

Novel topological optical lattices

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10:40 - 11:20, Monday, 20 November 2017

***Workshop on
Synthetic dimensions in quantum engineered systems
Zürich, 20-23 November 2017***

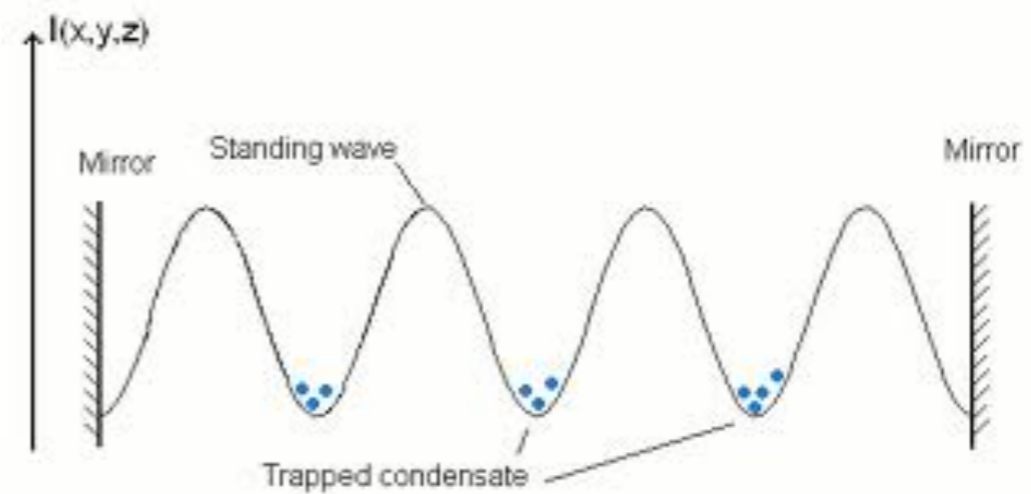
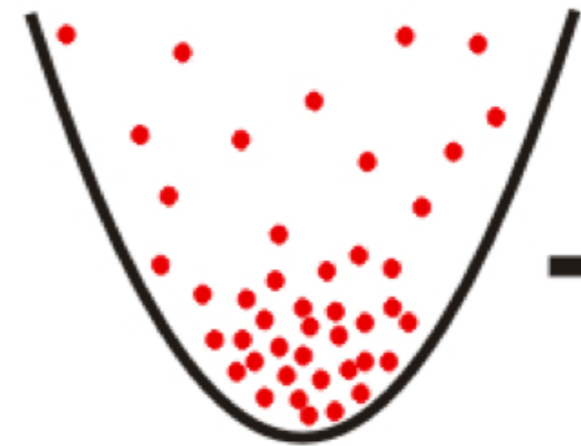
Outline

- Background:
 - Optical lattices
 - Magnetic flux in conventional optical lattices
- Part I: Topological lattices using multi-frequency radiation
- Part II. Optical Lattices using synthetic dimensions
 - Square geometry
 - Non-square geometry
- Conclusions

Background

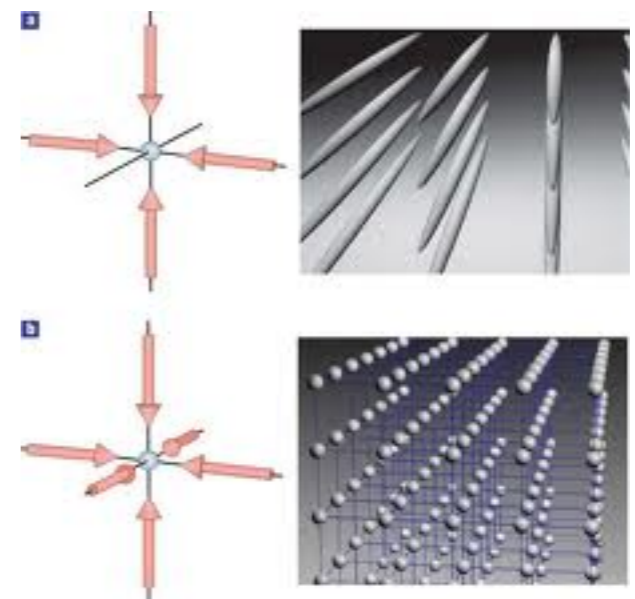
Ultracold atoms are trapped using:

- (Parabolic) trapping potential produced by magnetic or optical means
- Optical lattice (periodic potential)



Optical lattices (ordinary)

- A set of laser beams (off resonance to the atomic transitions)
- Atoms are trapped at intensity minima (or intensity maxima) of the interference pattern (depending on the sign of atomic polarisability)
- 2D square optical lattice
- 3D cubic optical lattice
- **Tunneling elements real**
→ **No magnetic flux**



Picture from I. Bloch, NP 2005

Artificial magnetic flux in optical lattices

Theory:

- D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
- J. Dalibard and F. Gerbier, New J. Phys. **12**, 033007 (2010)

See also:

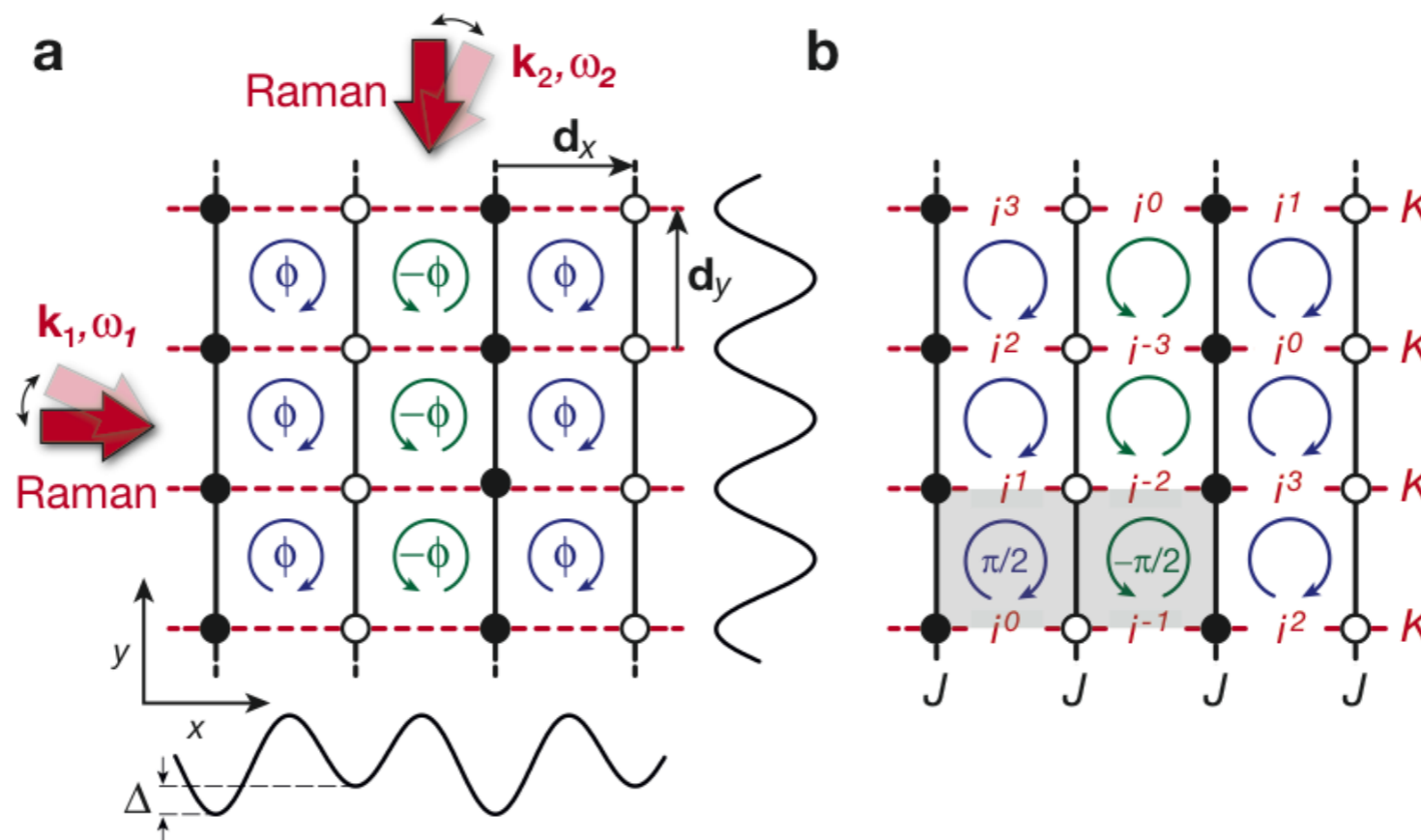
J. Ruostekoski, G.V. Dunne, and J. Javanainen,
Phys. Rev. Lett. **88**, 180401 (2002);

J. Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg,
Rev. Mod. Phys. **83** 1523 (2011).

- Laser-assisted tunnelling between lattice sites
(with a recoil in another direction):
Complex valued tunnelling matrix elements →
Non-zero magnetic flux

Artificial magnetic fields in optical lattices

- Optical square lattice: Laser-assisted tunnelling along x
- Ordinary tunnelling along y
- Experiment: M. Aidelsburger, M. Atala, S. Nascimbène, S. Trotzky, Yu-Ao Chen and I. Bloch, PRL **107**, 255301 (2011).

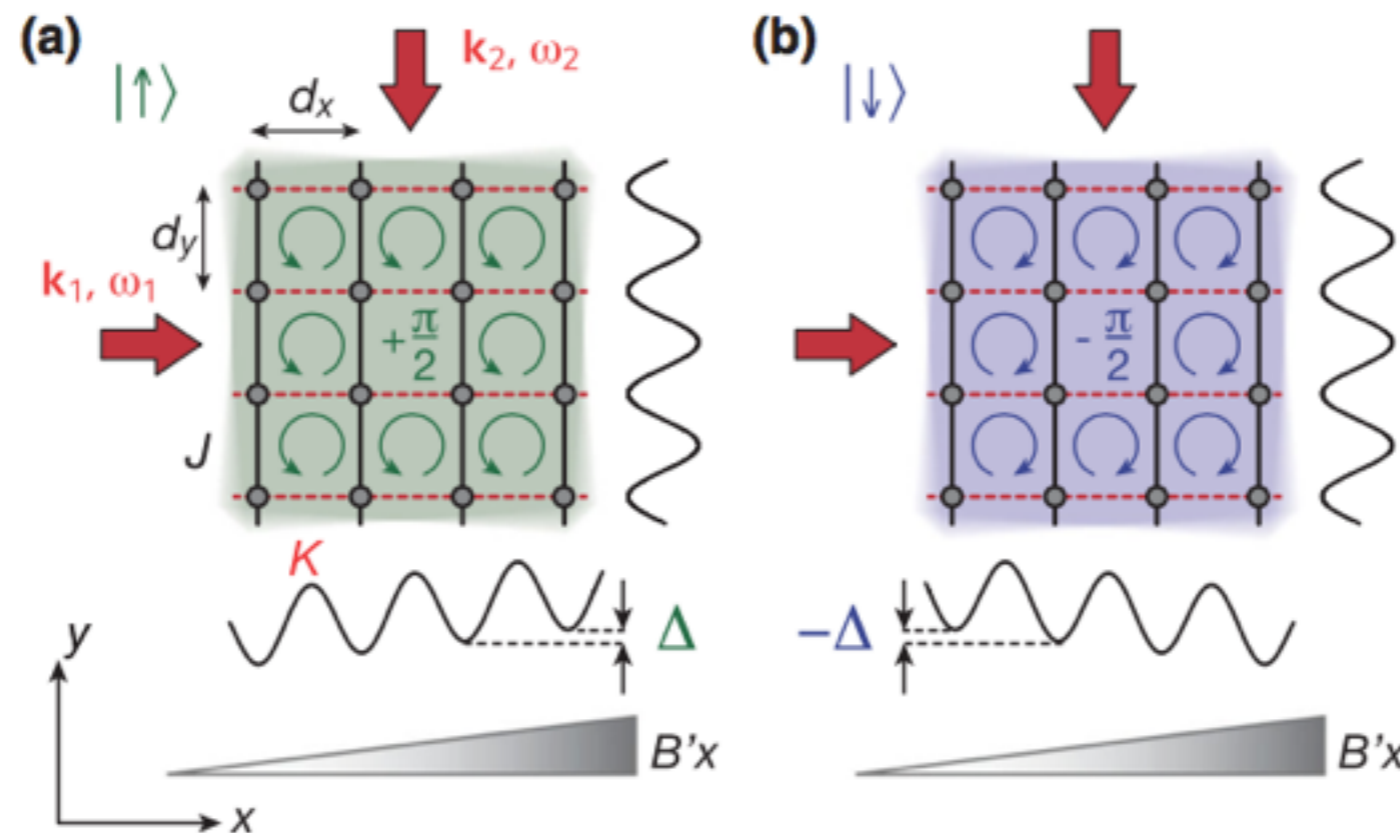


Complex valued tunnelling matrix elements along x

Double-well potential \rightarrow Staggered flux!

Artificial magnetic fields in optical lattices

- Optical square lattice: Laser-assisted tunnelling along x
- Ordinary tunnelling along y
- Experiments: M. Aidelsburger et al PRL **111**, 185301 (2013); H. Miyake et al PRL **111**, 185302 (2013).



Complex valued tunnelling matrix elements along x

Tilted potential



Non-staggered flux!

Artificial magnetic flux in optical lattices

- Lattice shaking

J. Struck, C. Ölschläger, M. Weinberg, P. Hauke, J. Simonet, A. Eckardt, M. Lewenstein, K. Sengstock and P. Windpassinger, Phys. Rev. Lett. **108**, 225304 (2012).

T. Uehlinger, D. Greif, and T. Esslinger, Nature **515**, 237 (2014).

N. Fläschner, B. S. Rem, M. Tarnowski, D. Vogel, D.-S. Lühmann, K. Sengstock, C. Weitenberg, Science **352**, 1091 (2016).

...

Complex valued tunnelling matrix elements

Here:

**Non-staggered magnetic flux
without (conventional)
optical lattices**

Part I: Topological lattices using multi-frequency radiation

Tomas Andrijauskas*, Ian Spielman** & Gediminas Juzeliūnas*

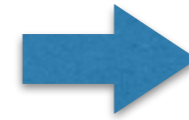
* Institute of Theoretical Physics and Astronomy, Vilnius University, Lithuania

** Joint Quantum Institute, NIST, Gaithersburg, USA

arXiv:1705.11101




Magnetic flux without usual optical lattices

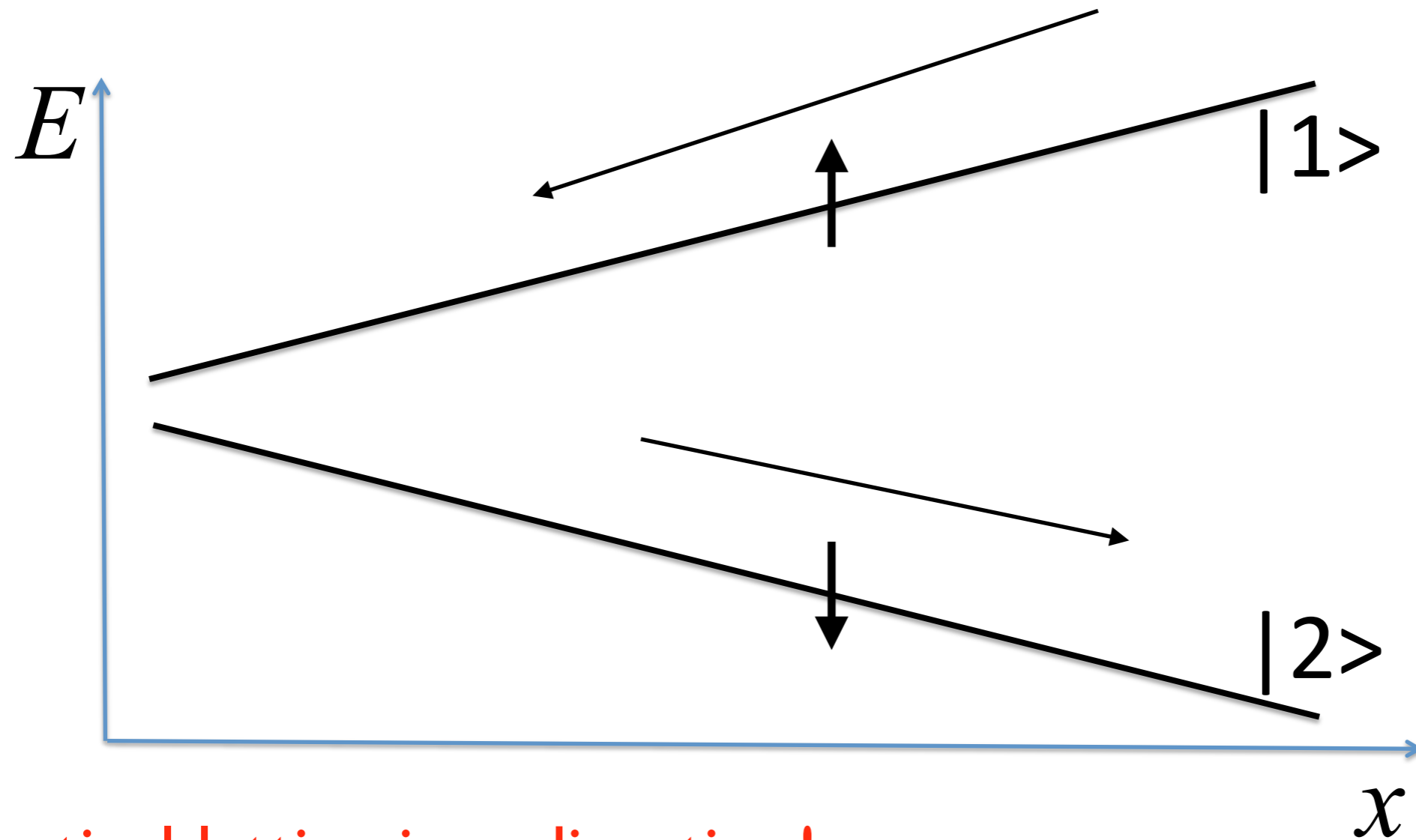
- Magnetic field gradient along the x axis:
- Two different atomic spin states (up and down)



E.g. two hyperfine atomic states with different magnetic momenta

Magnetic flux without usual optical lattices


- Magnetic field gradient along the x axis: 
- Two different atomic spin states (up and down)  



No optical lattice in x direction!

