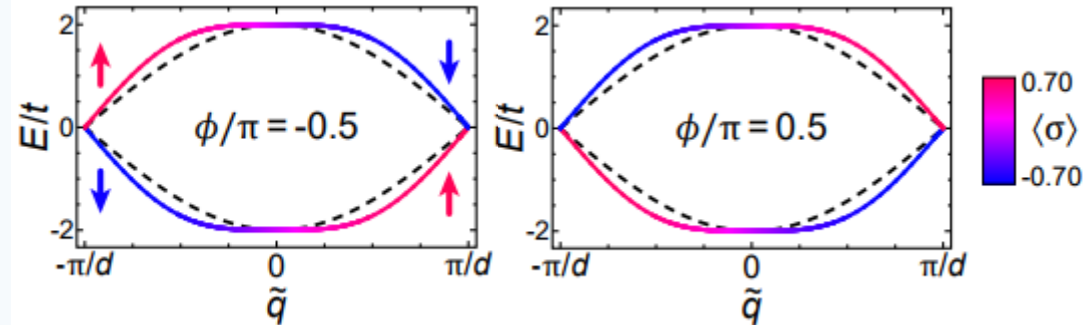
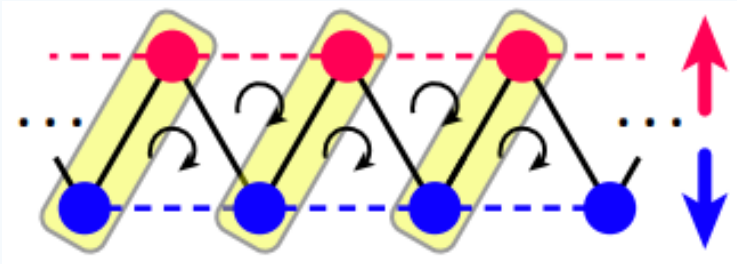
recent proposal:

Anisimovas, et al. [Gediminas Juzeliunas] PRA 94, 063632 (2016)

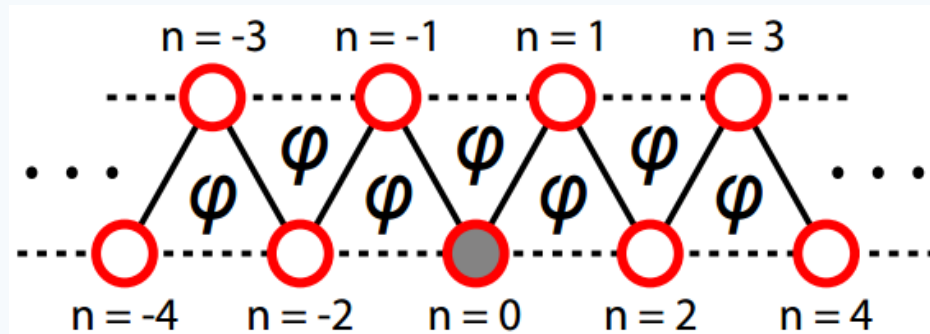


$p / 2 \hbar k$

# Aside: chiral quantum walks & spin-momentum locking



$$|t'/t| \approx 0.5$$



$$\phi / \pi = 0.5$$

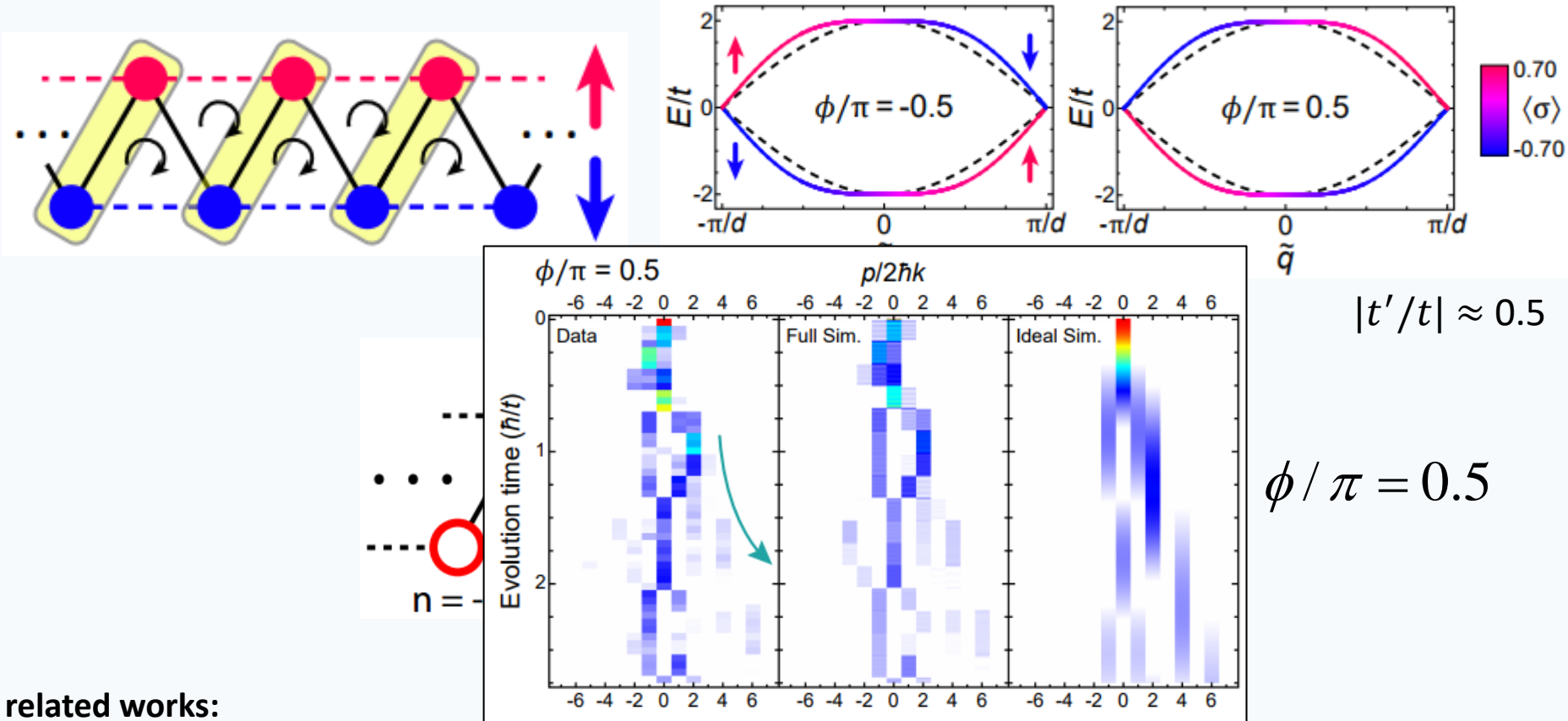
## related works:

Fallani group & Ye group with synthetic dimensions (Yb/Sr optical transitions)  
 Bloch group, superlattices + Raman-assisted tunneling (RAT) ; Greiner group, RAT  
 Fallani group & Spielman group, synthetic dimensions Raman

F. A. An, E. J. Meier, and BG, arXiv:1705.09268 (2017)

[also: F. A. An, E. J. Meier, and BG. *Science Advances* **3**, e1602685 (2017)]