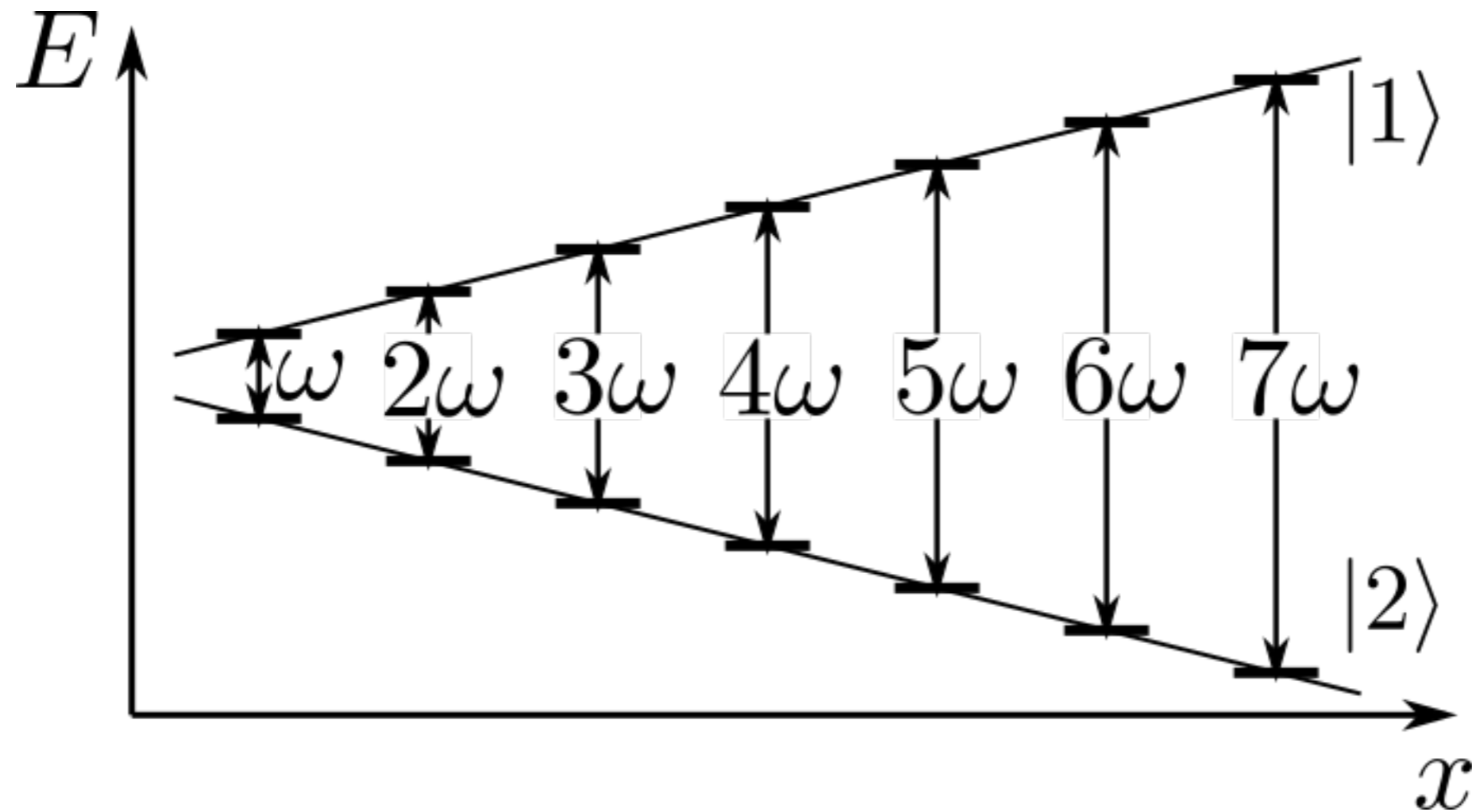
Spin-dependent potential gradient  Spin-dependent acceleration

Magnetic flux without usual optical lattices

- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states


Raman coupling with many frequencies

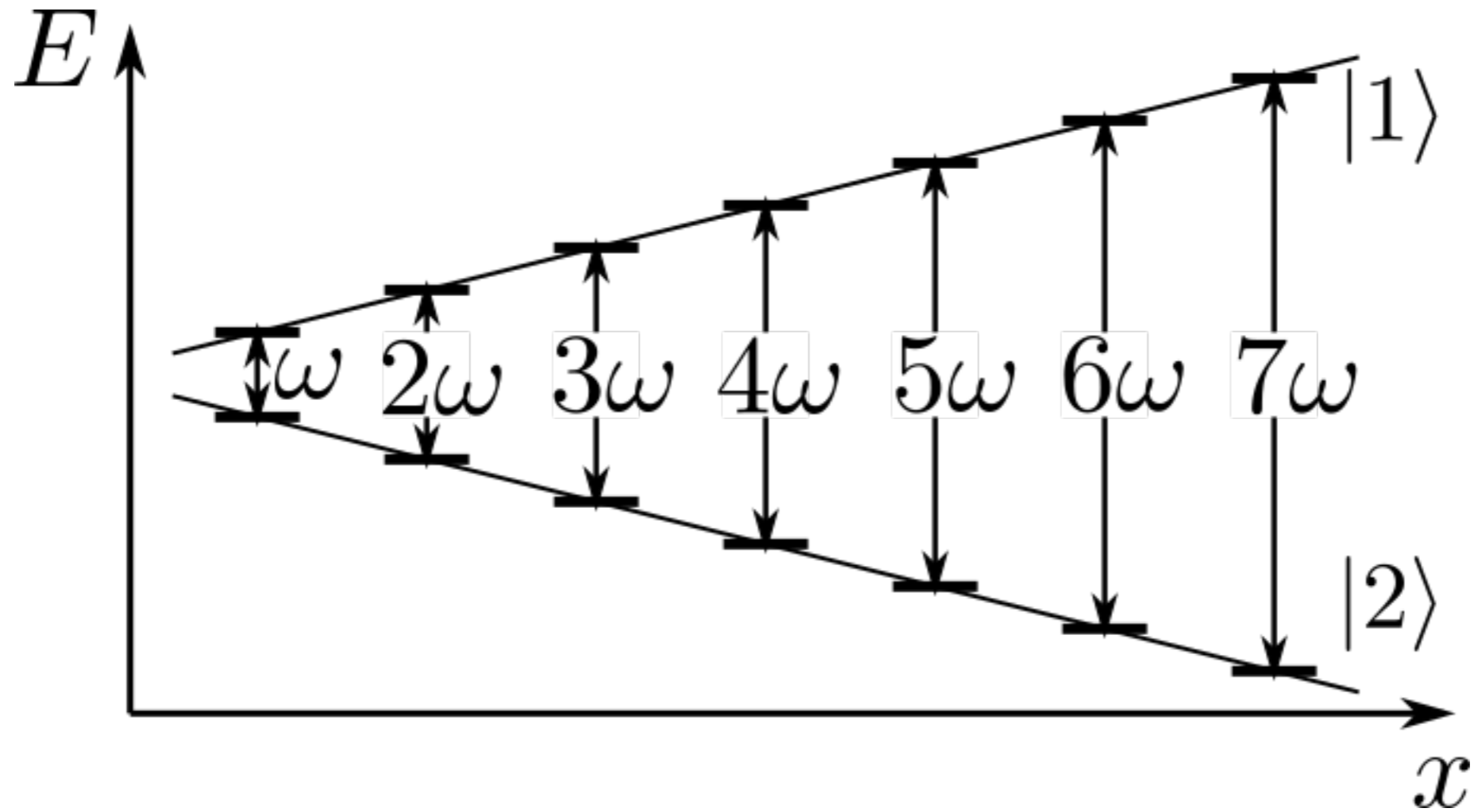
induce resonant transitions at different spatial locations x



Spin-dependent acceleration of atoms is interrupted


Magnetic flux without usual optical lattices

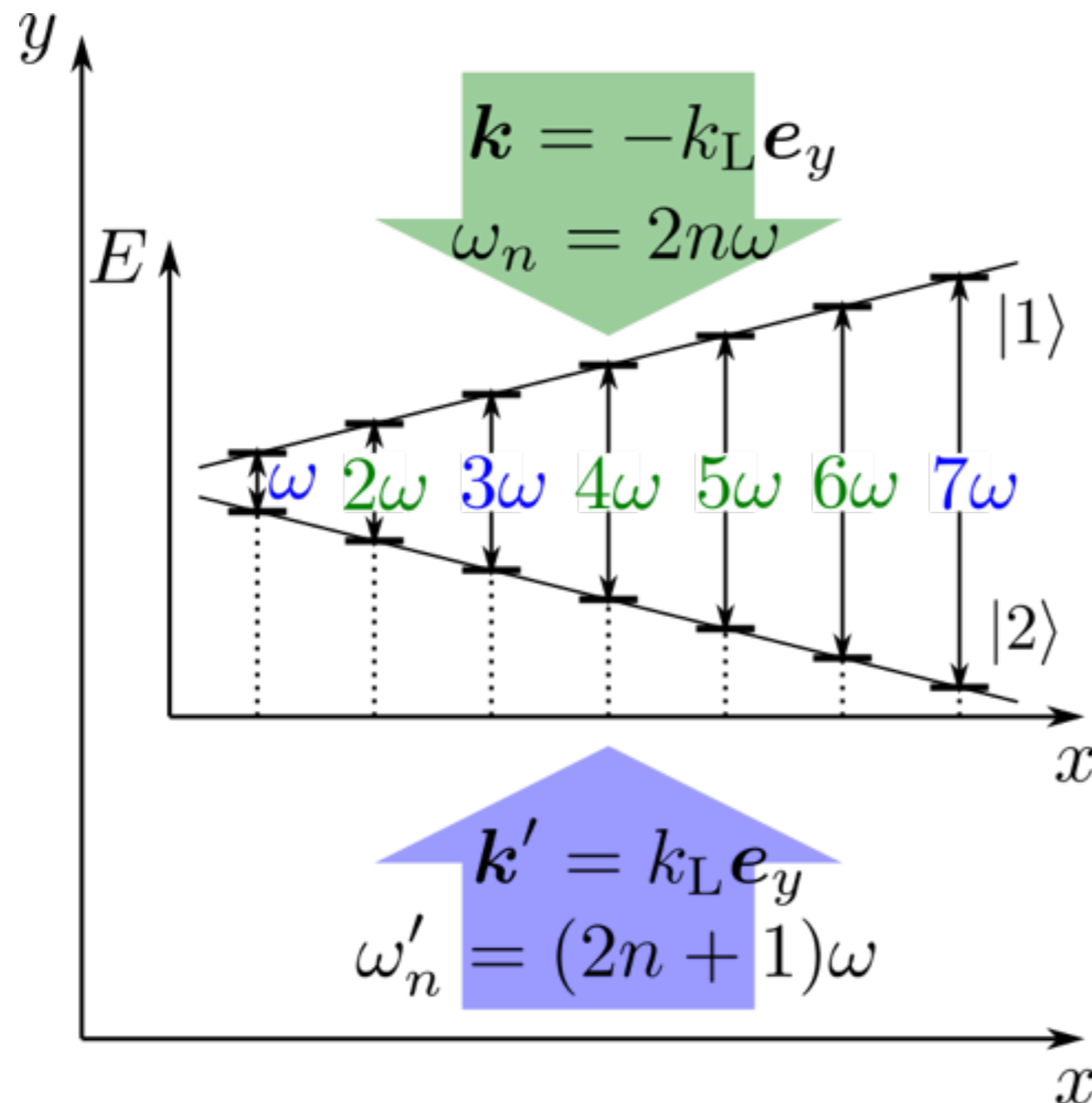
- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Frequency-comb (multi-frequency) Raman transitions between the spin states with a recoil kick in an orthogonal (y) direction



Spin-dependent acceleration of atoms is interrupted

Magnetic flux without usual optical lattices


- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Raman transitions with **odd frequencies**: Recoil along y axis
- Raman transitions with **even frequencies**: Recoil along -y axis

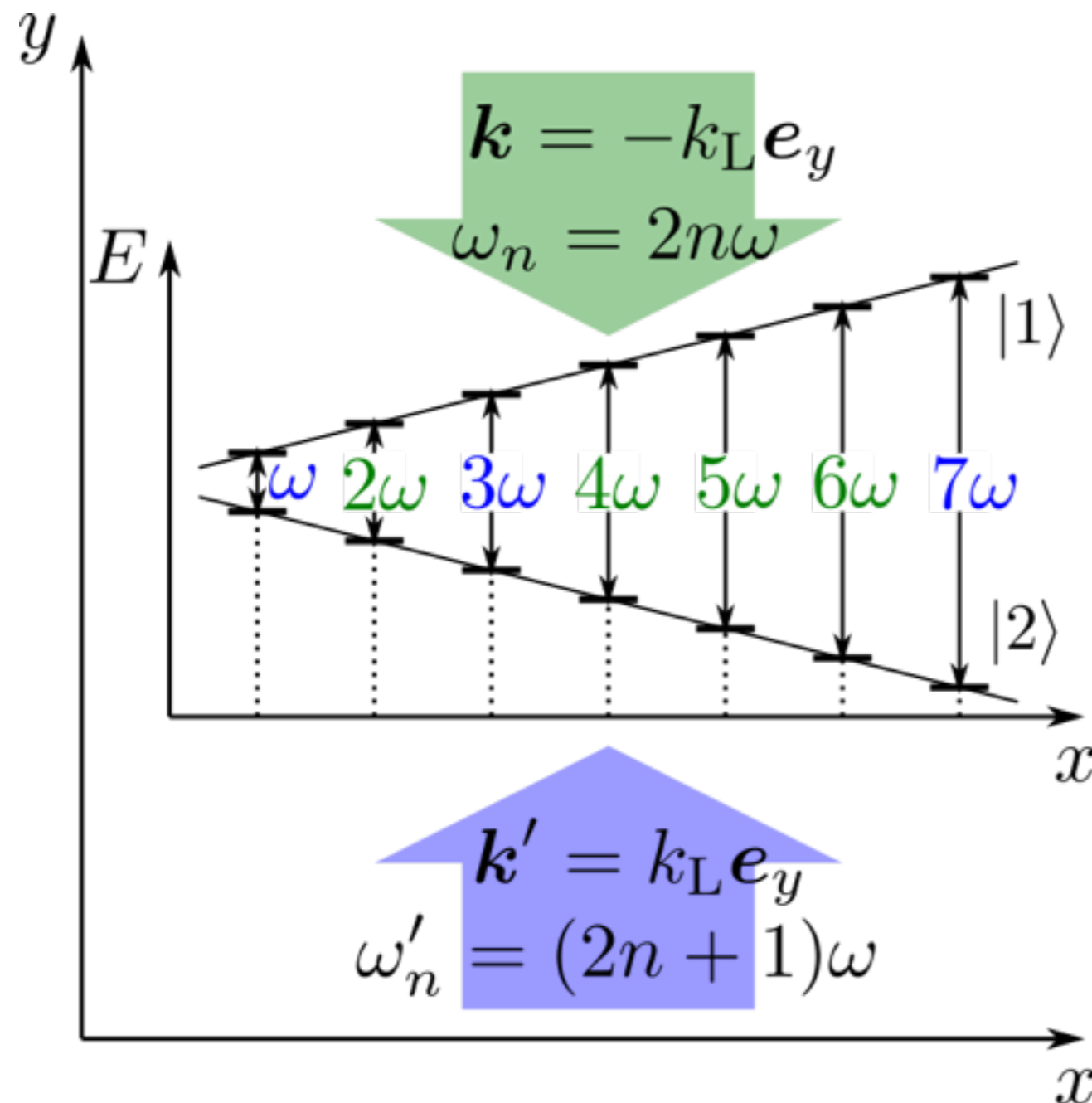


(Even frequencies)

(Odd frequencies)

Magnetic flux without usual optical lattices


- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Raman transitions with **odd** and **even** frequencies address atoms at alternating x_n with alternating **recoil** (along y or -y)