# Aside: chiral quantum walks & spin-momentum locking



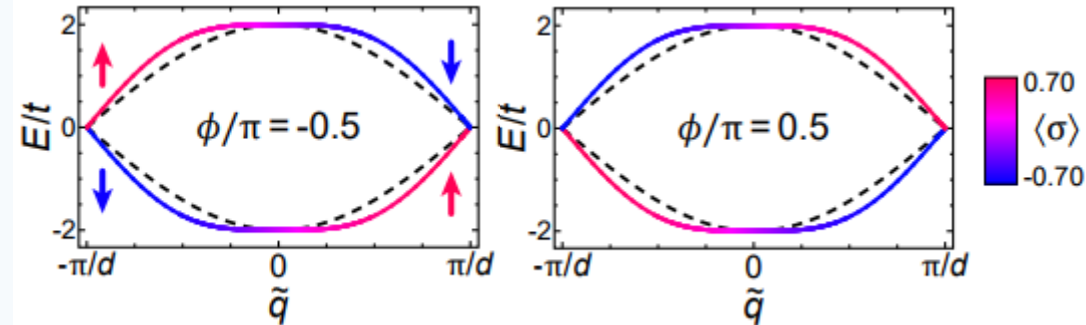
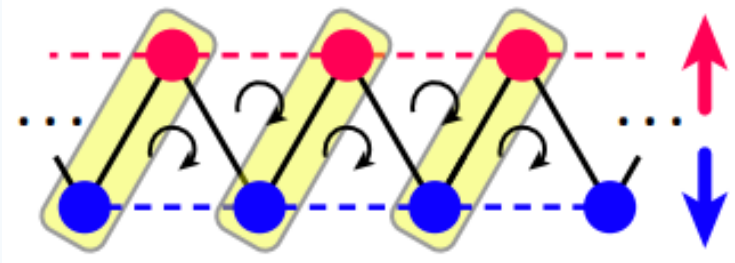
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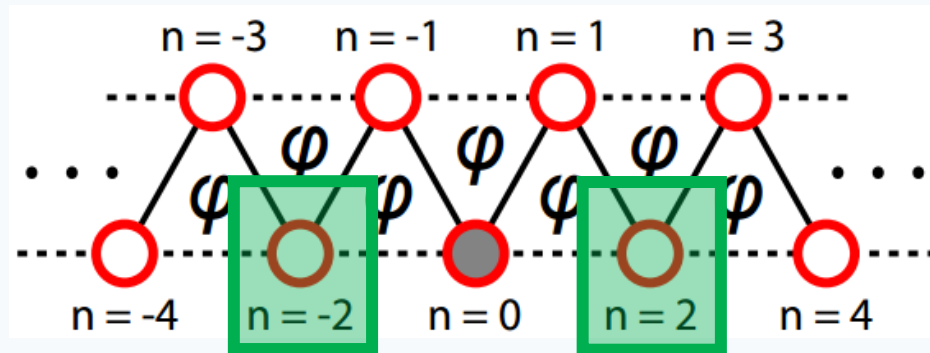
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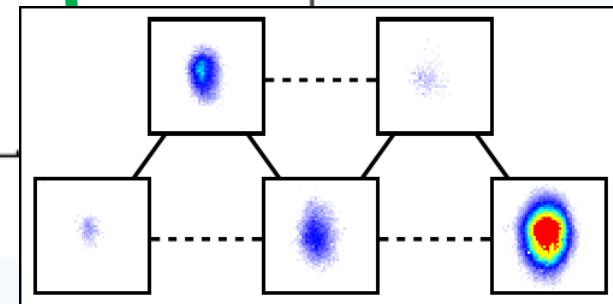
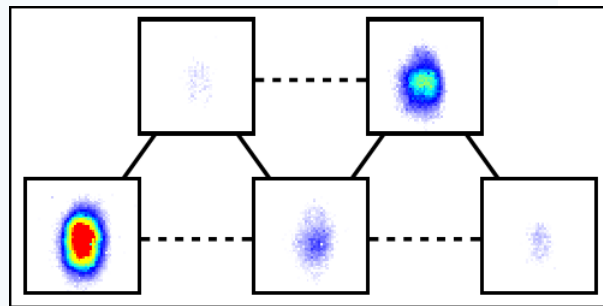
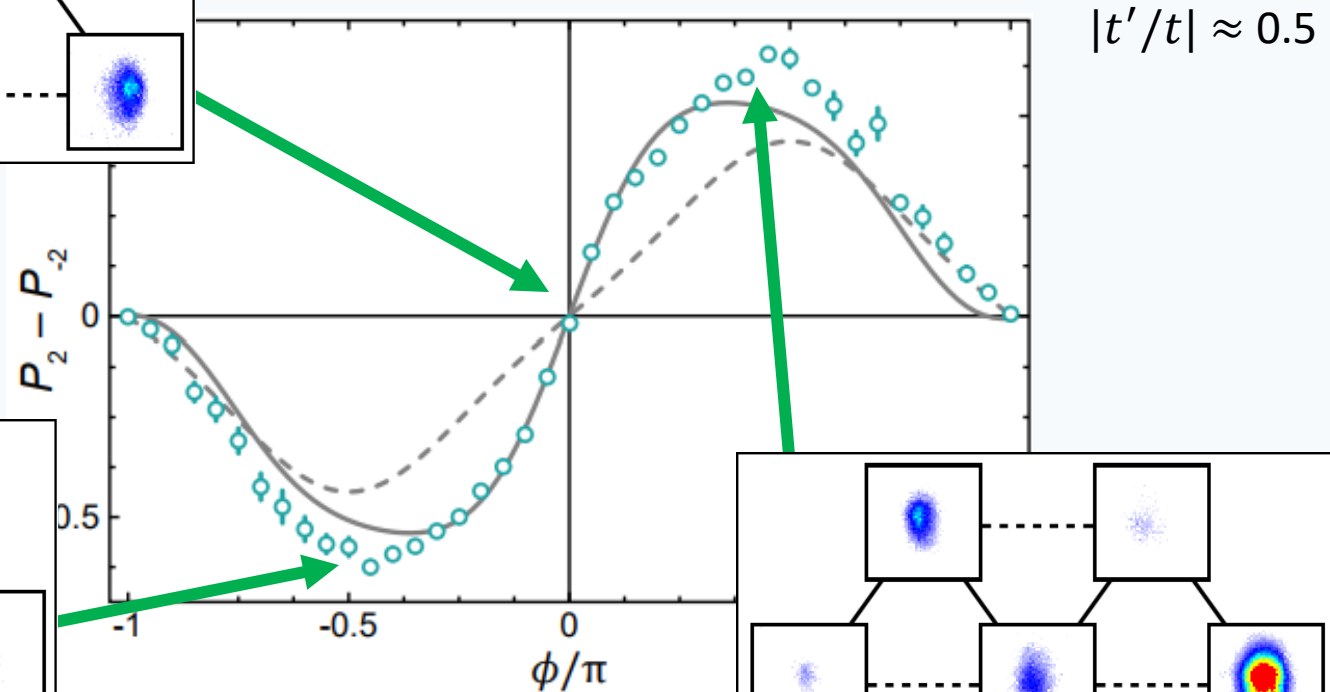
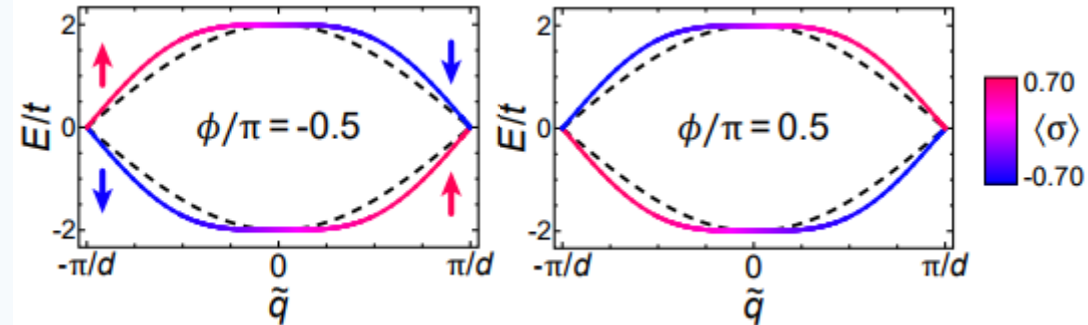
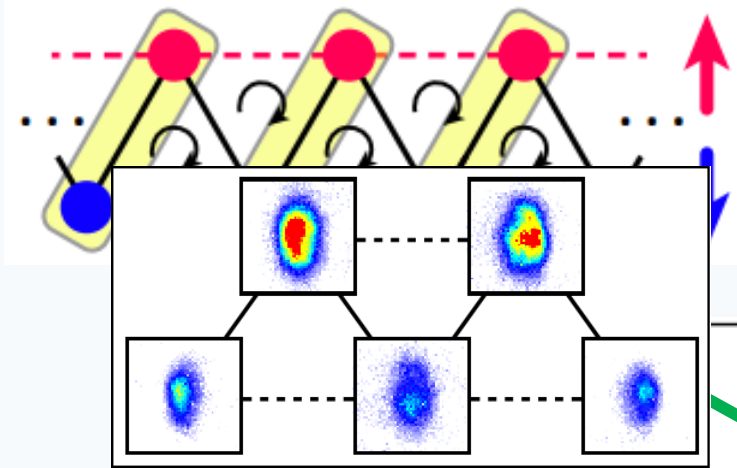
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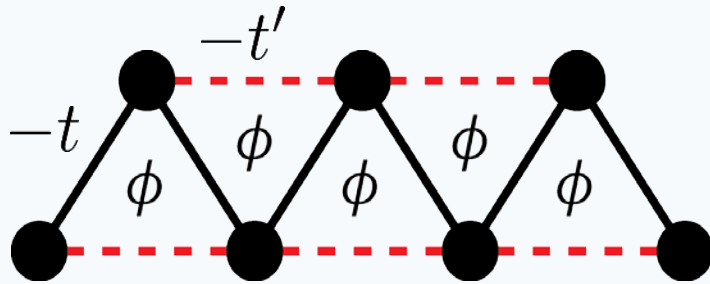
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# Aside: chiral quantum walks & spin-momentum locking





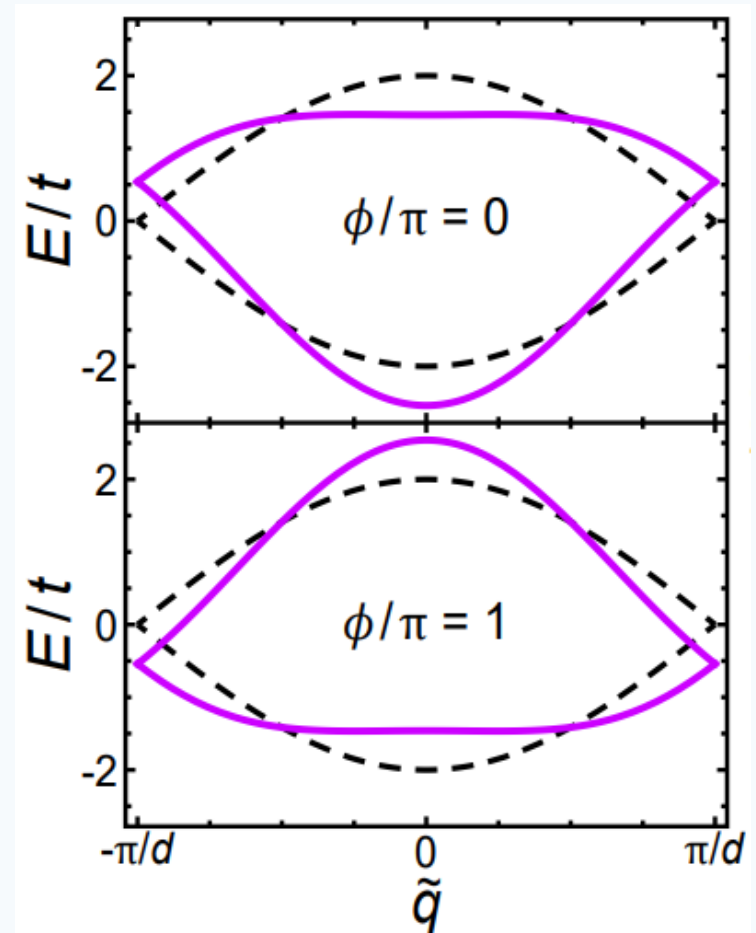
# Dispersion engineering



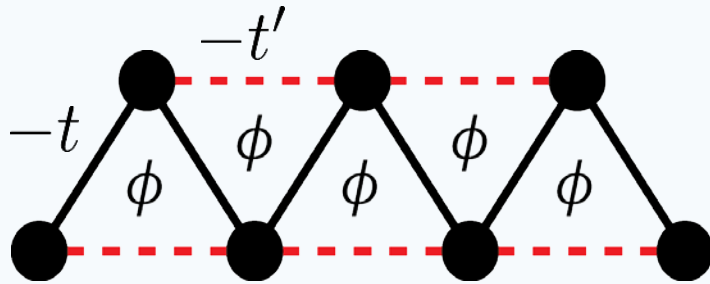
**Kinetic frustration gives quartic,  
nearly flat dispersions**

$$|t'/t| \approx 0.25$$

for  $t'/t = 0$ , normal lattice band structure  
(dashed line)



# Dispersion engineering



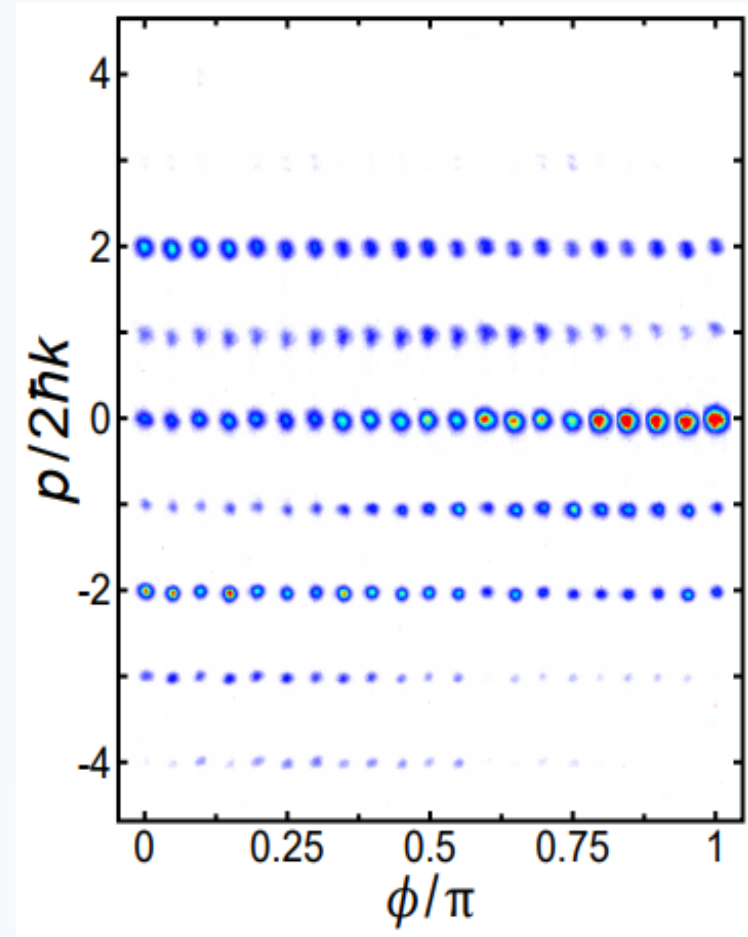
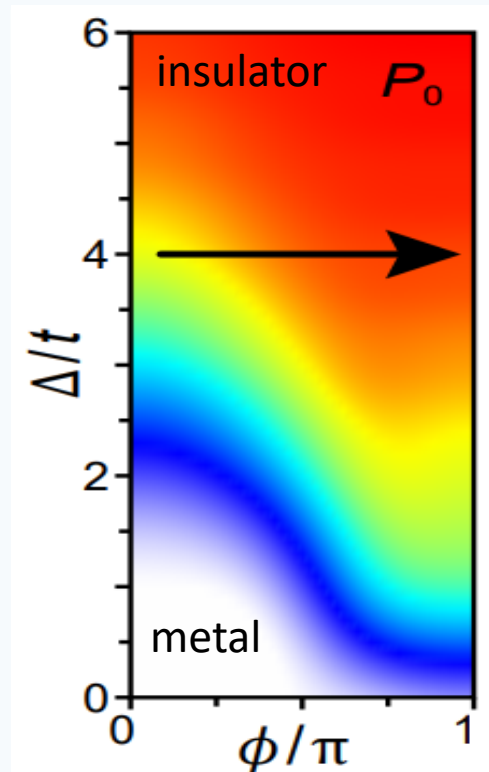
$$|t'/t| \approx 0.25$$

for  $t'/t = 0$ , normal lattice band structure  
(dashed line)

Localization properties  
of ground state

flux-dependent  
localization

ground state localization  
properties are suggestive  
of a flux-dependent  
mobility edge

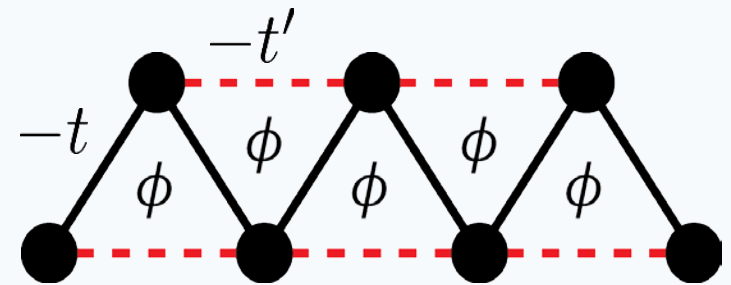


# Dispersion engineering

Can we push this dispersion engineering further?

By adding longer- and longer-ranged tunneling terms, we should be able to engineer a **completely** flat band

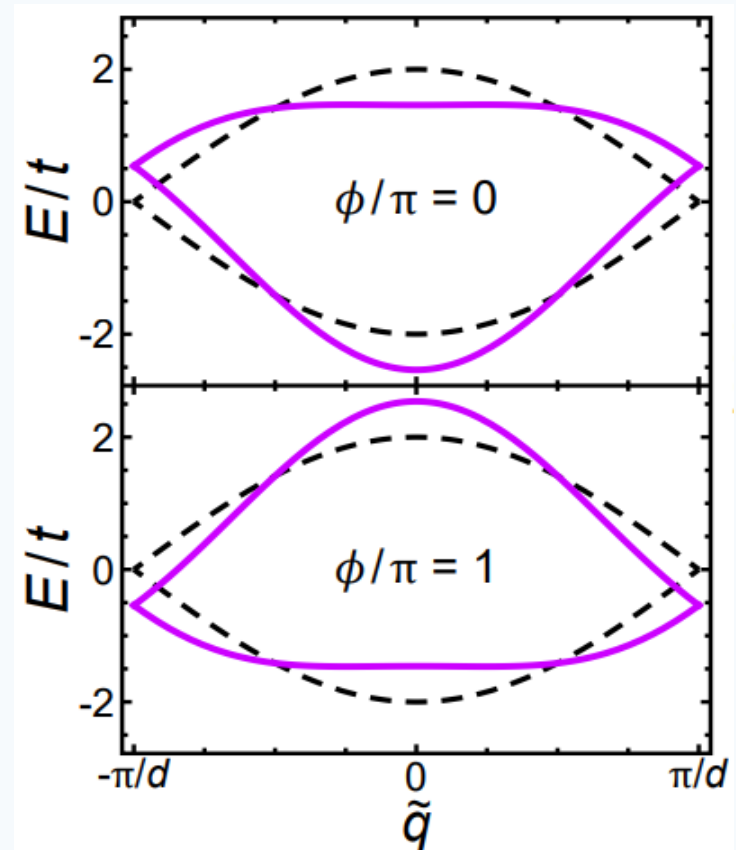
~ “Fourier synthesis of bands”



this would be technically challenging...



duality of the AA model is forgiving!  
Adding higher-order tunneling terms is **dual** to adding higher harmonics of the  $\Delta \cos(2\pi\beta n + \varphi)$  potential