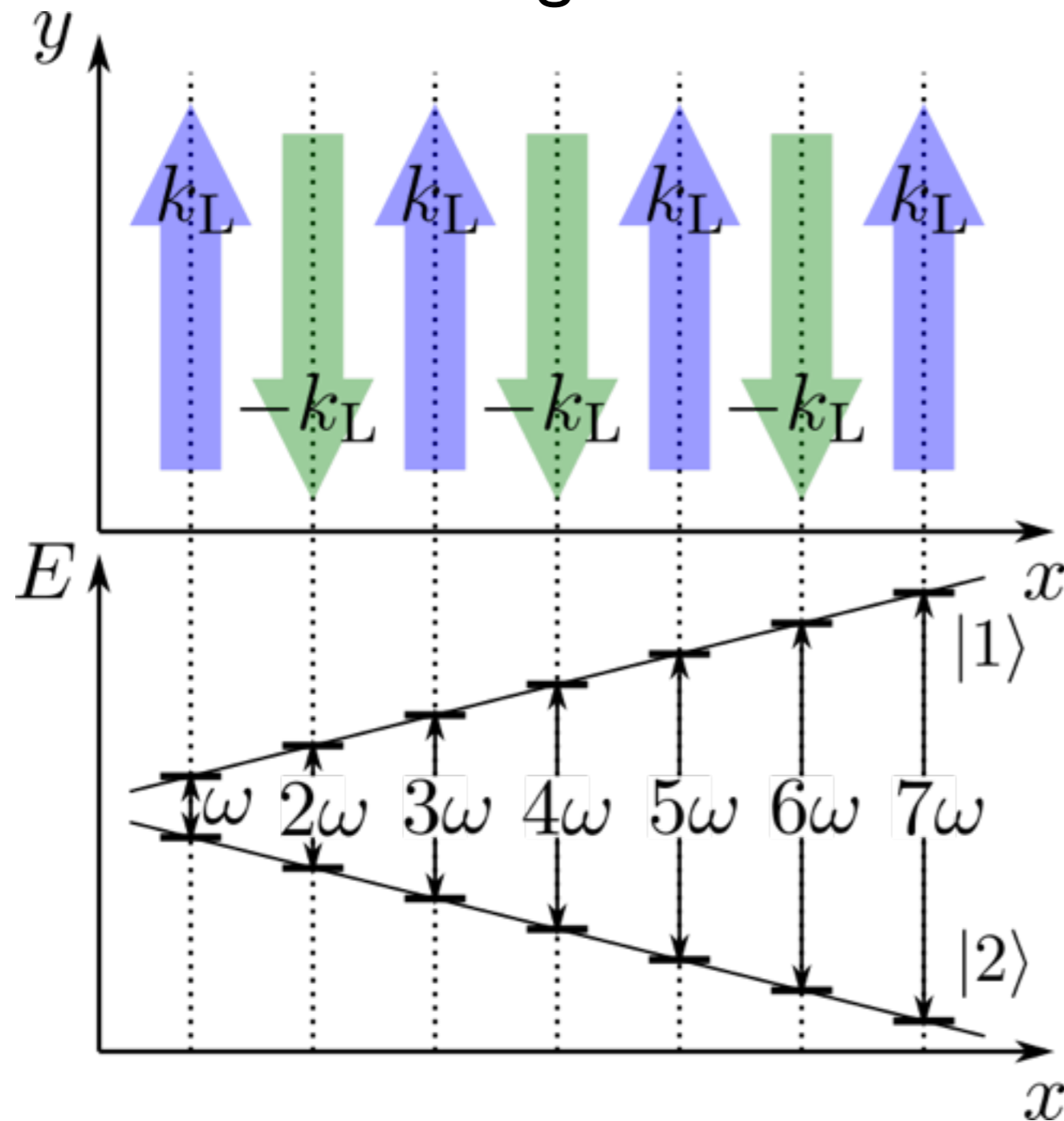


(Even frequencies)

(Odd frequencies)

Magnetic flux without usual optical lattices

- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Raman transitions with **odd** and **even** frequencies address atoms at alternating x_n with alternating **recoil** (along y or $-y$)




(Even frequencies)

(Odd frequencies)

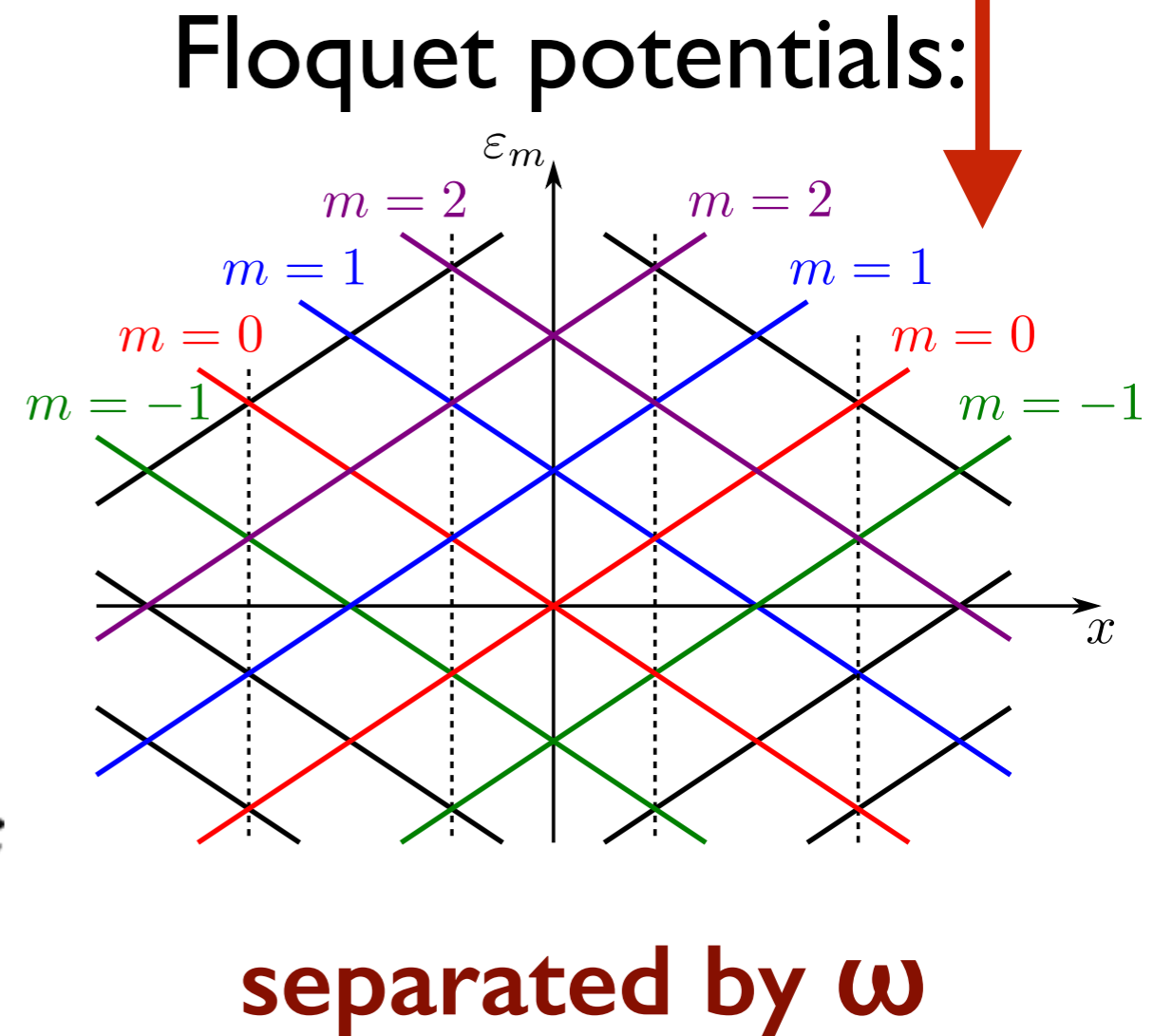
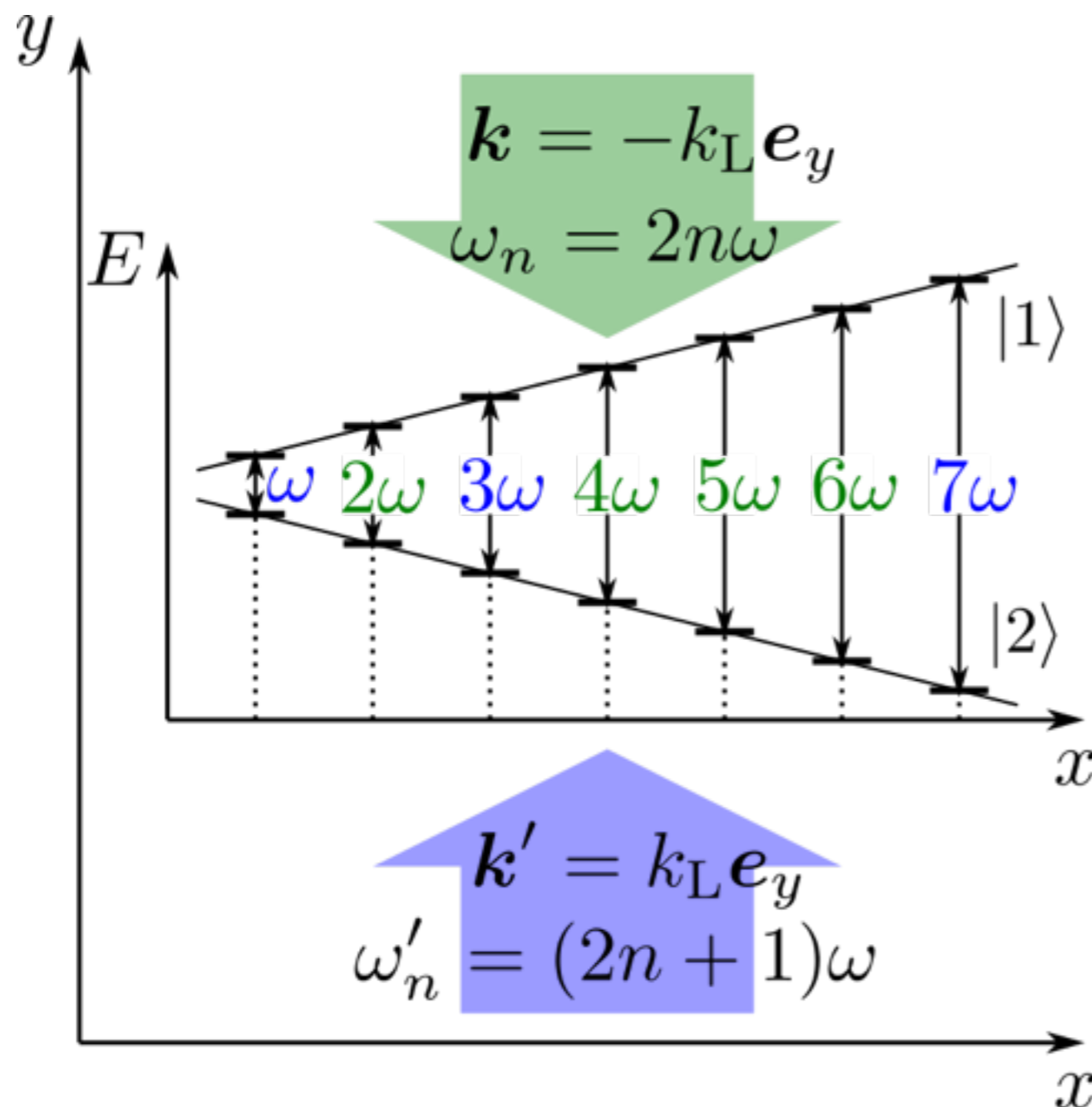
Imitates the
Lorentz force

Optical lattice &
non-staggered magn. flux


Magnetic flux without usual optical lattices

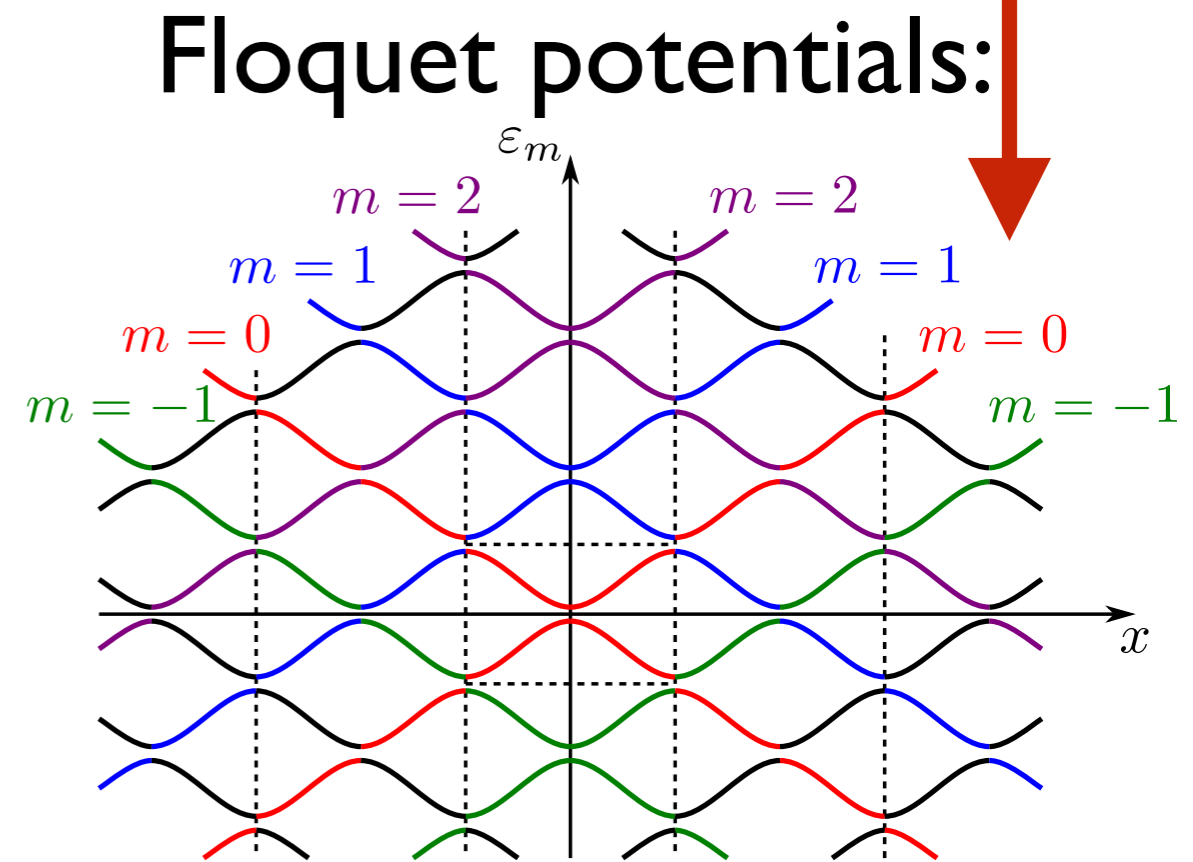
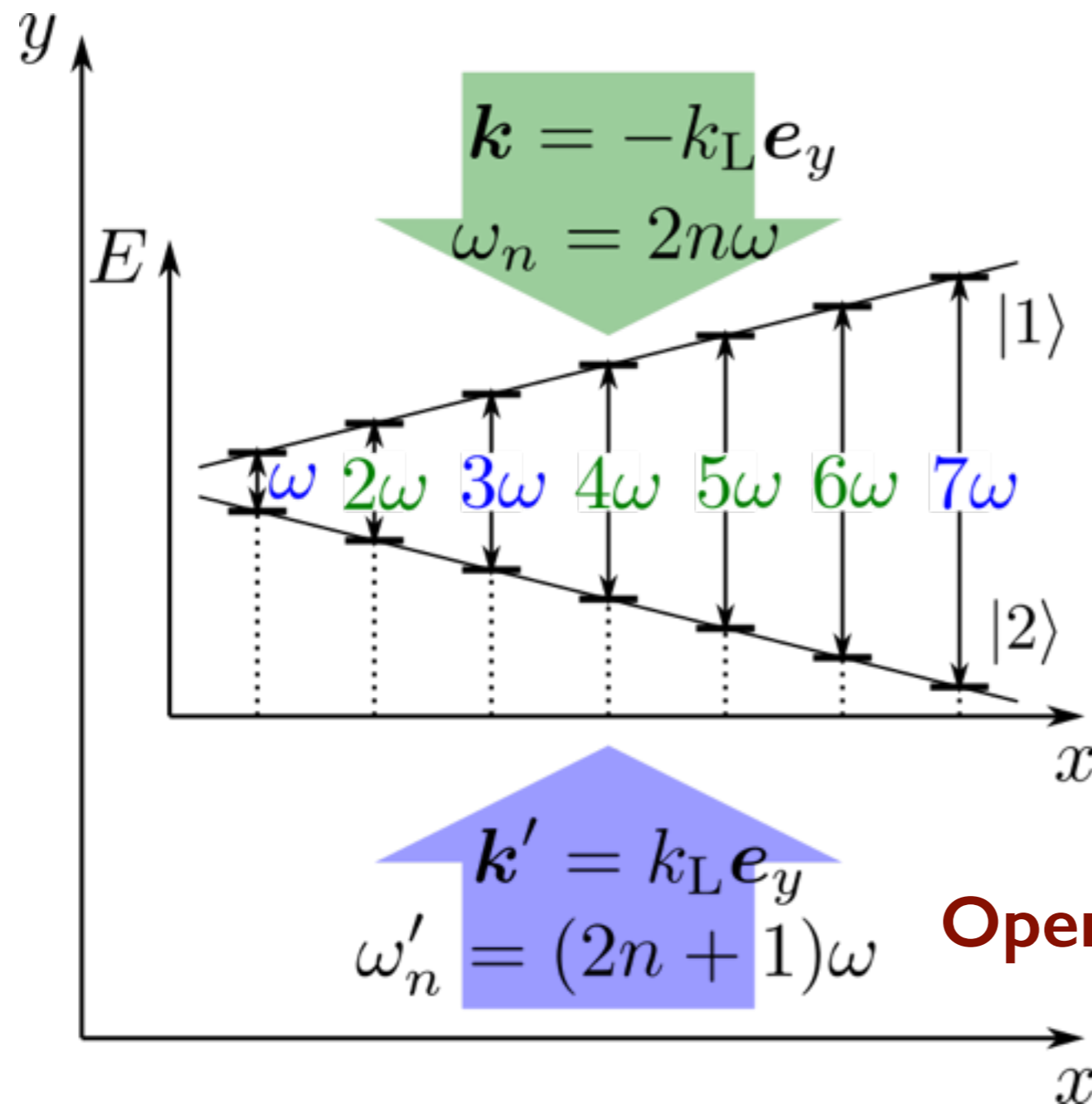
- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states

- Frequency-comb Raman coupling - periodic driving at ω




Magnetic flux without usual optical lattices

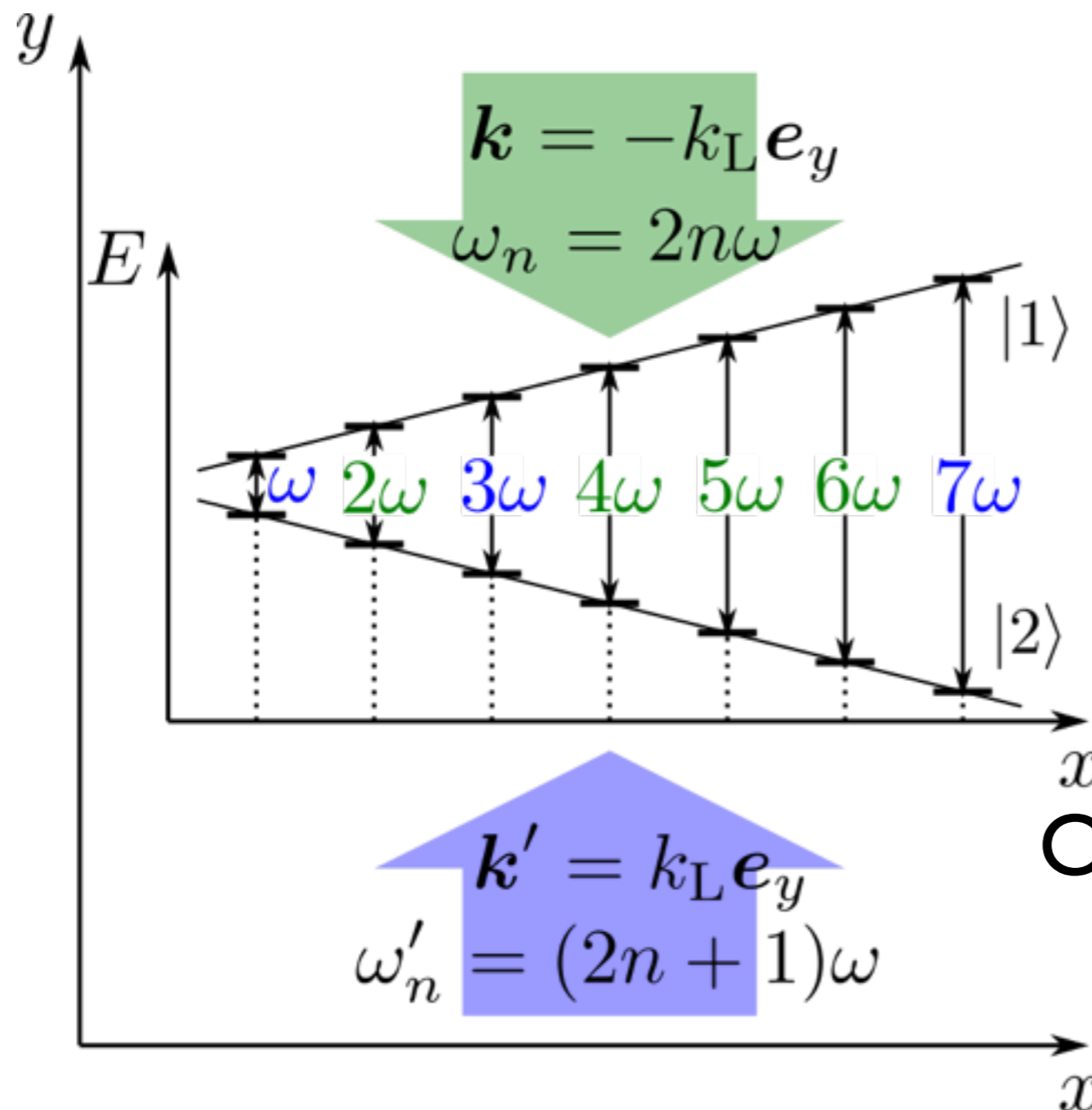
- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)



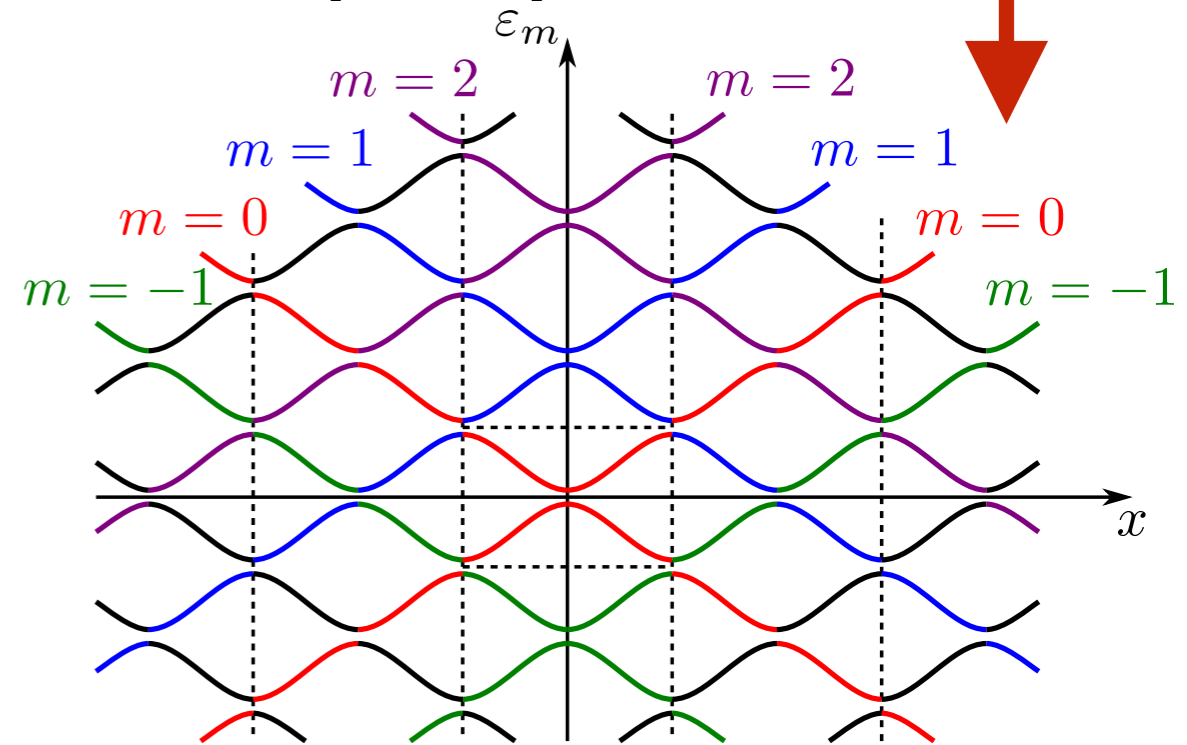
Opens gaps in the Floquet potentials (in a topologically non-trivial way)

Magnetic flux without usual optical lattices

- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)




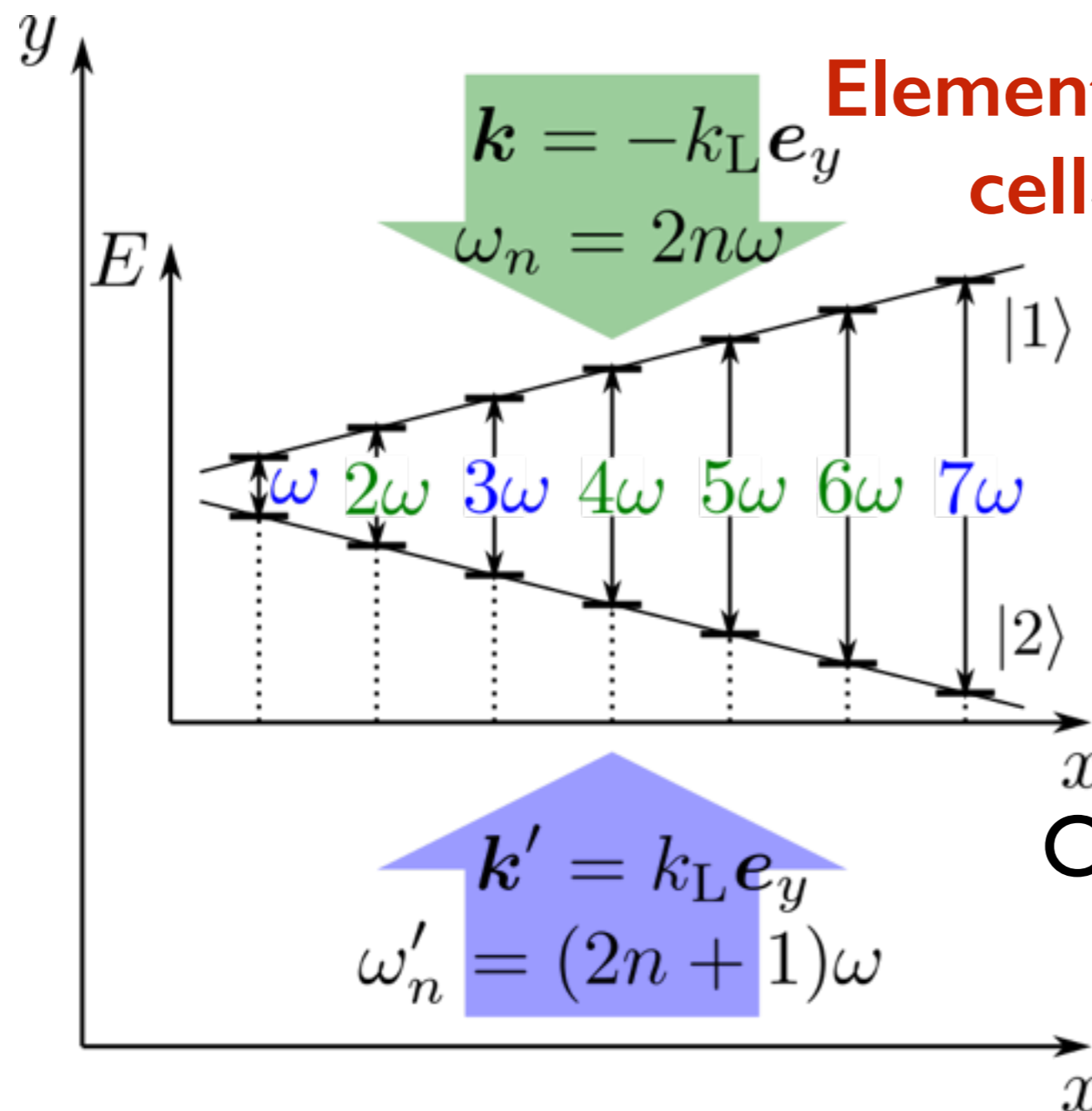
Floquet potentials:



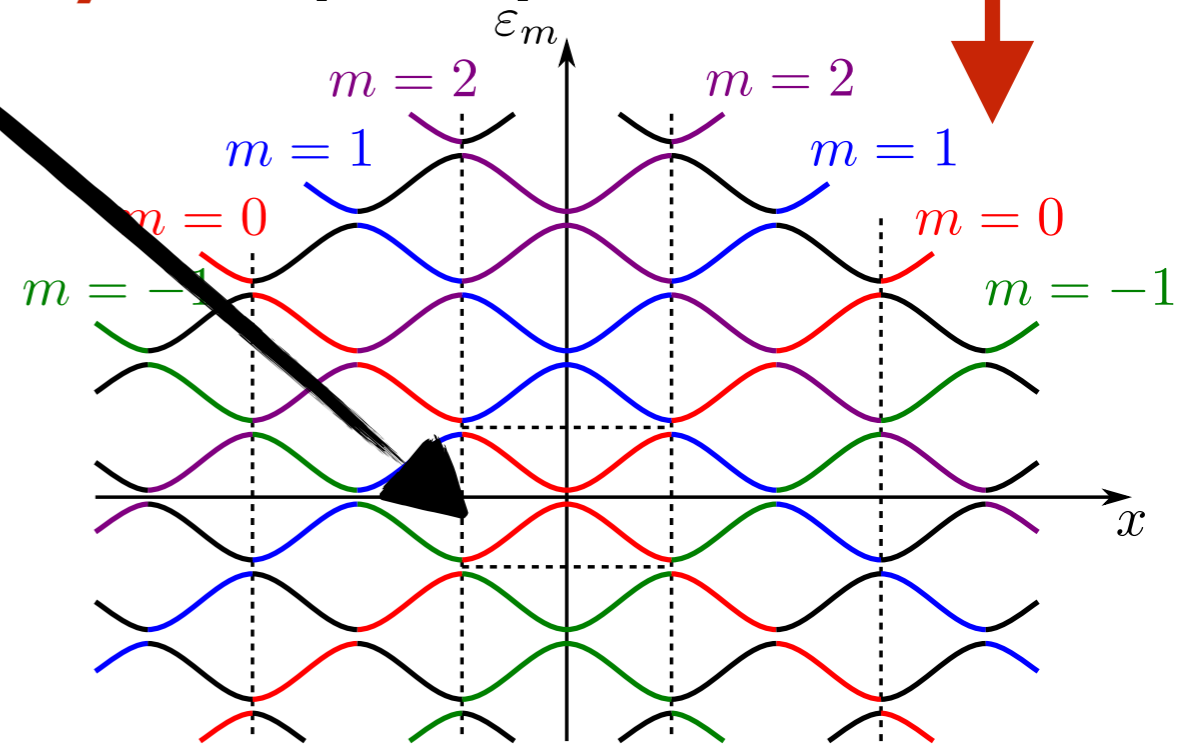
Optical lattice (period. pot.) & **non-staggered magn. flux for adiabatic motion**

Magnetic flux without usual optical lattices

- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)




Elementary Floquet potentials:
cell

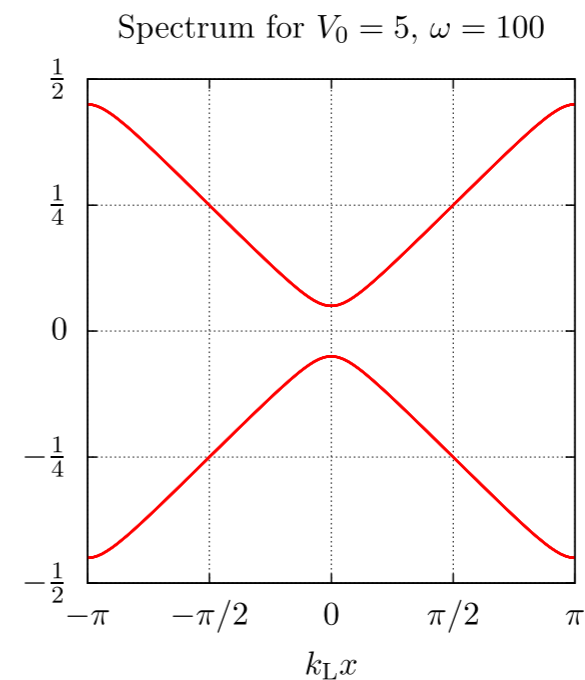
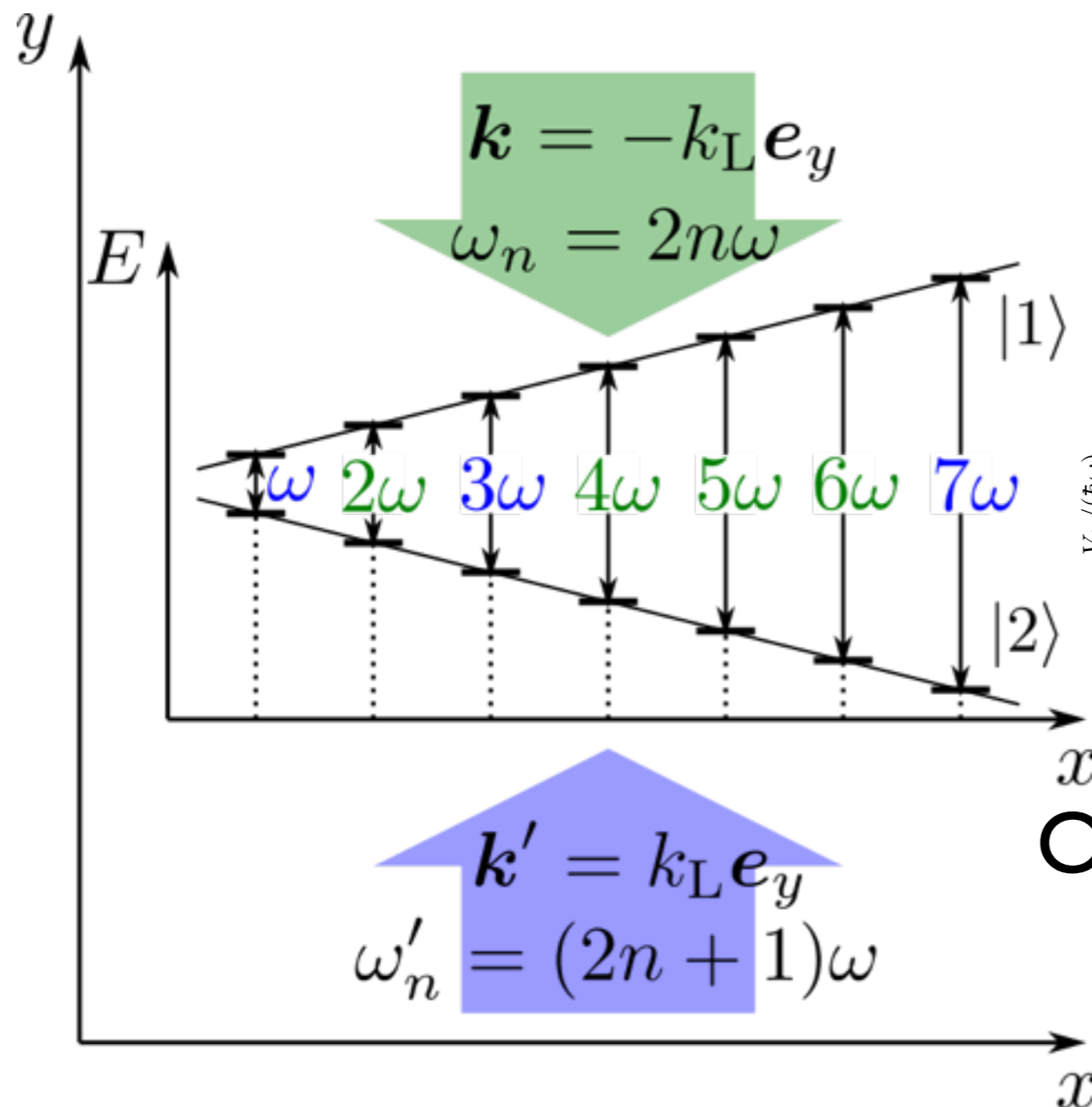