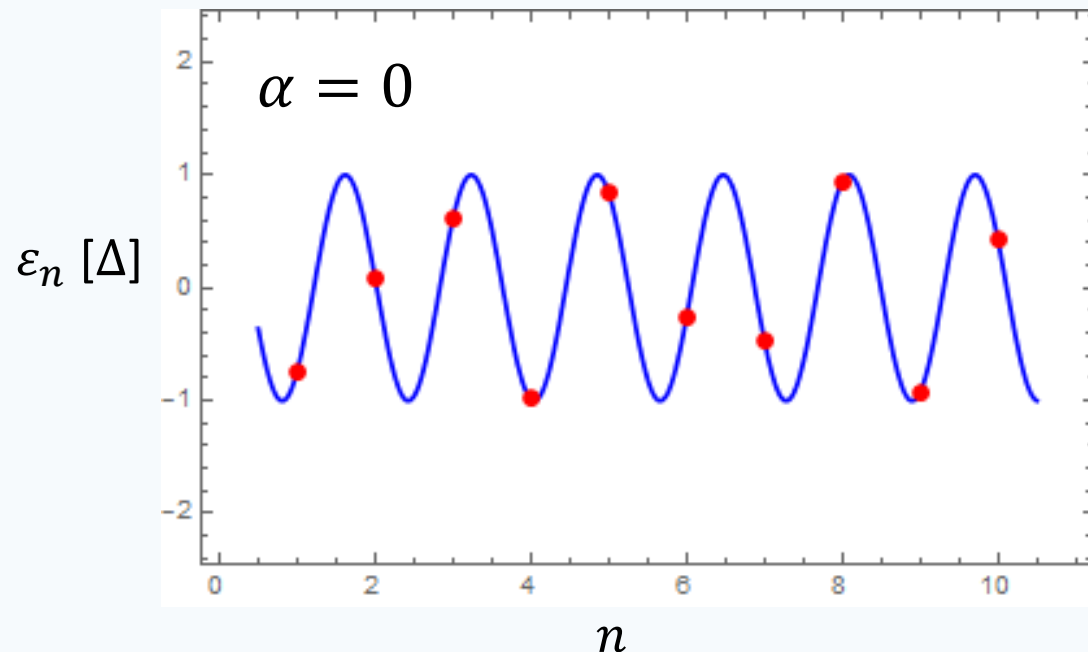
# Modified AA model with SPME

Ganeshan, Pixley, and Das Sarma, PRL (2015)

$$H = \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n - t \sum_n (\hat{c}_{n+1}^\dagger \hat{c}_n + \hat{c}_n^\dagger \hat{c}_{n+1})$$

$$\varepsilon_n = \frac{\Delta \cos(2\pi b n + \varphi)}{1 - \alpha \cos(2\pi b n + \varphi)} = \Delta \sum_{j=1}^{\infty} \alpha^{j-1} [\cos(2\pi b n + \varphi)]^j$$

$$b = \frac{\sqrt{5} - 1}{2}$$



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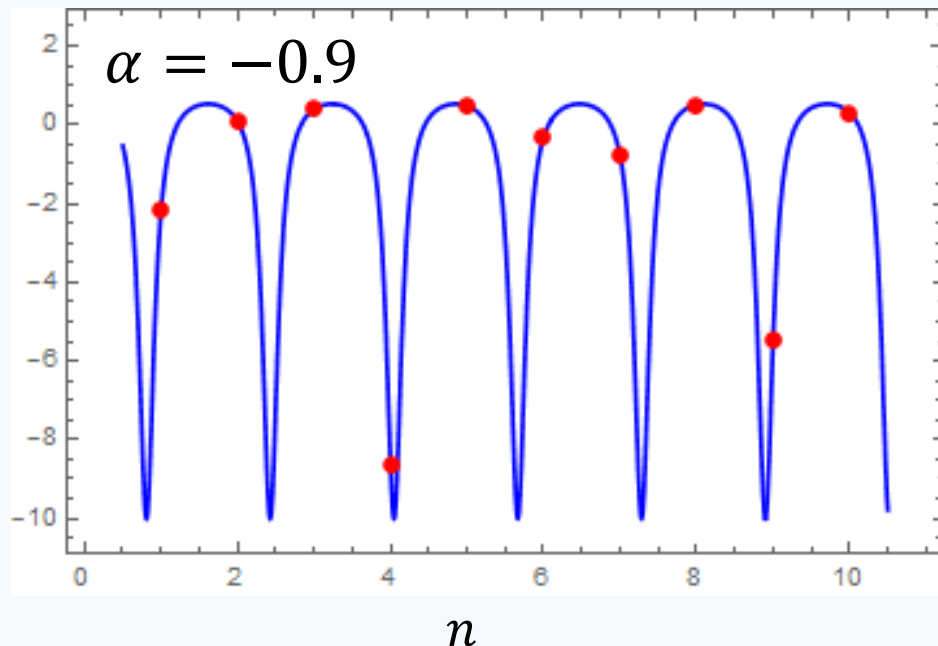
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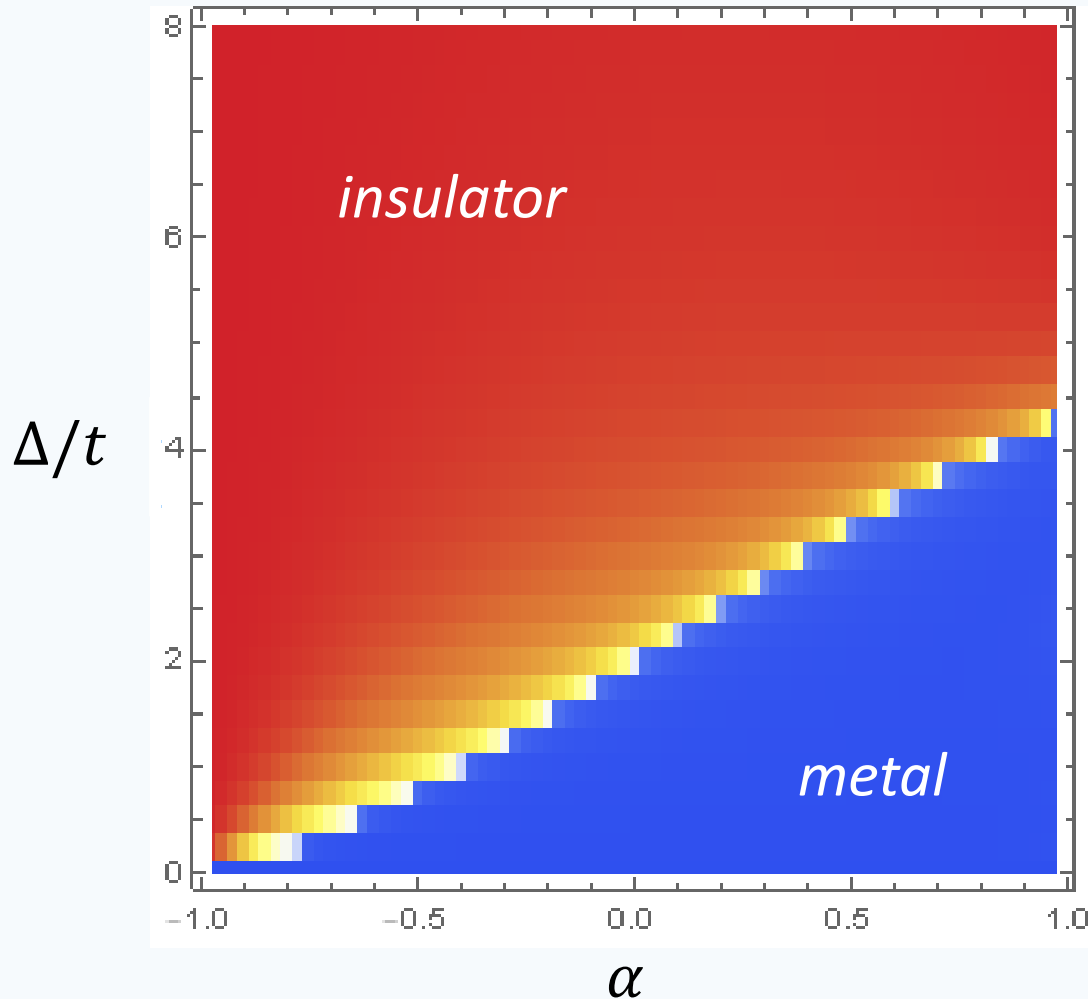
**Modifies the density of states  
(now lower at low energy,  
higher at high energy)**

$\varepsilon_n [\Delta]$



# Mobility edge vs. $\alpha$

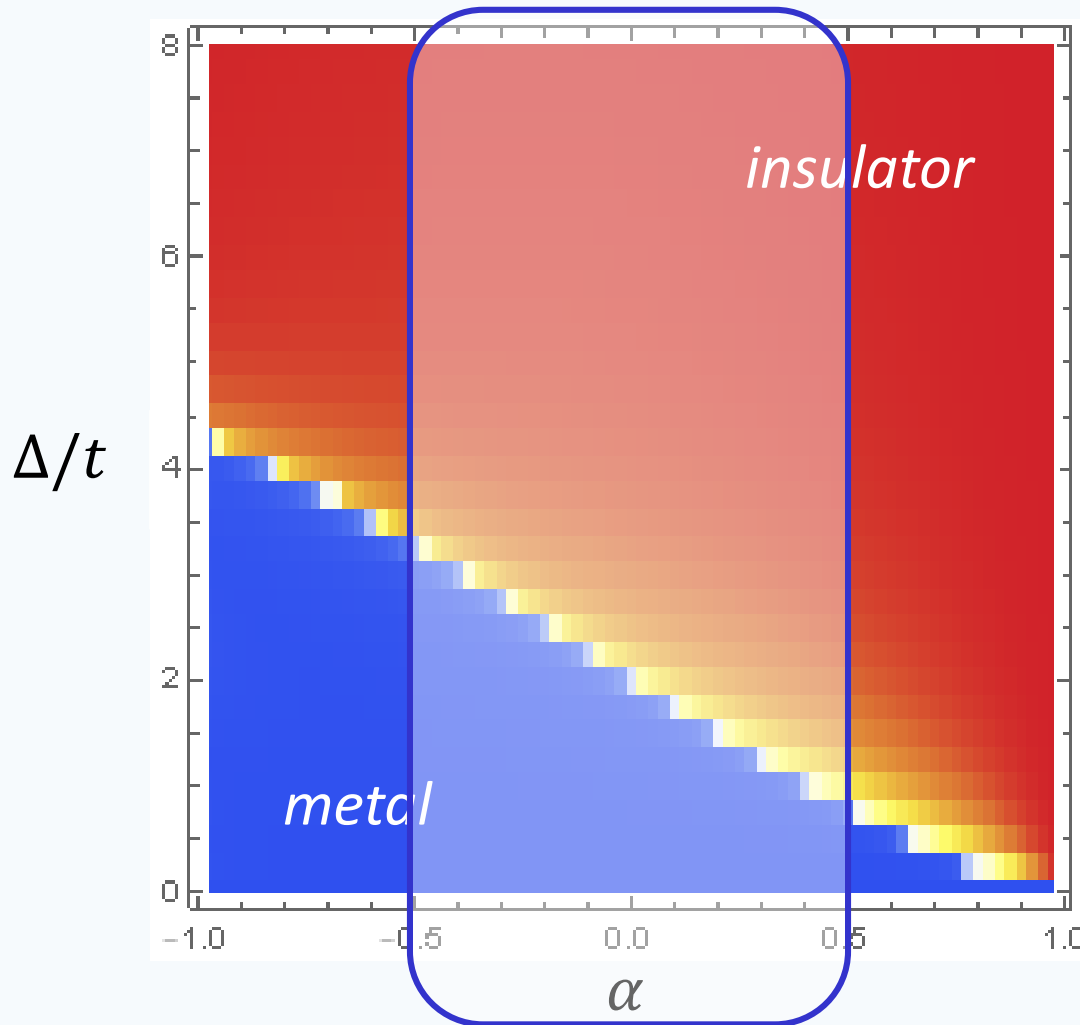
Ground state localization properties ( $\varphi = 0$ )



$$\varepsilon_n = \frac{\Delta \cos(2\pi b n + \varphi)}{1 - \alpha \cos(2\pi b n + \varphi)}$$