Optical lattice (period. pot.) &
non-staggered magn. flux
for adiabatic motion

Magnetic flux without usual optical lattices

- Magnetic field gradient along the x axis: 
- Position-dependent detuning between the spin up and down states
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)

Floquet potentials:



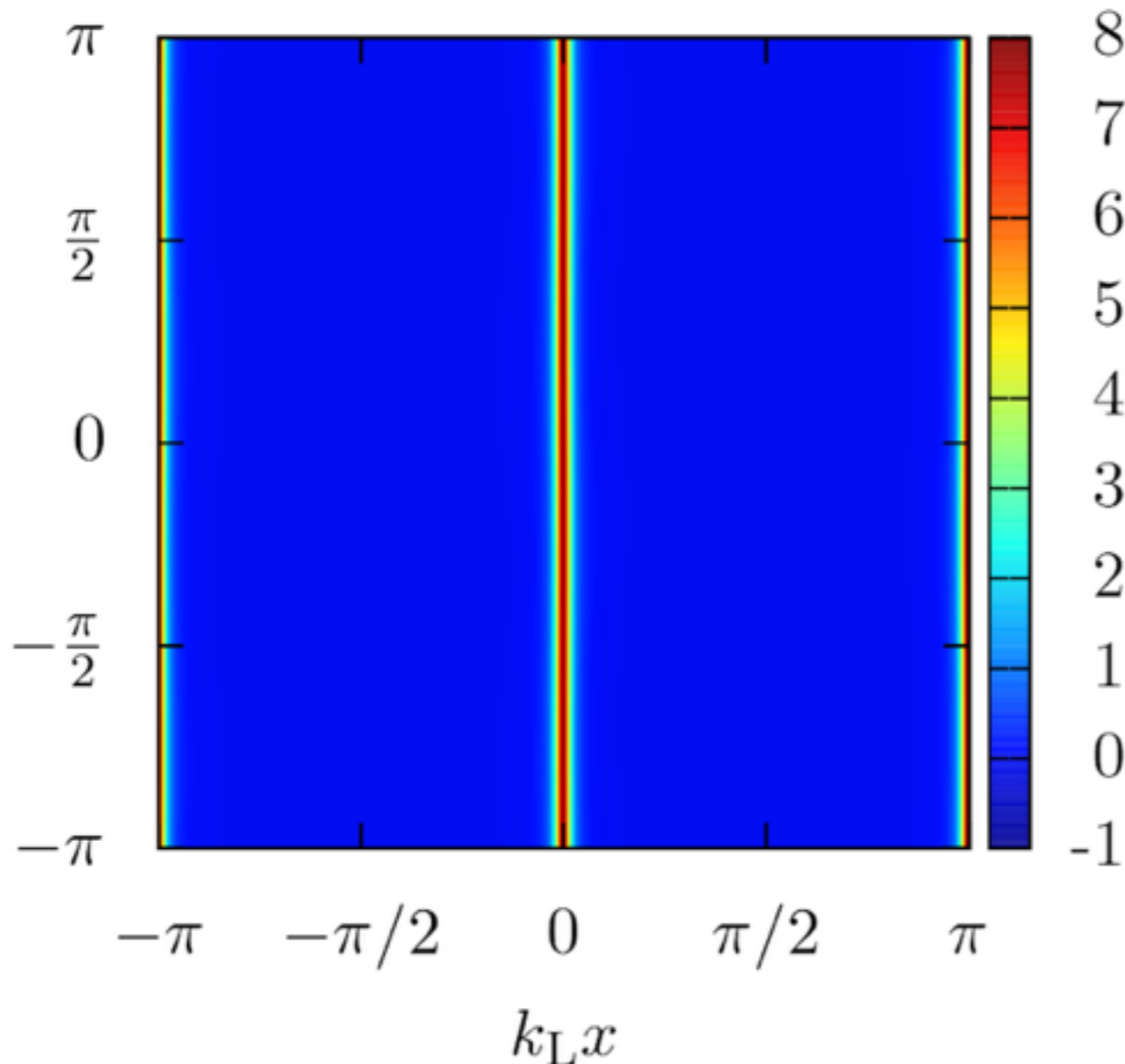
Elementary cell:
Adiabatic potential

Optical lattice (period. pot.) & **non-staggered magn. flux** for adiabatic motion

Very weak Raman coupling

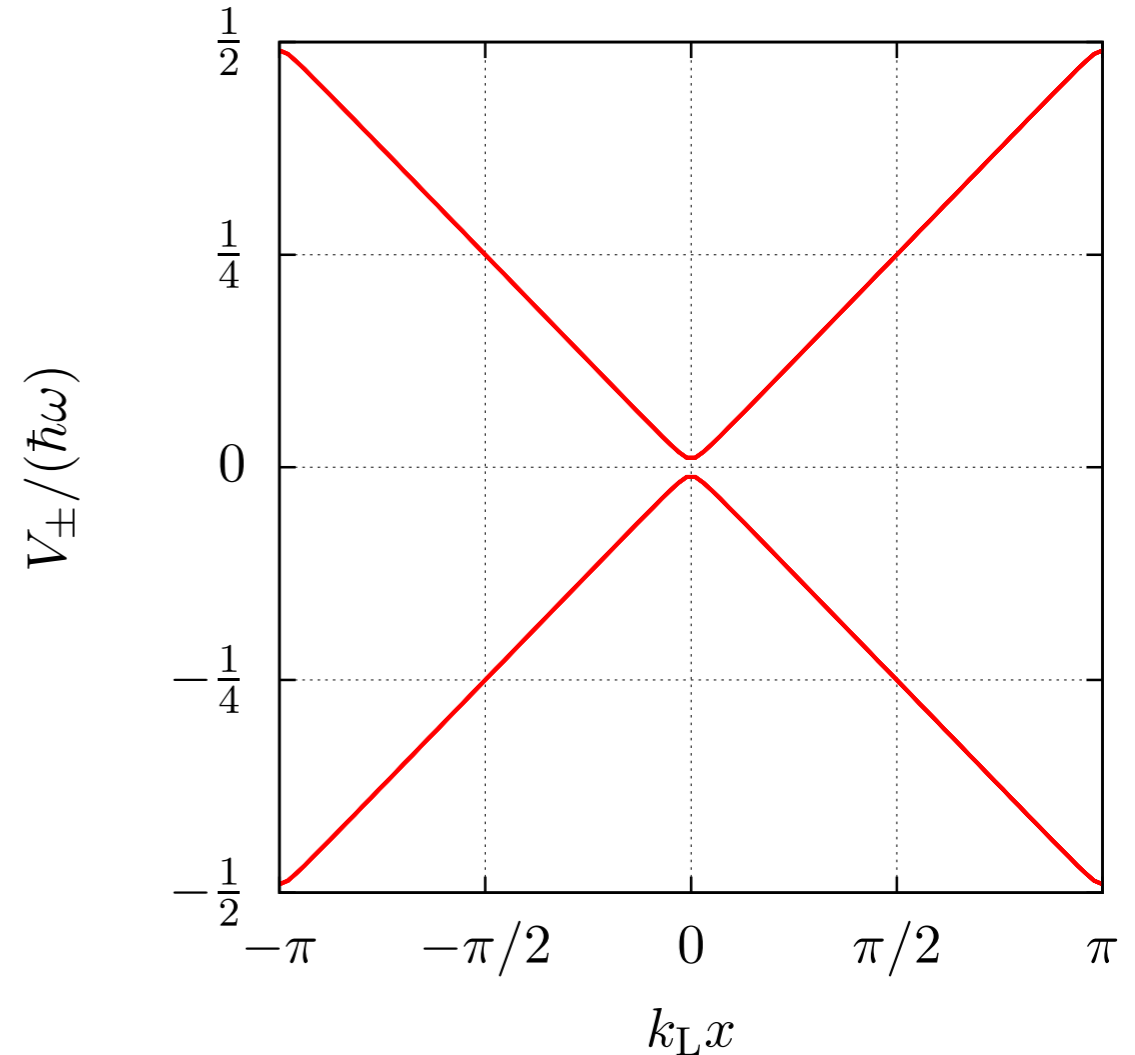
Magnetic flux in elementary cell:

$$B_z(x, y) \text{ for } V_0 = 1, \omega = 100$$



Adiabatic potential in elementary cell:

$$\text{Spectrum for } V_0 = 1, \omega = 100$$



Adiabatic potential:

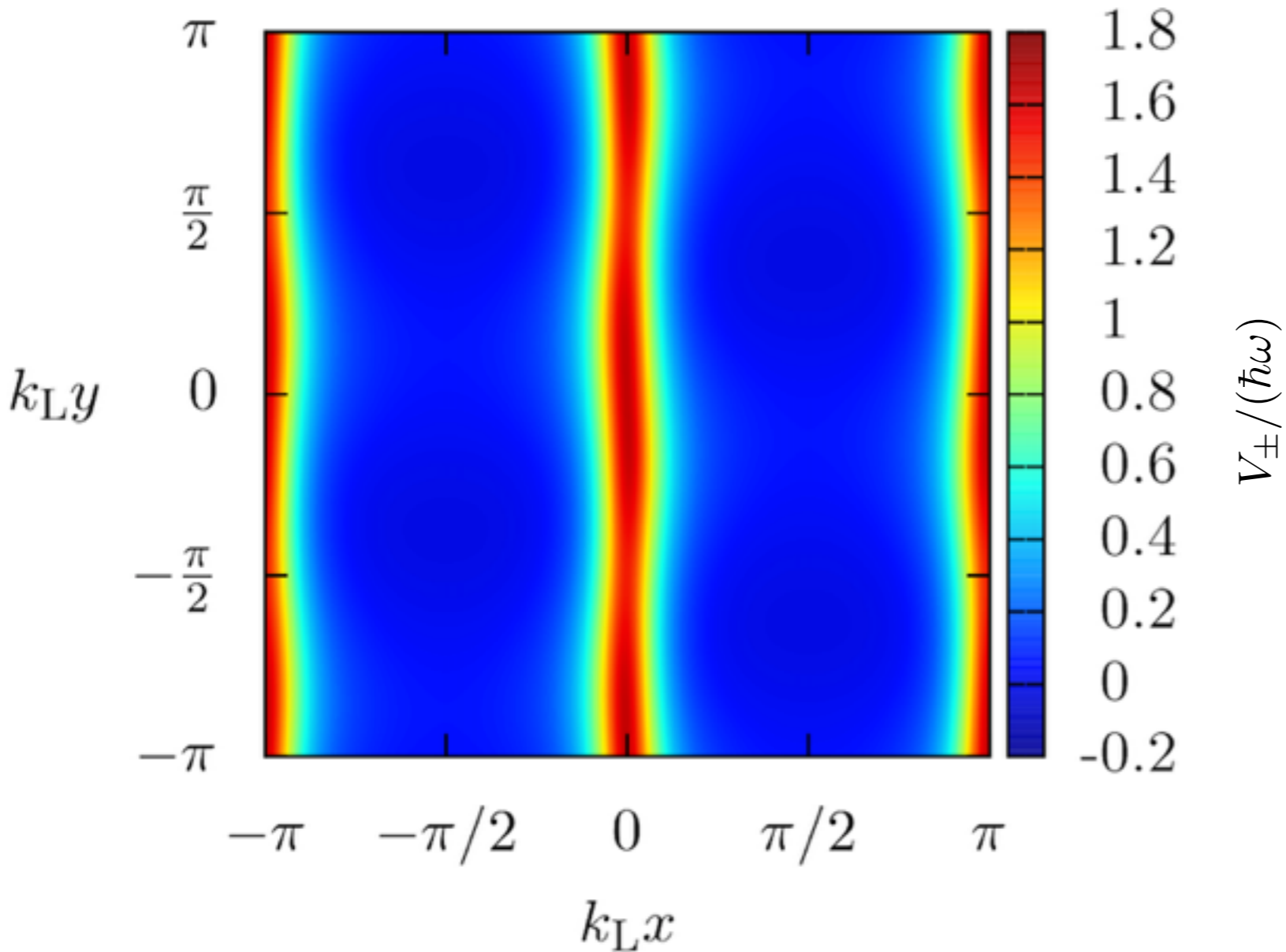
Very narrow gap

**Magnetic flux: Very narrow stripes at
Floquet band intersections: No y dependence,**

A little stronger Raman coupling

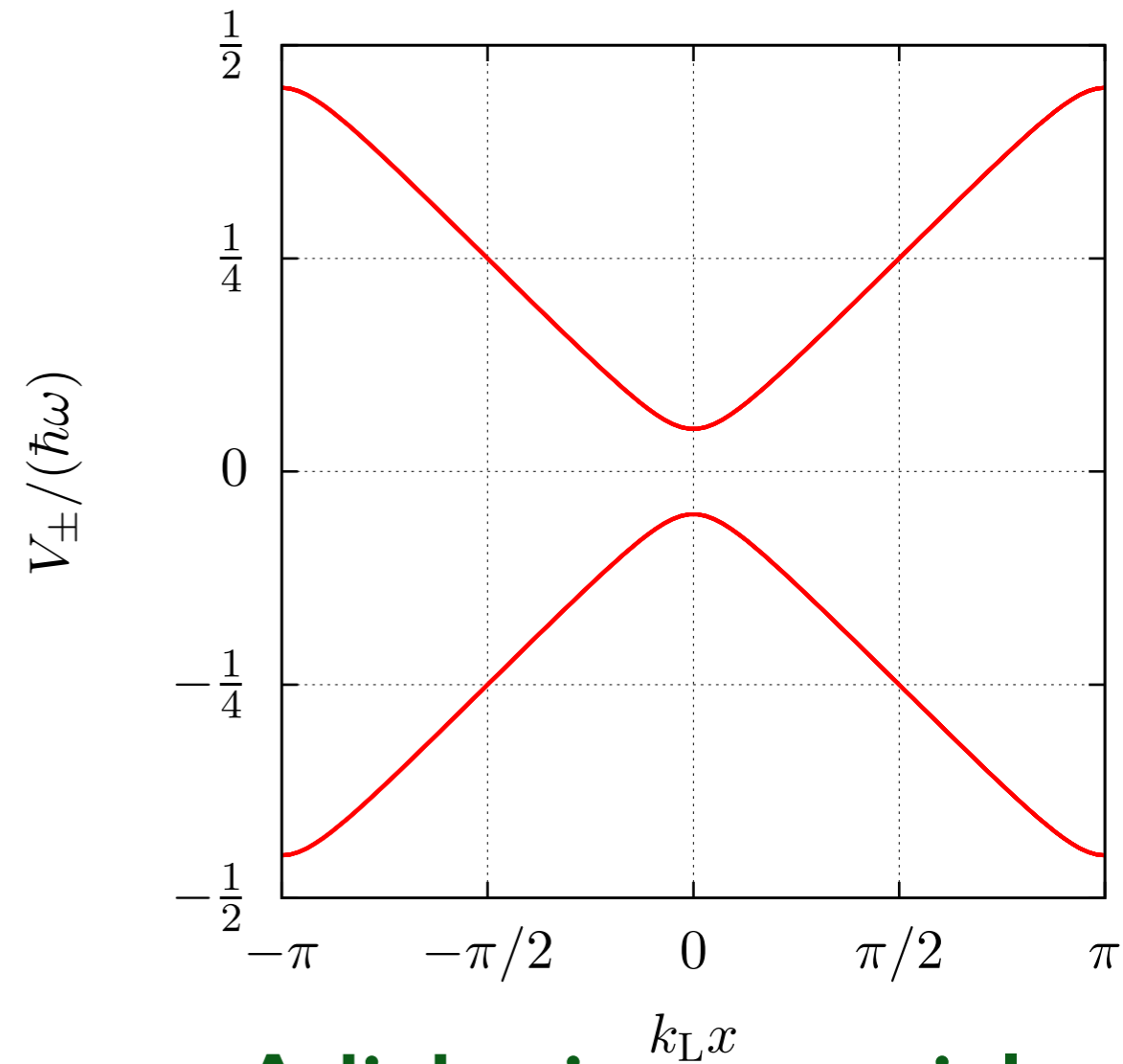
Magnetic flux in elementary cell:

$$B_z(x, y) \text{ for } V_0 = 5, \omega = 100$$



Adiabatic potential in elementary cell:

$$\text{Spectrum for } V_0 = 5, \omega = 100$$



Magnetic flux: Slightly broader stripes

at Floquet band intersections: Small y dependence

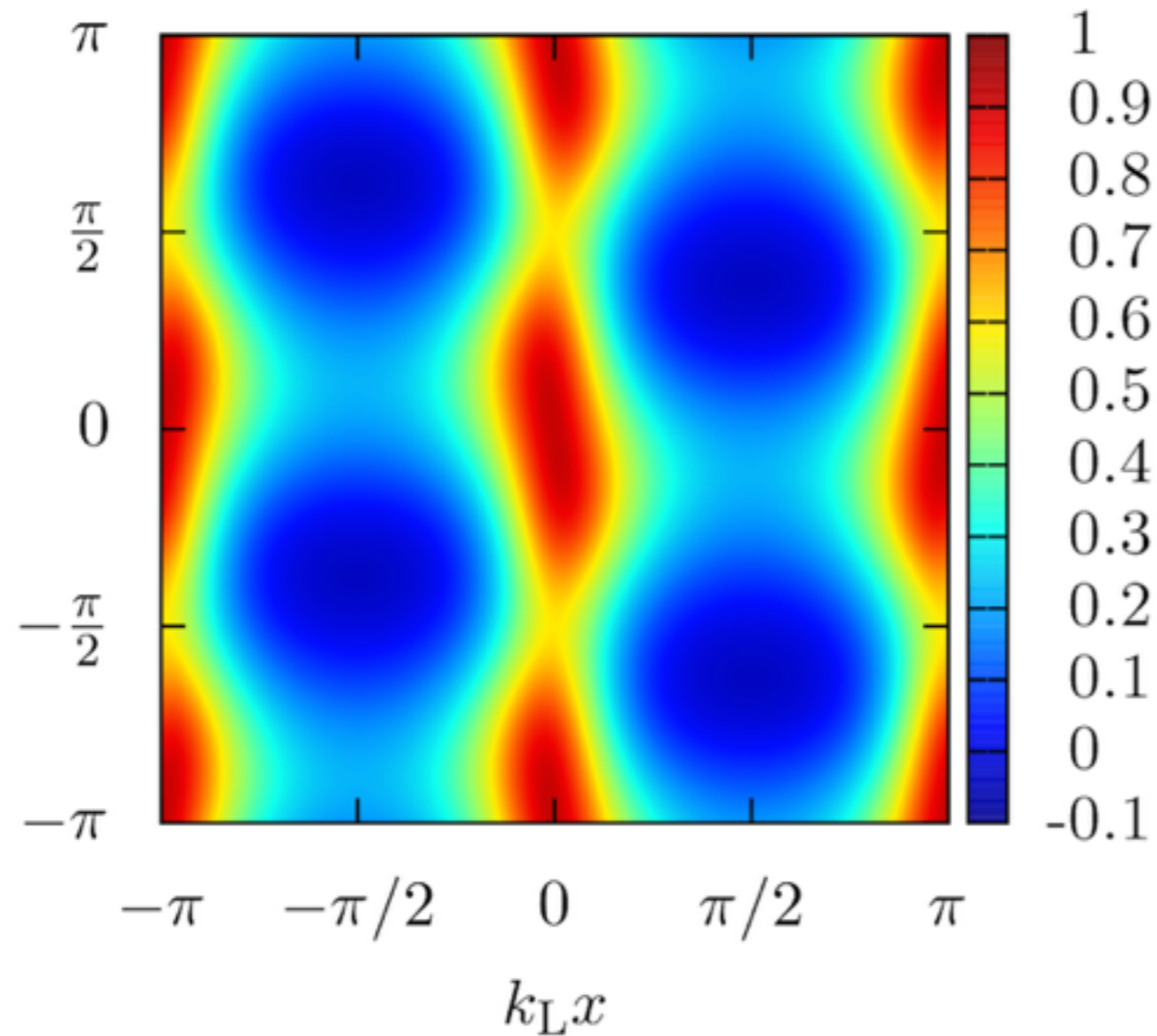
Adiabatic potential:

Slightly wider gap

Stronger Raman coupling

Magnetic flux in elementary cell:

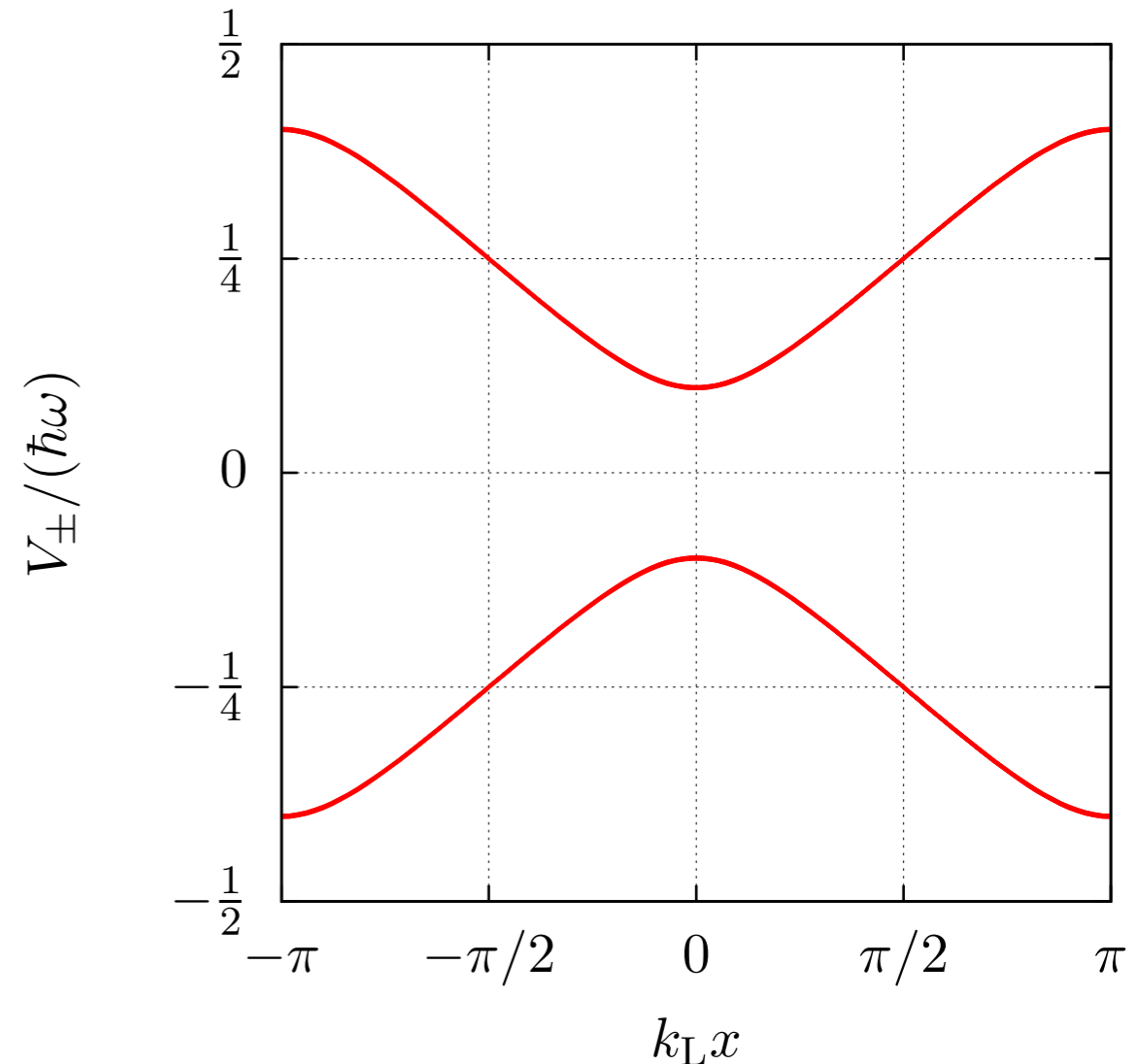
$B_z(x, y)$ for $V_0 = 10$, $\omega = 100$



Magnetic flux: Broader stripes at Floquet band intersections; obvious y dependence

Adiabatic potential in elementary cell:

Spectrum for $V_0 = 10$, $\omega = 100$

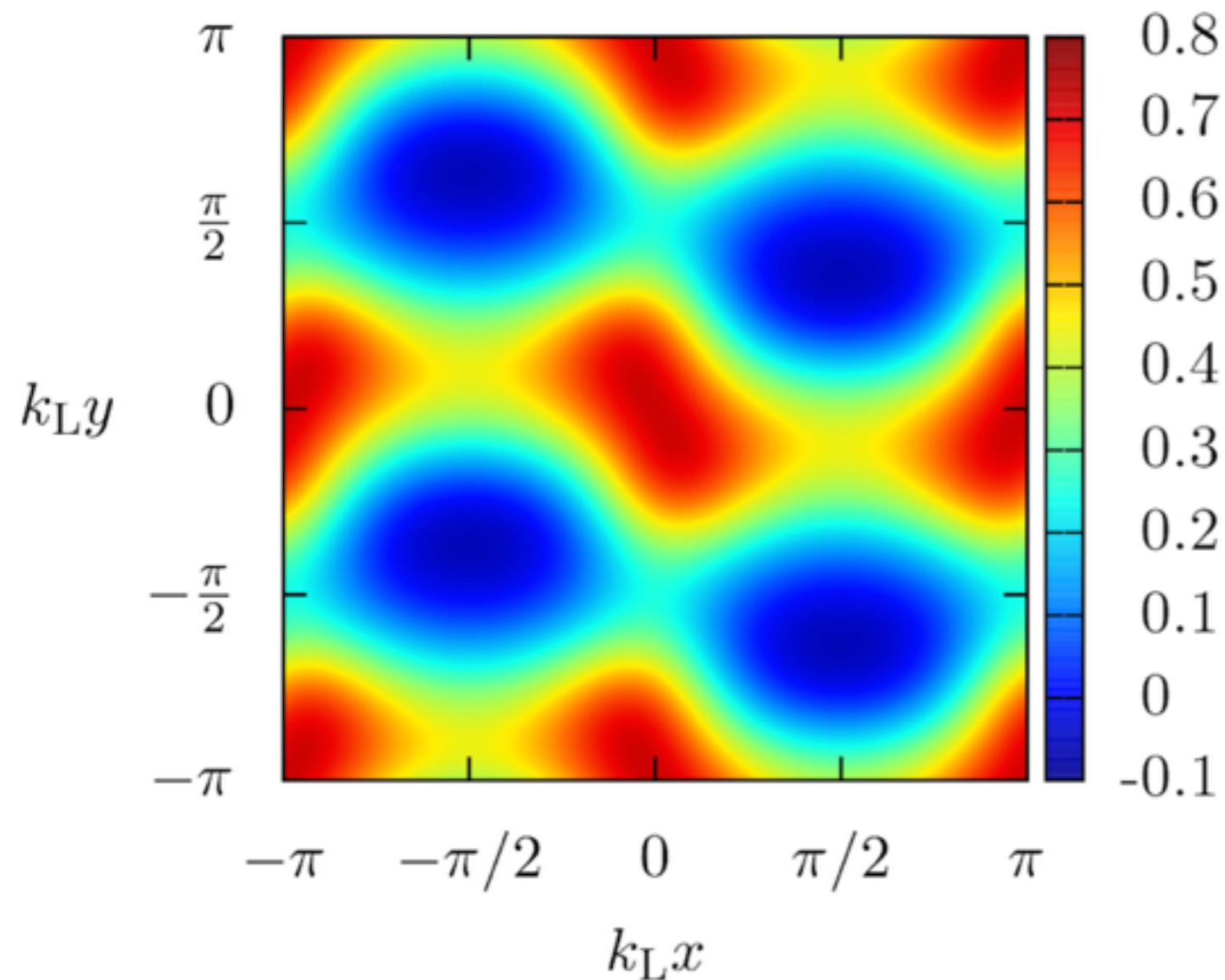


**Adiabatic potential:
Wider gap**

Even stronger Raman coupling

Magnetic flux in elementary cell:

$B_z(x, y)$ for $V_0 = 15$, $\omega = 100$

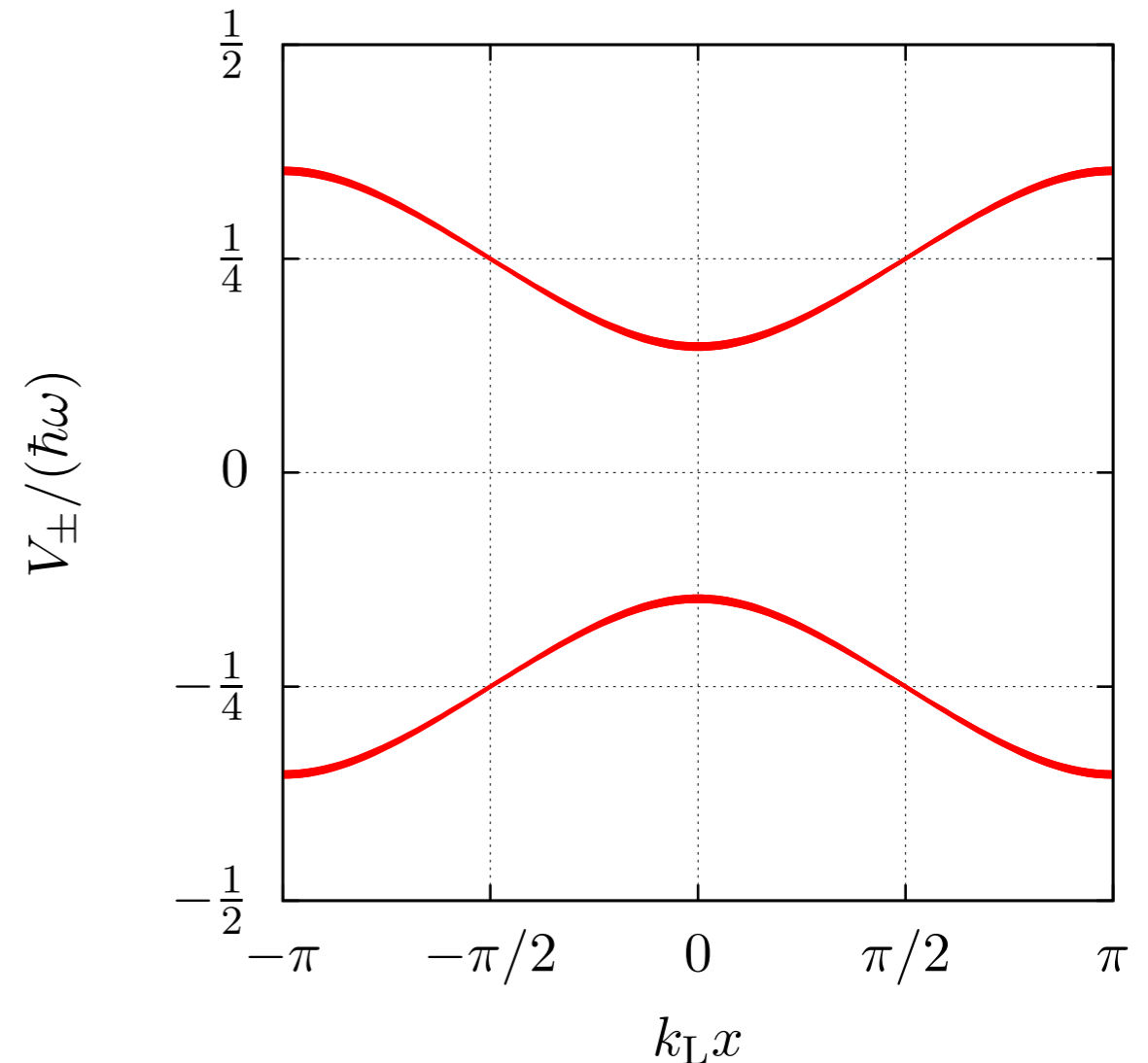


Magnetic flux: Broad magnetic flux

beyond intersections; Significant y dependence

Adiabatic potential in elementary cell:

Spectrum for $V_0 = 15$, $\omega = 100$



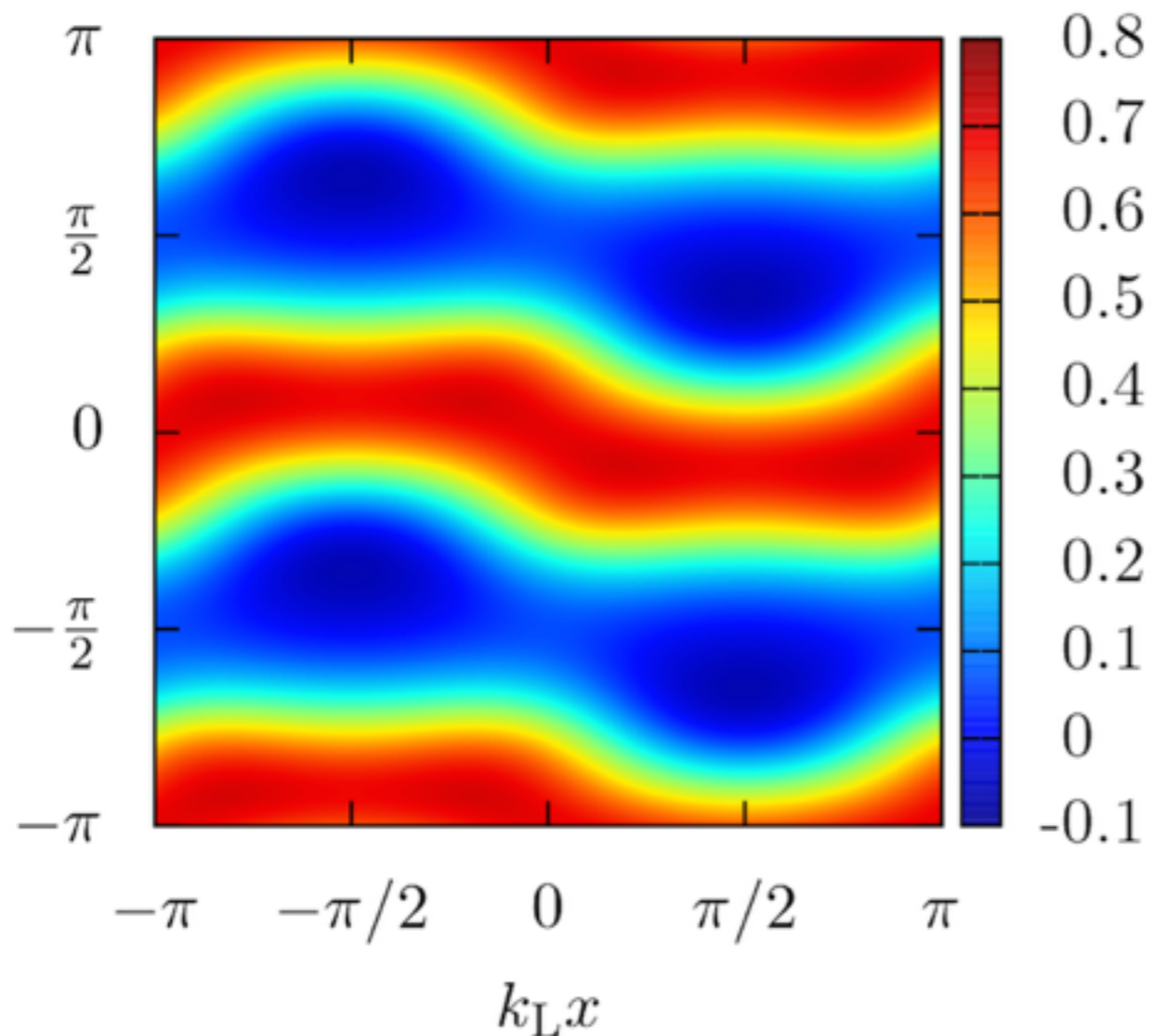
Adiabatic potential:

Becomes flatter

Very strong Raman coupling

Magnetic flux in elementary cell:

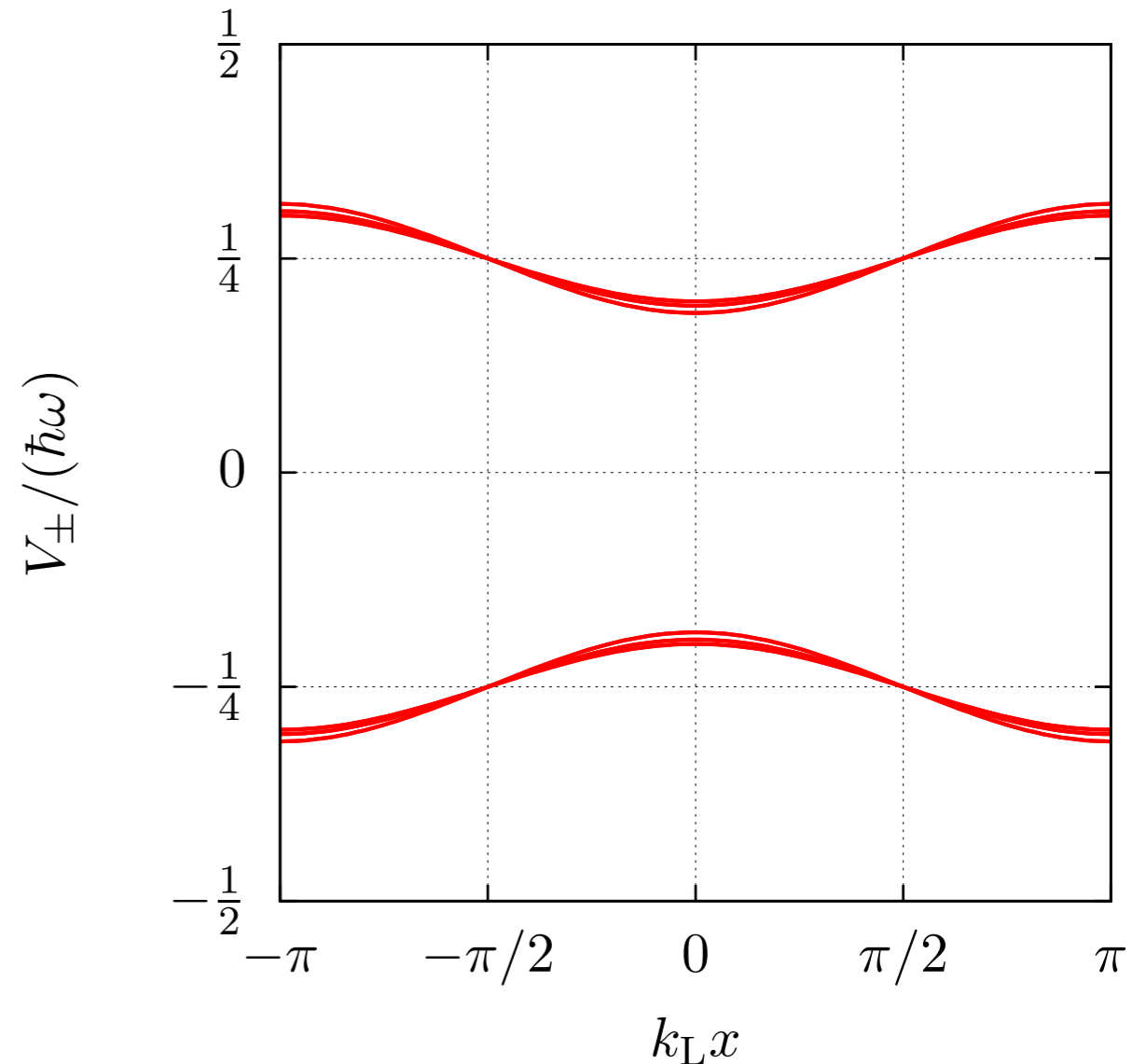
$B_z(x, y)$ for $V_0 = 20$, $\omega = 100$



Magnetic flux: Significant changes in the magnetic flux, reversing the stripes.

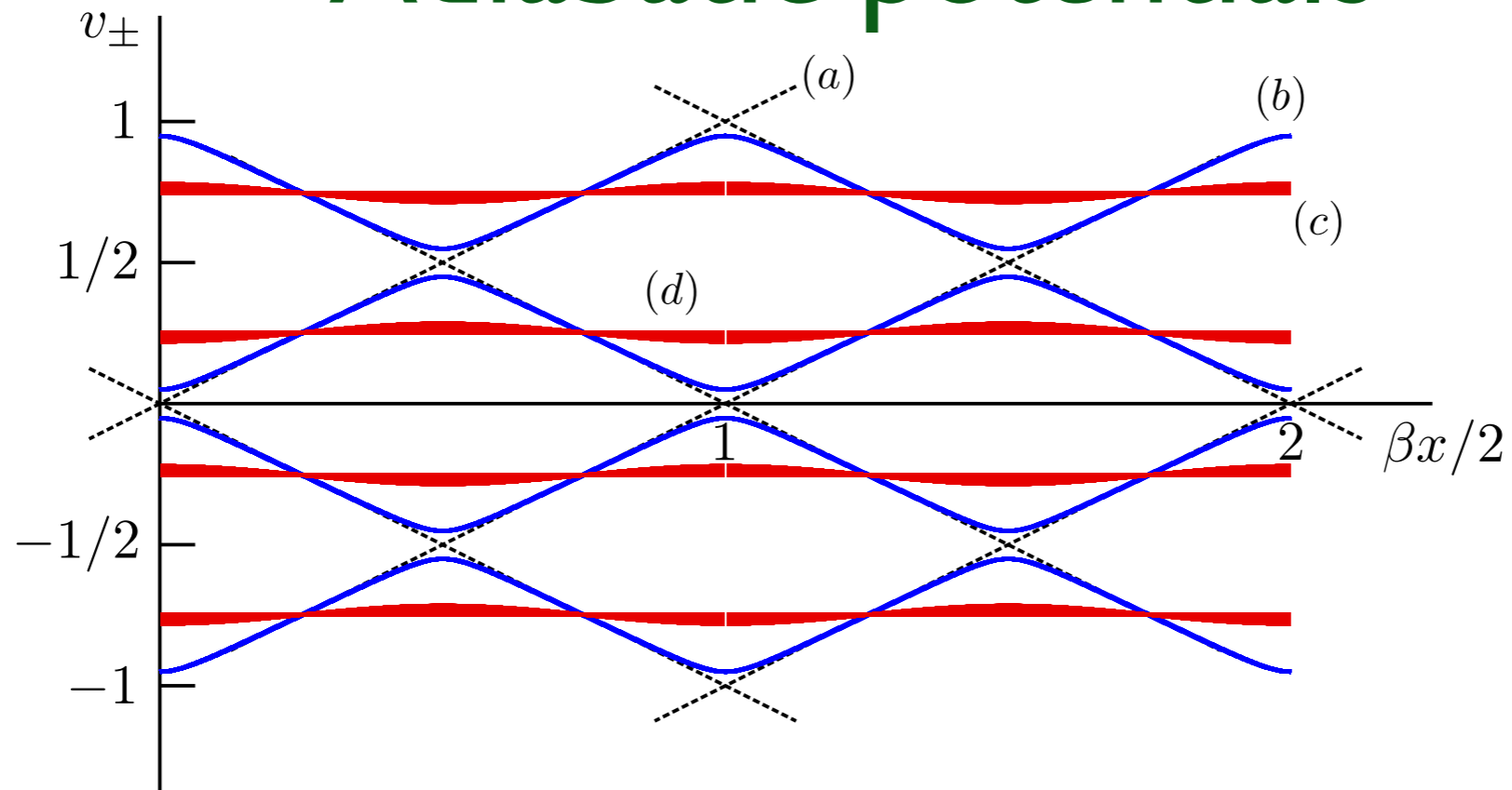
Adiabatic potential in elementary cell:

Spectrum for $V_0 = 20$, $\omega = 100$



Adiabatic potential:
Quite flat, some y dependence

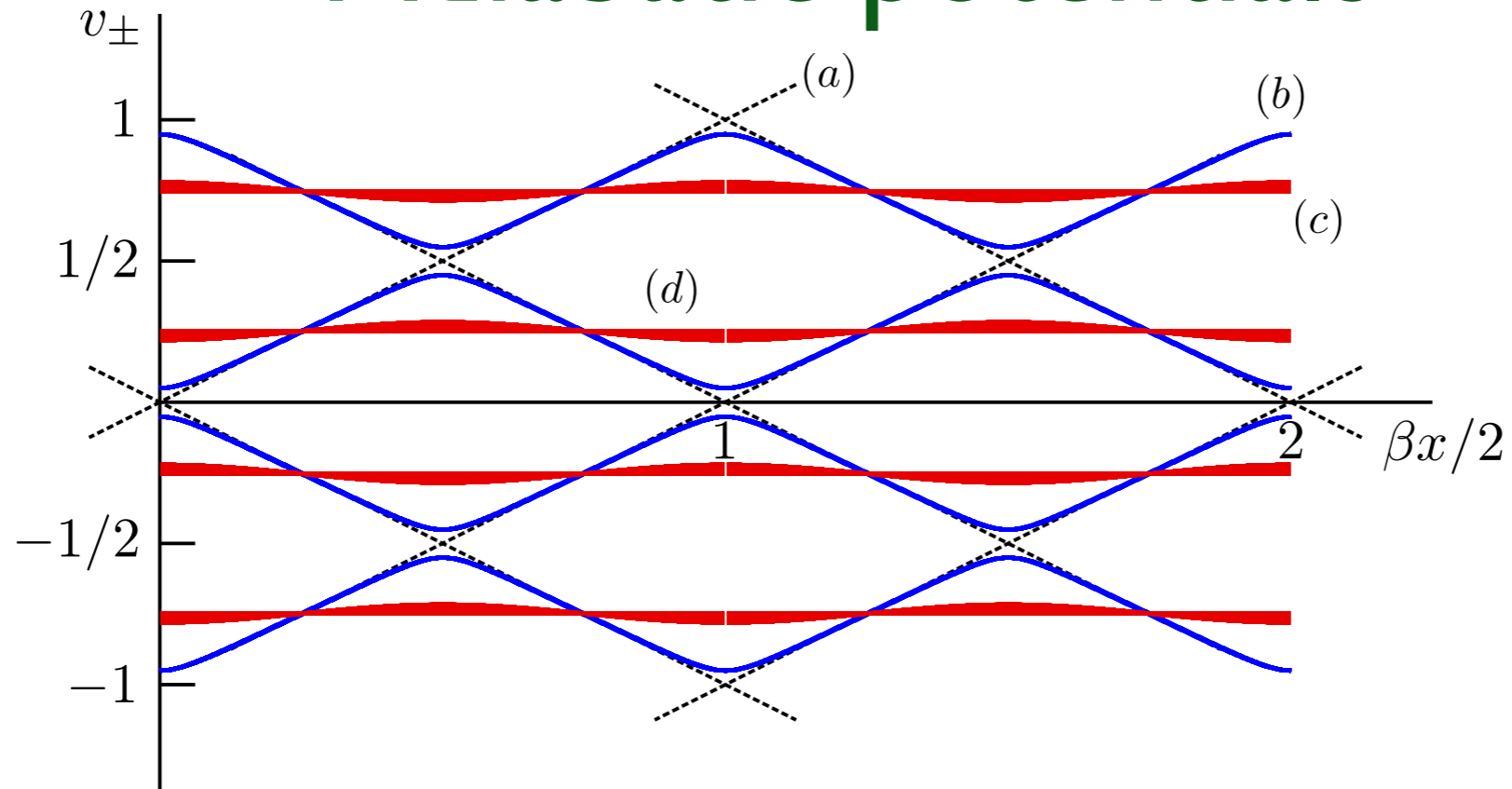
Adiabatic potentials



(Weak y
dependence
of adiabatic
potentials)

$V_0/\omega=0$: no coupling, $V_0/\omega=0.05$: weak coupling; $V_0/\omega=0.25$: flat adiabatic potentials

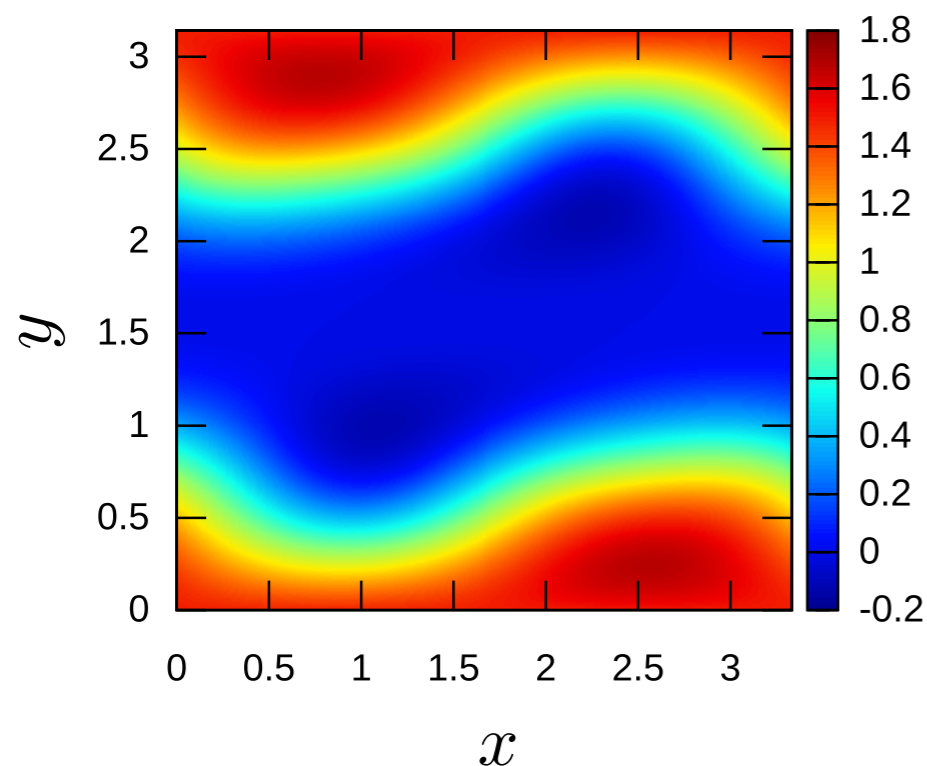
Adiabatic potentials



(Weak y dependence of adiabatic potentials)