# Mobility edge vs. $\alpha$

“max” excited state localization properties ( $\varphi = \pi$ )



$$\varepsilon_n = \frac{\Delta \cos(2\pi b n + \varphi)}{1 - \alpha \cos(2\pi b n + \varphi)}$$

excited state  $\rightarrow$  ground state

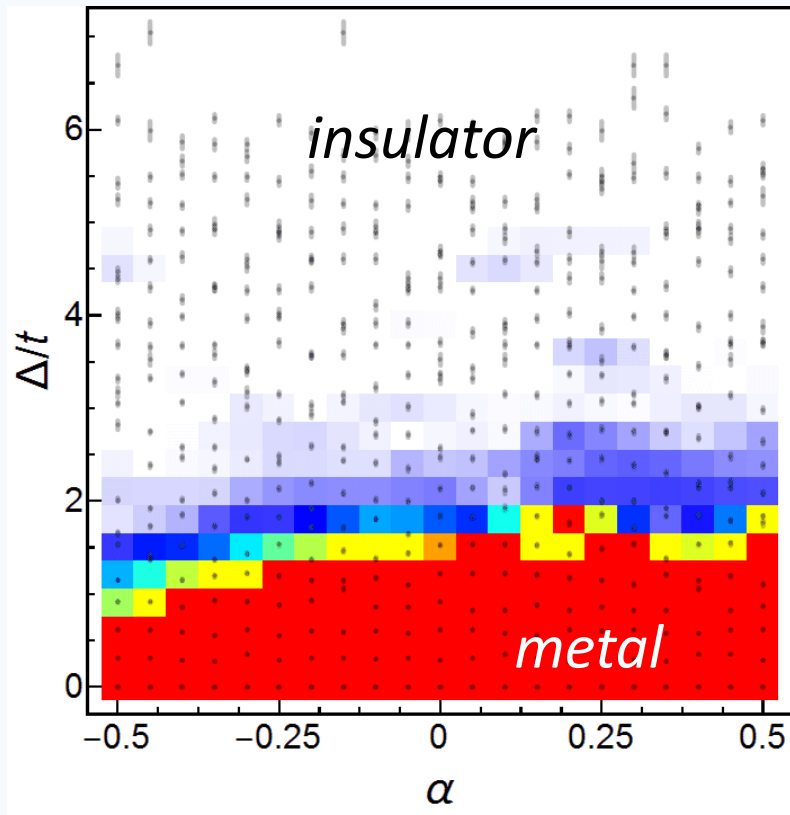
$$\varphi \rightarrow \varphi + \pi$$

$$\alpha \rightarrow -\alpha$$

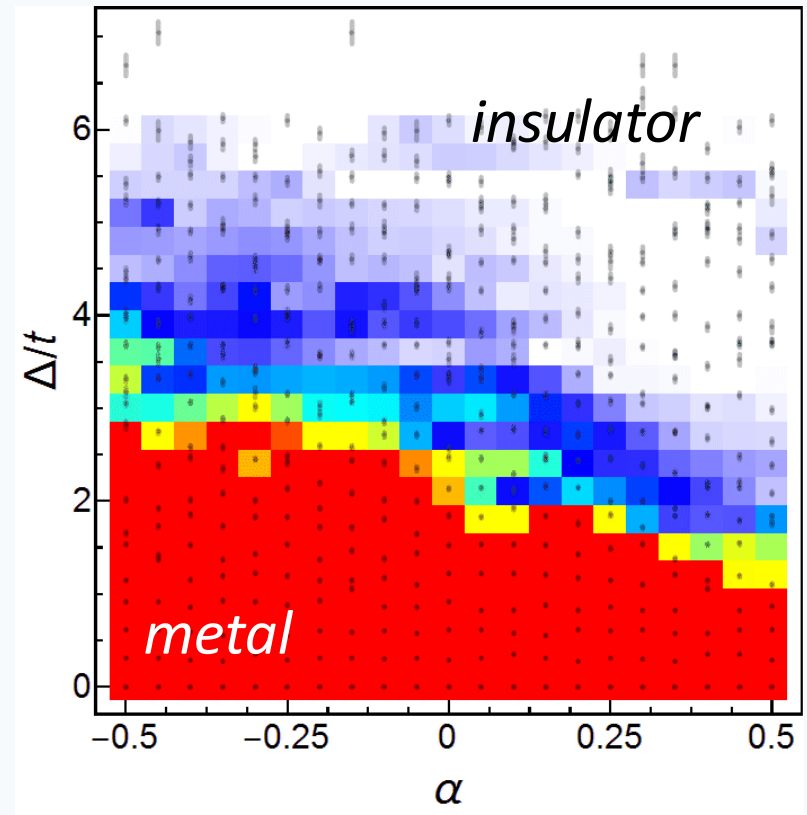
Note: values of  $\alpha$  near  $\pm 1$   
lead to infinite potential

# So what do we see in experiment?

Ground state s.d. ( $\varphi = 0$ )



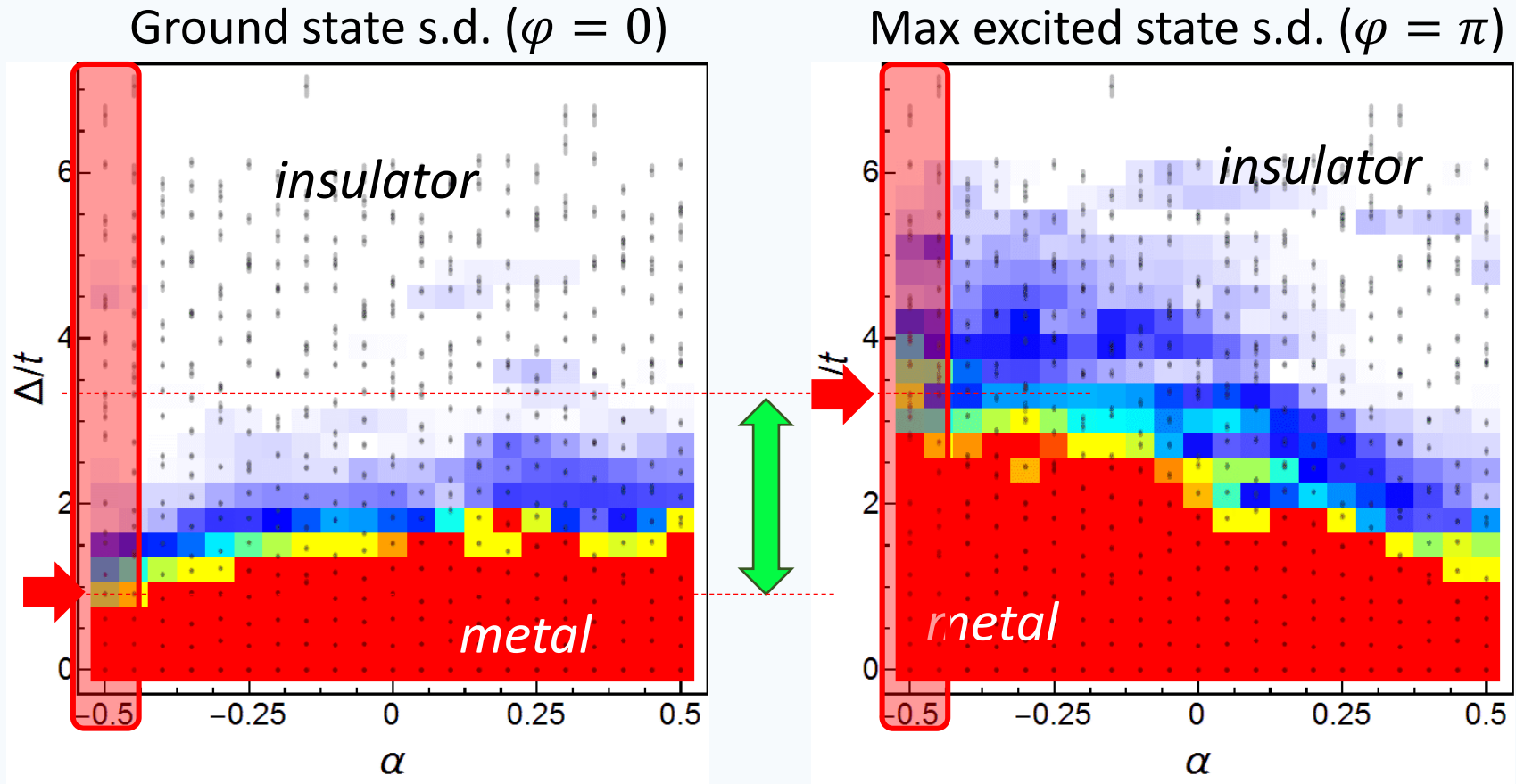
Max excited state s.d. ( $\varphi = \pi$ )





# So what do we see in experiment?

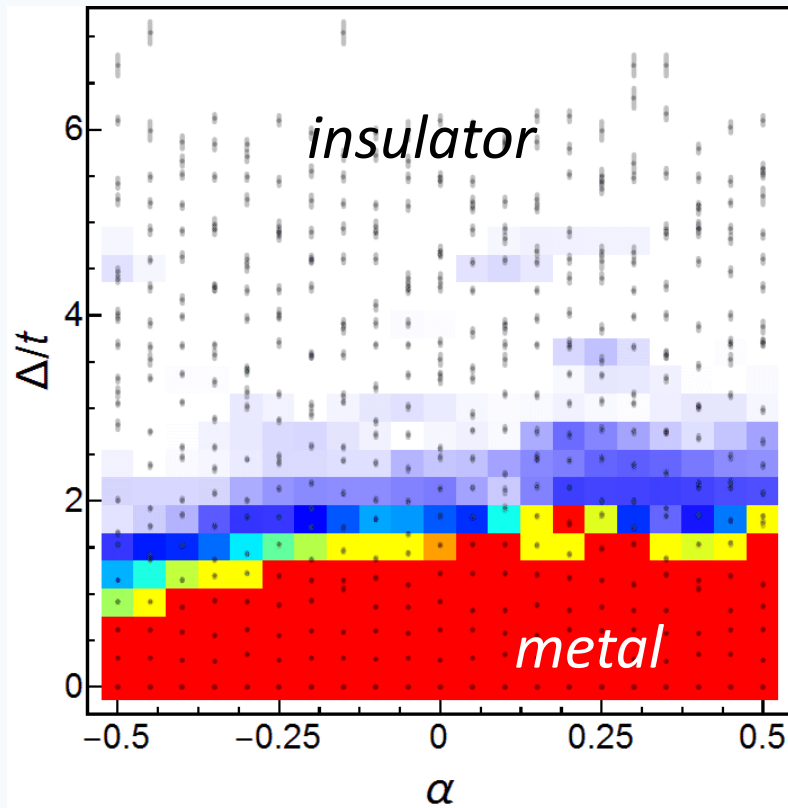
## Some definite evidence for a mobility edge



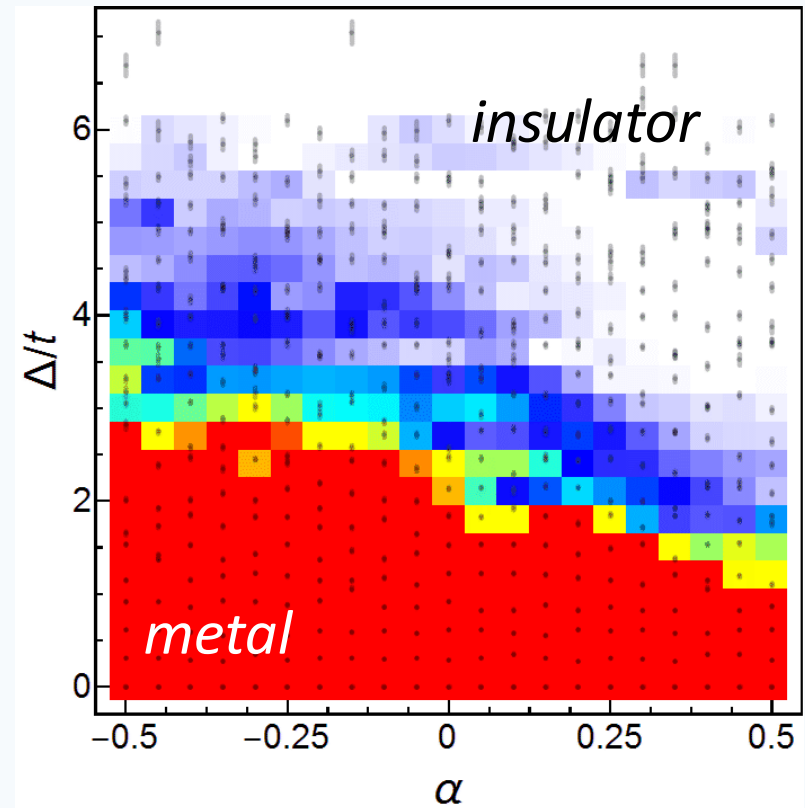
# So what do we see in experiment?

but these don't look like mirror images...

Ground state s.d. ( $\varphi = 0$ )



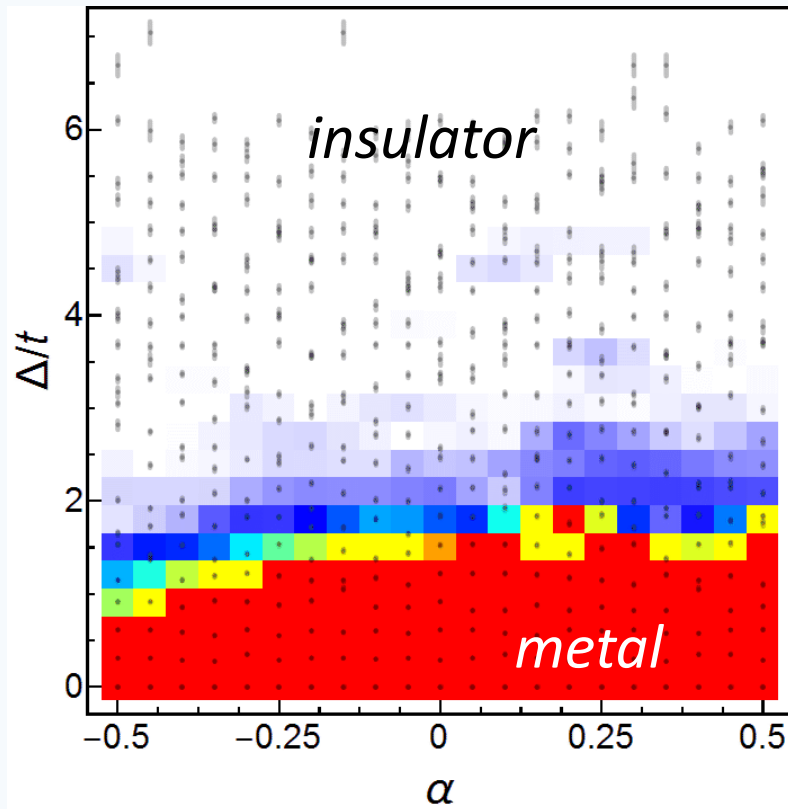
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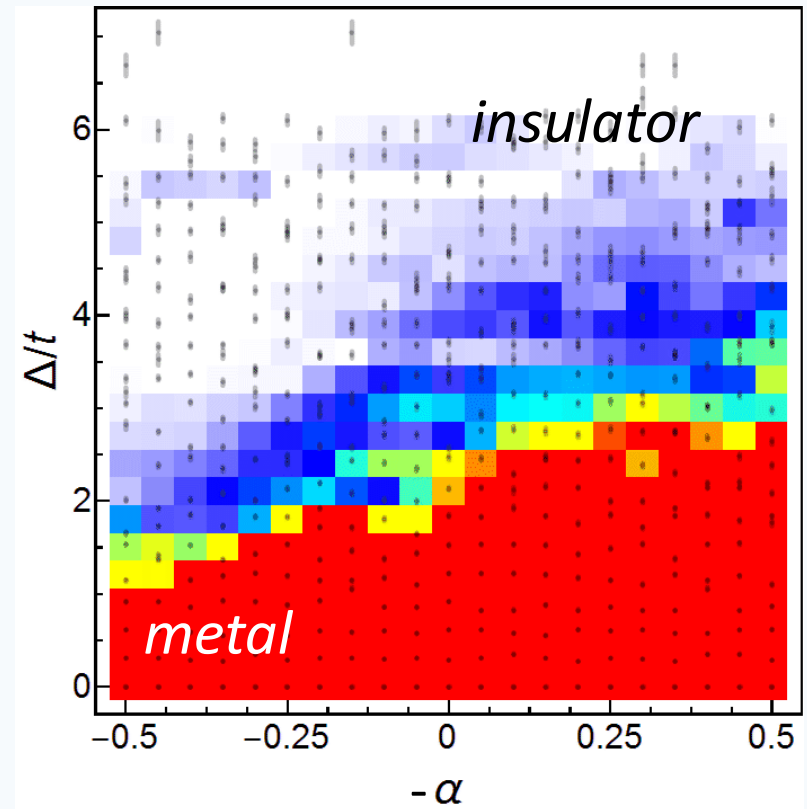
# So what do we see in experiment?

OK, definitely not mirror images...what gives?

Ground state s.d. ( $\varphi = 0$ )



Max excited state s.d. ( $\varphi = \pi$ )



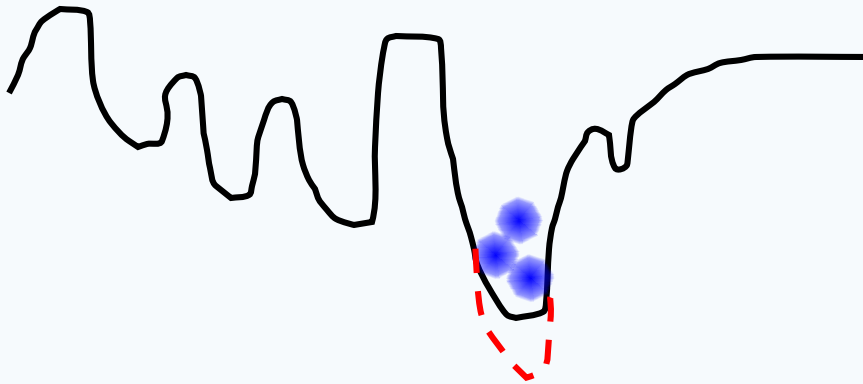
**it's the interactions!**

# Influence of interactions

## Influence of effectively *attractive* interactions

ground state ( $\varphi = 0$ ):

starts at potential minimum – interactions bring it further away in energy  
from other wells



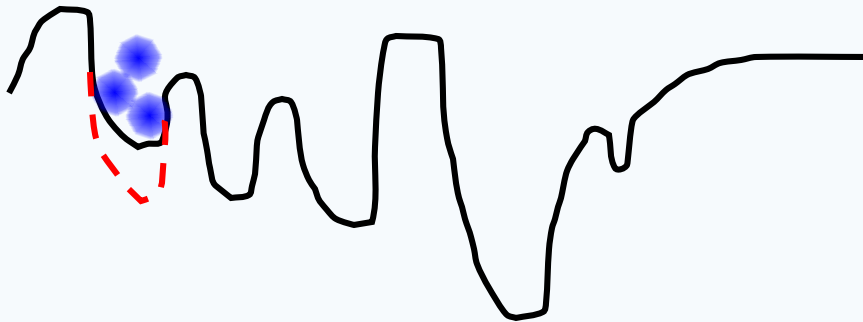
**more likely to localize!**

# Influence of interactions

## Influence of effectively *attractive* interactions

“Max” excited state ( $\varphi = \pi$ ):

starts at potential maximum – interactions bring it closer in energy to  
the other wells



**more likely to delocalize!**

# Excited state for $U < 0 \rightarrow$ Ground state for $U > 0$

At single-particle level:  $H \rightarrow -H$

effectively for:  $\varphi \rightarrow \varphi + \pi$

$$\alpha \rightarrow -\alpha$$

To negate entire Hamiltonian  
would require :  $U \rightarrow -U$

$\rightarrow$  the localization properties for some parameters  $\{\varphi, \alpha, U\}$  should be equivalent to those for  $\{\varphi + \pi, -\alpha, -U\}$

Our interactions are effectively **attractive**, however we can probe physics related to **repulsive** interactions by reversing the rest of the Hamiltonian (a feature of not studying the thermodynamic ground state)

# Outlook

# Some possible directions for exploration

## squeezing / higher-order correlations

- can access richer physics by measuring the off-diagonal correlations between different sites, not just their mean occupations

## flat-band physics with interactions

- exploration of “sawtooth” and other flat-band lattices

## increased control over interactions

- switching over to  $^{39}\text{K}$  for broad Feshbach resonance

## access multiple internal states

- allows access to local loss (w/ resonant light removing from one state) and the creation of synthetic  $U(2)$  gauge fields

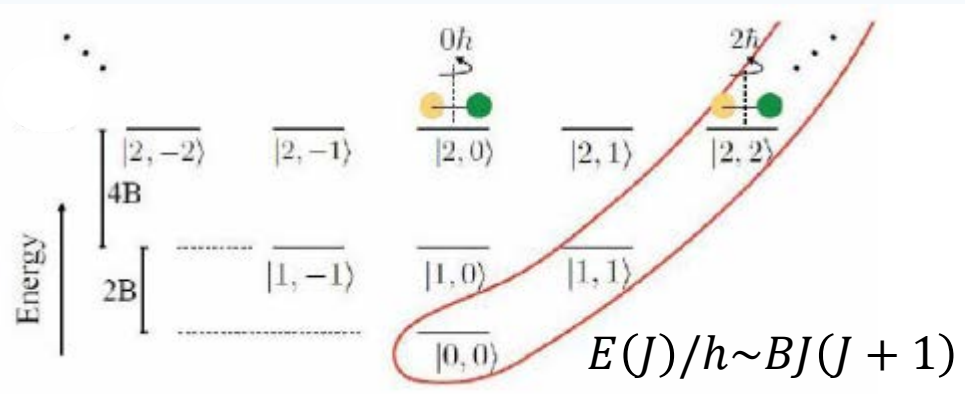
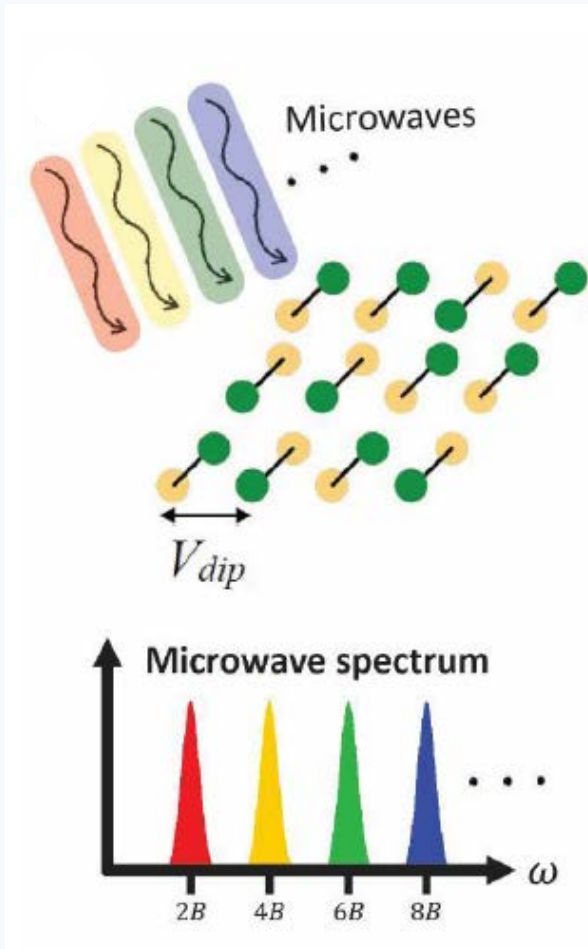
## synthetic dimensions for boson sampling

- can the realization of random Hamiltonians in synthetic lattices be useful for certifying quantum supremacy?



# Synthetic lattices on a molecule

## Similar idea, but with molecules (strongly correlated)



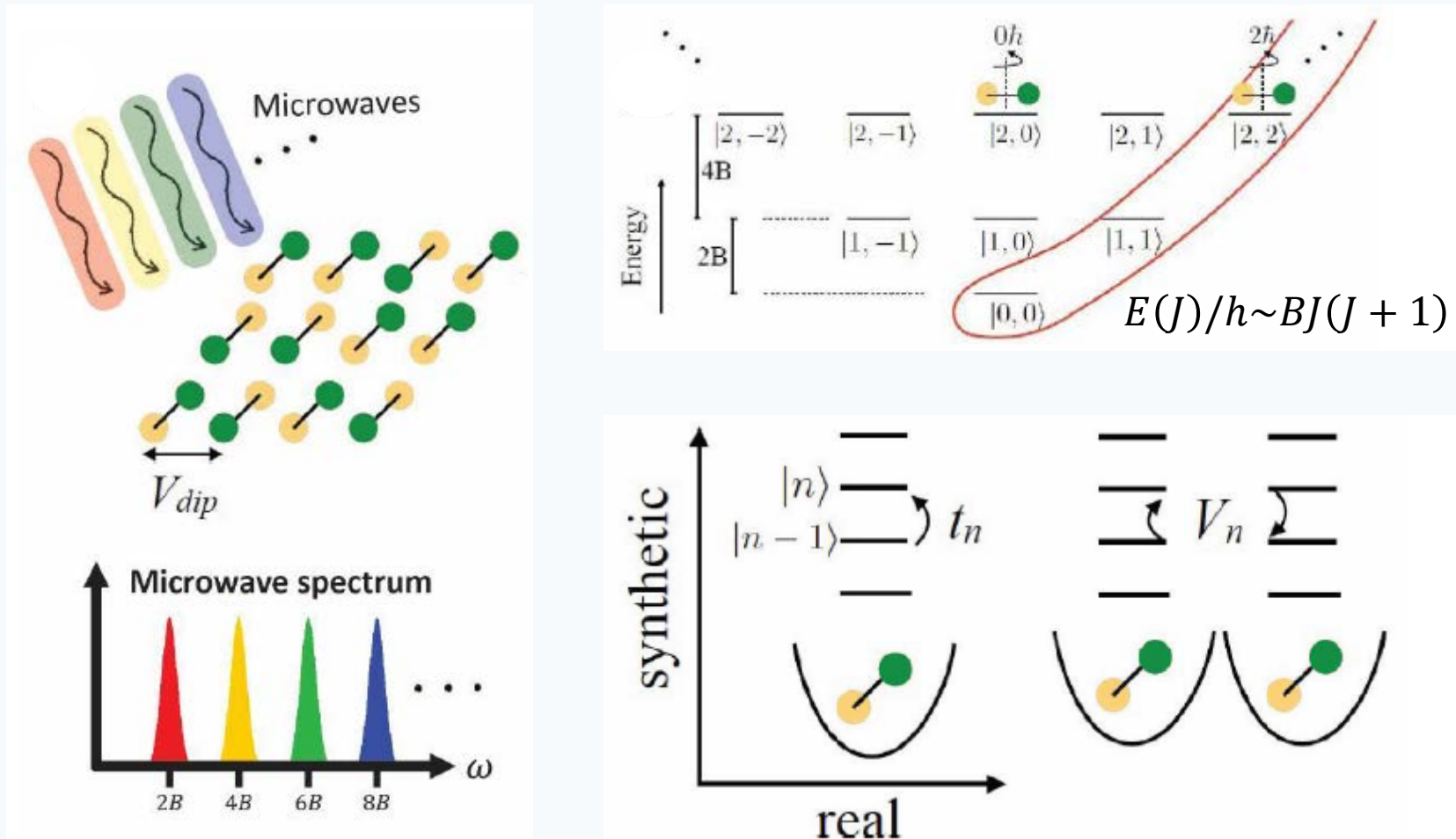
Rotational states are almost perfect “clock” states [same hyperfine quantum numbers, singlet state]

Families of states have common “magic” polarization

Can use full manifold of states ( $J_{max}^2$  states)

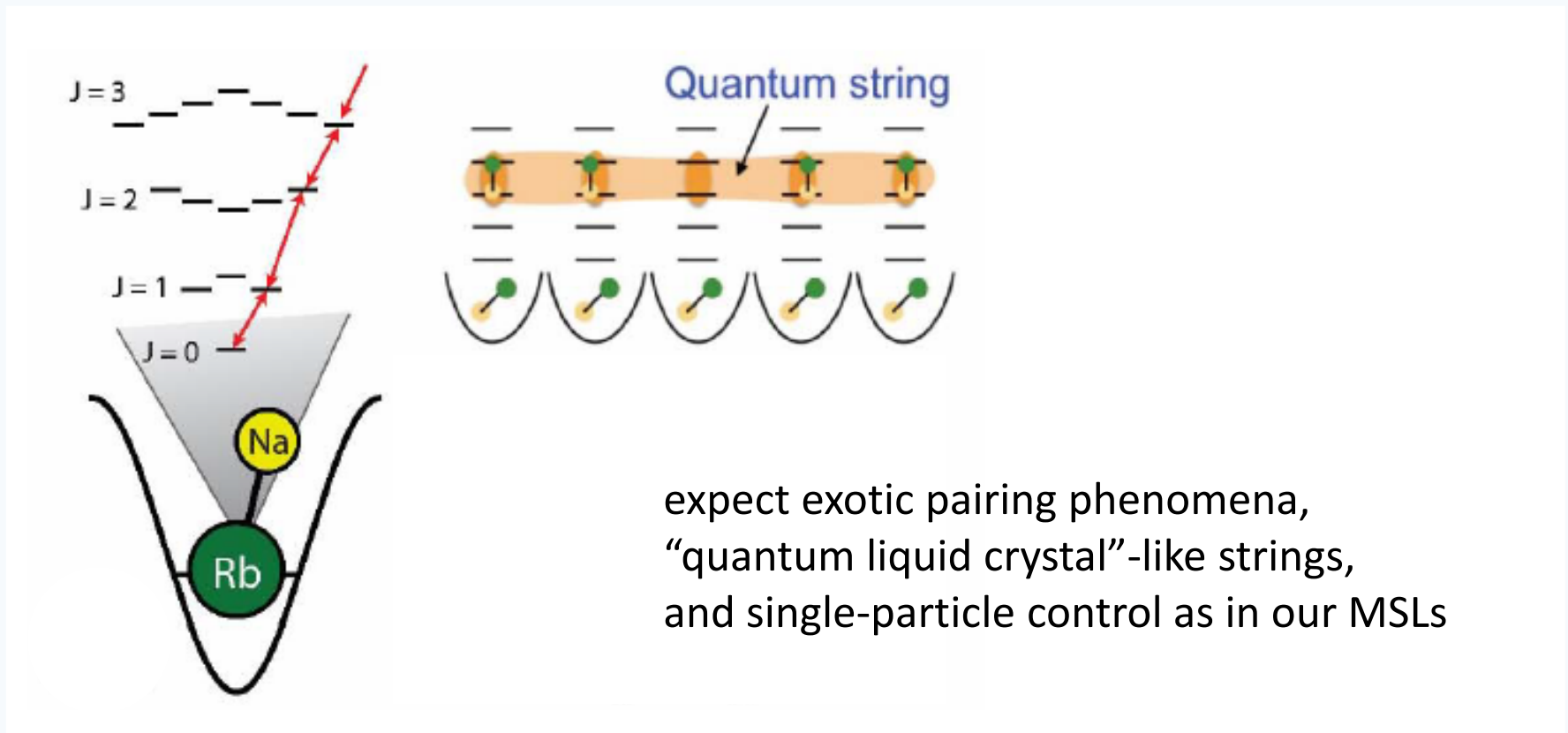
# Synthetic lattices on a molecule

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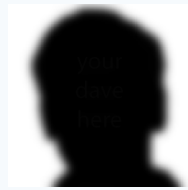
# Synthetic lattices on a molecule

## Similar idea, but with molecules (strongly correlated)



# Thanks! Questions?

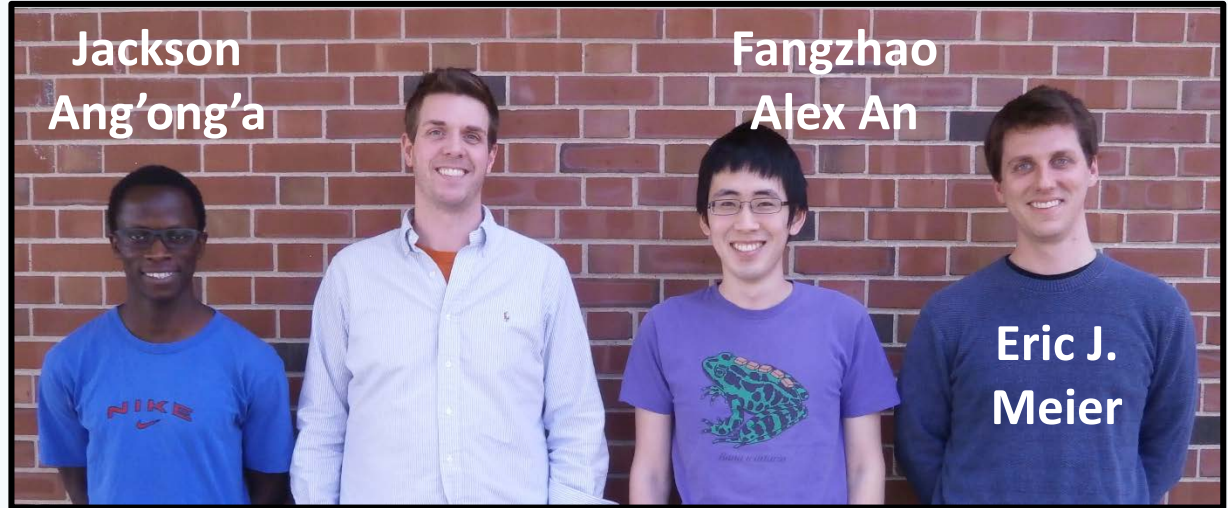
## Acknowledgments



Muyan Du



Michael Highman

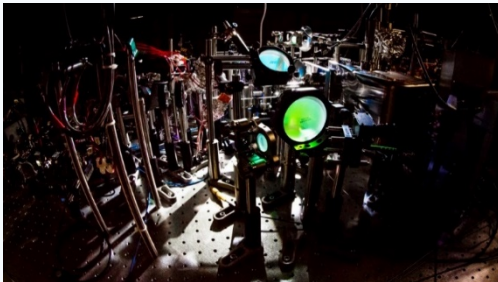


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Eric J.  
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## Theory friends & collaborators:

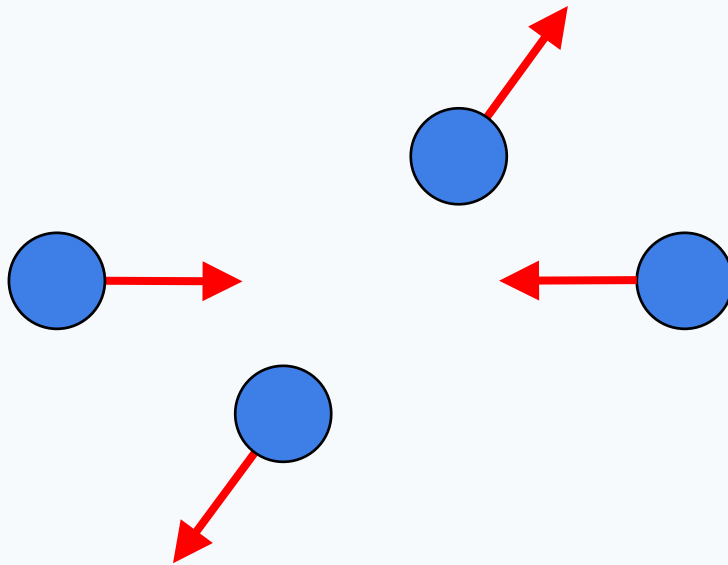
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- Kaden Hazzard (Rice University)
- Taylor Hughes (UIUC)
- Smitha Vishveshwara (UIUC)





# Elastic scattering “loss”

Momentum & energy conservation allow for pairwise scattering into  $4\pi$  steradians (i.e. s-wave halos)



Elastic loss rate  $\sim n\sigma v \propto n \times a^2$

Interaction energy  $\sim gn \propto n \times a$

different scaling with scattering length

→ Ratio can in principle be tuned

**Ratio of “bad” to “good” processes:**

$$R = 2|k|a \sim 0.1 \quad \text{for} \quad k = 2k_L$$

Trippenbach, Band, Julienne.  
PRA **62**, 023608 (2000)