$V_0/\omega=0$: no coupling, $V_0/\omega=0.05$: weak coupling; $V_0/\omega=0.25$: flat adiabatic potentials

Total flux = 2π

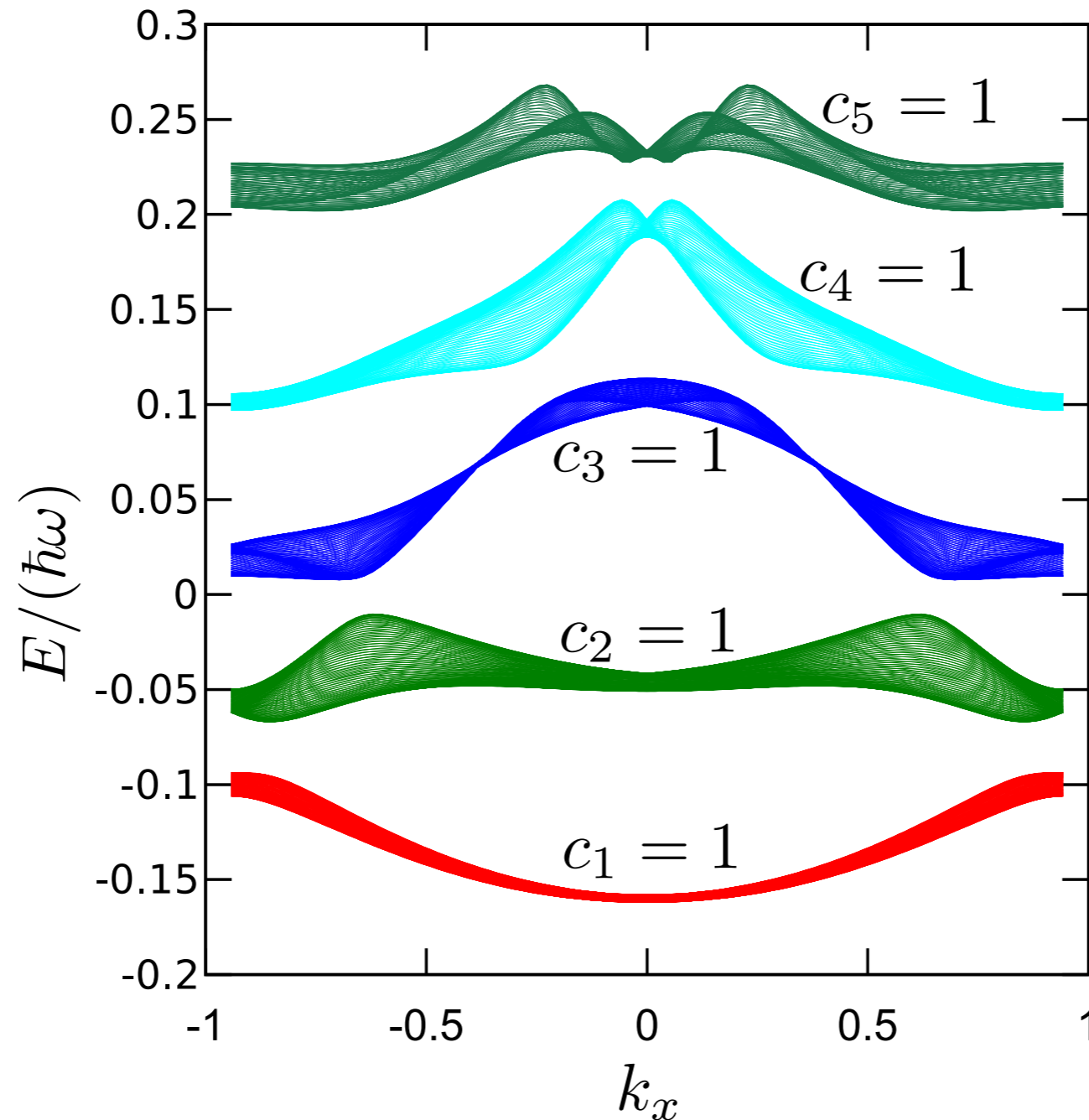


Non-staggered magnetic flux over an elementary cell
($V_0/\omega=0.25$; $\beta=0.6$)

Topology of energy bands?

Band structure and Chern numbers

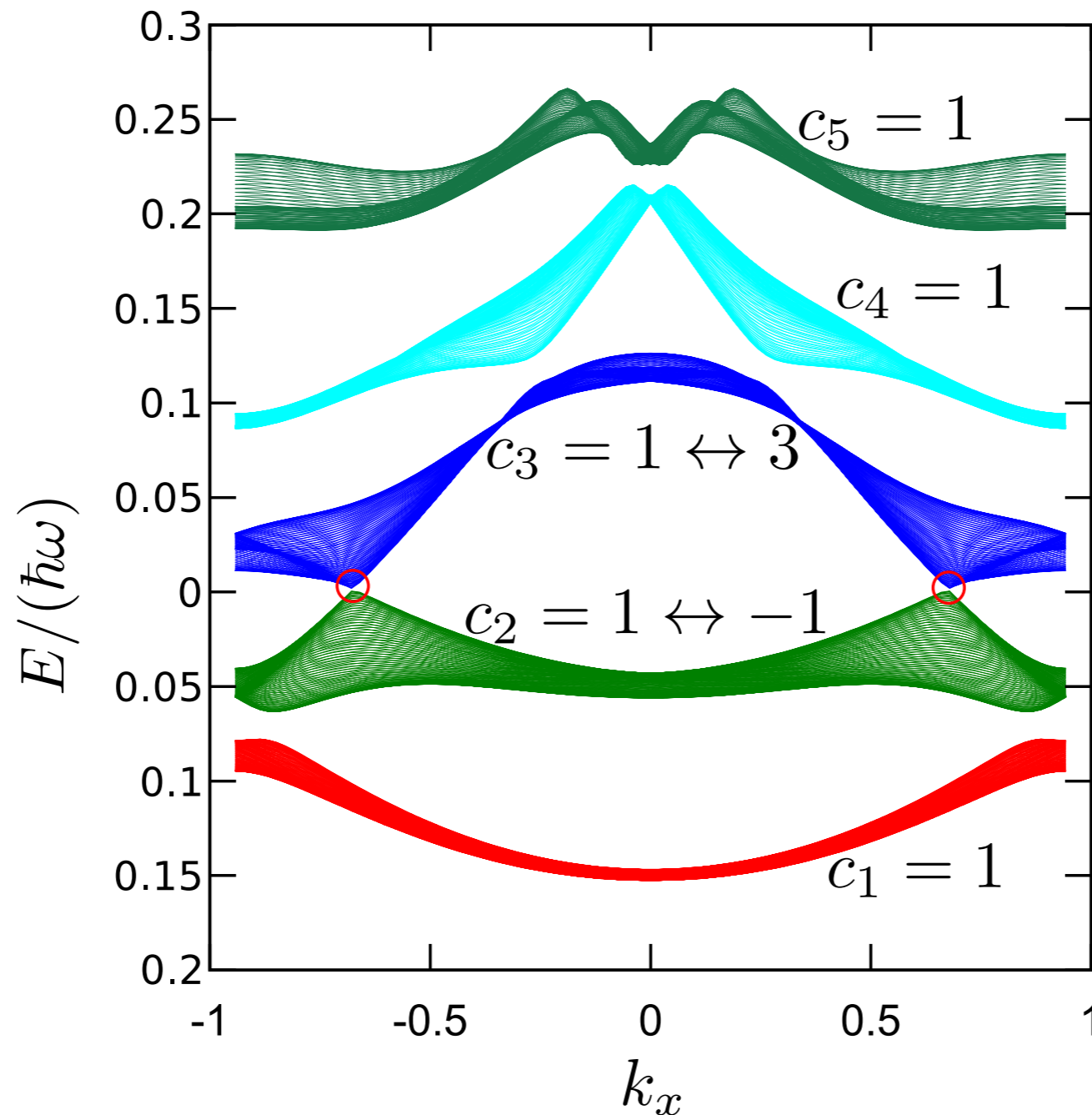
$V_0 = 0.25\omega$ (Strong coupling)



All five bands are topological with unit Chern numbers (like in the integer quantum Hall effect)

Band structure and Chern numbers

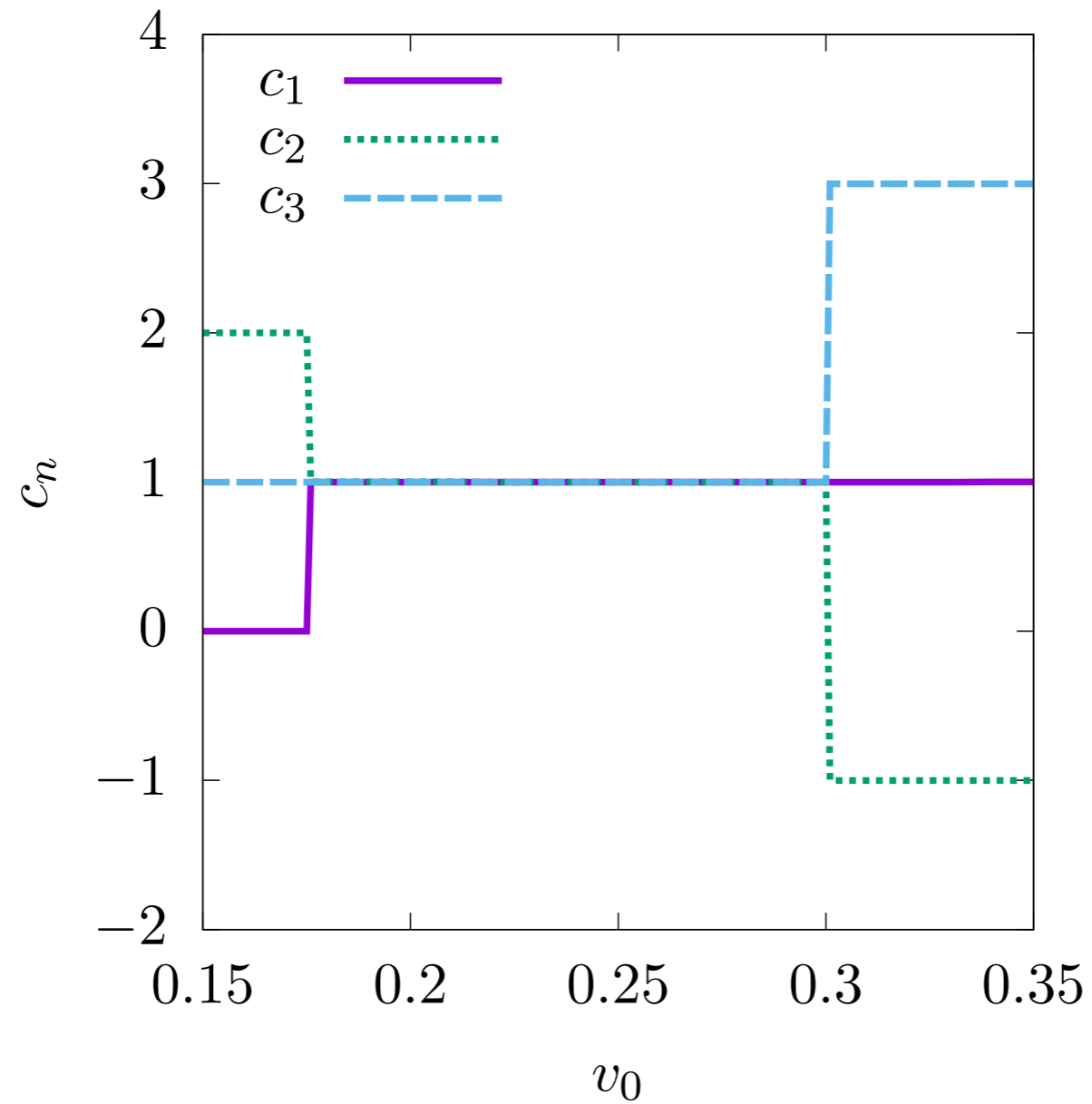
$V_0 = 0.3\omega$ (A little stronger coupling)




Band touching - topological phase transition

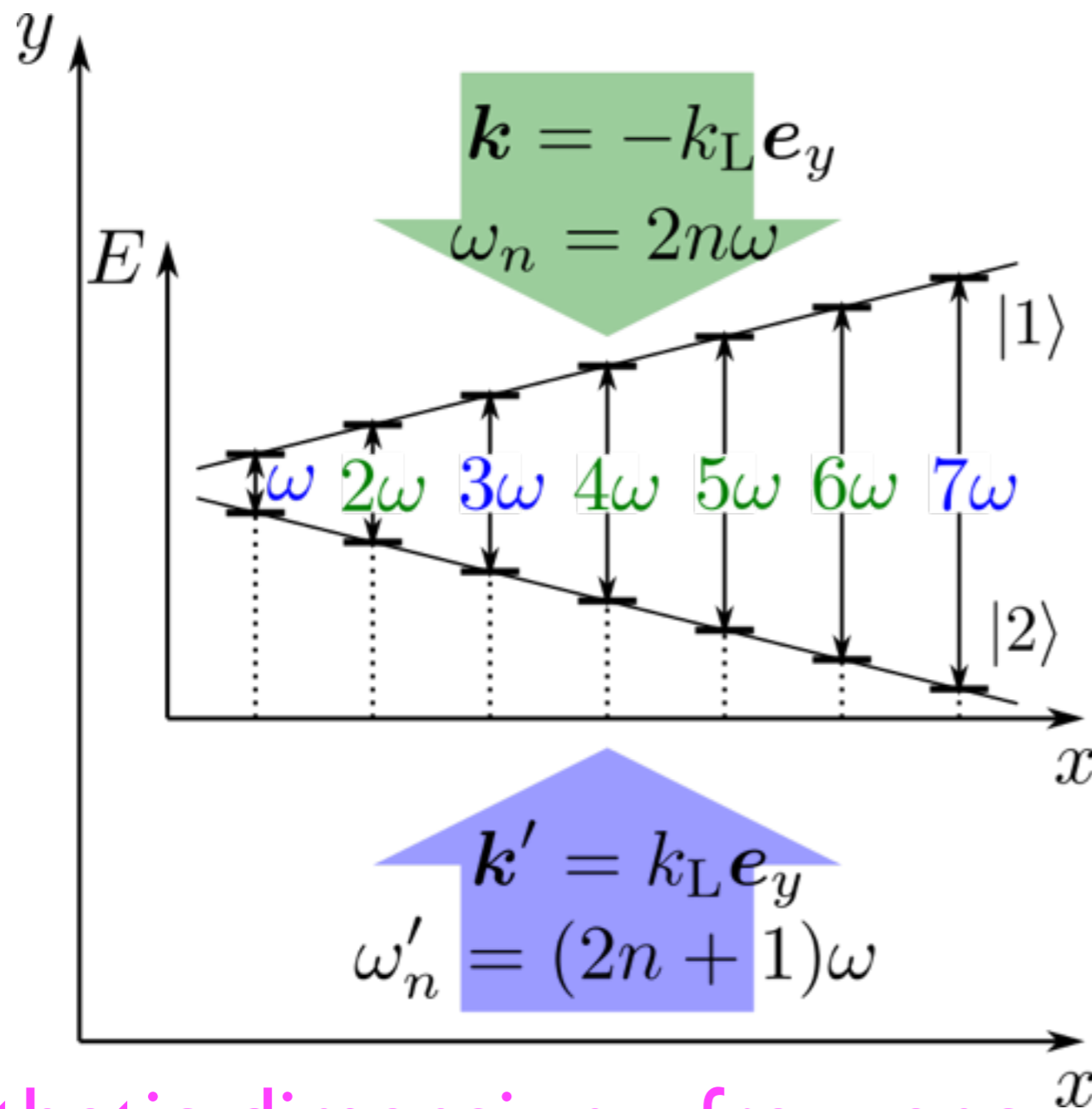
Chern numbers of the first three bands

Topological phase transitions



CONCLUSIONS (For part I):

- Magnetic field gradient along the x axis: 
- *Position-dependent detuning between the spin up and down states*
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)



Optical lattice & **non-staggered magnetic flux** for adiabatic atomic motion

Topological bands with unit Chern numbers can be formed

Synthetic dimension - frequency domain

Conclusions (for Part I)*

- Artificial magnetic field can be created combining the magnetic field gradient and the counter-propagating frequency comb radiation
- This produces a 2D lattice affected by a non-staggered magnetic flux.
- The distribution of the magnetic flux can be **controlled** by the strength of the Raman coupling
- Topological bands with unit Chern numbers can be formed (like in the integer quantum Hall effect)

* T. Andrijauskas, I.B. Spielman and G. Juzeliūnas, arXiv:1705.11101

State-dependent, addressable subwavelength lattices with cold atoms

Optical lattice is chopped

W Yi¹, A J Daley, G Pupillo and P Zoller

New Journal of Physics **10** (2008) 073015

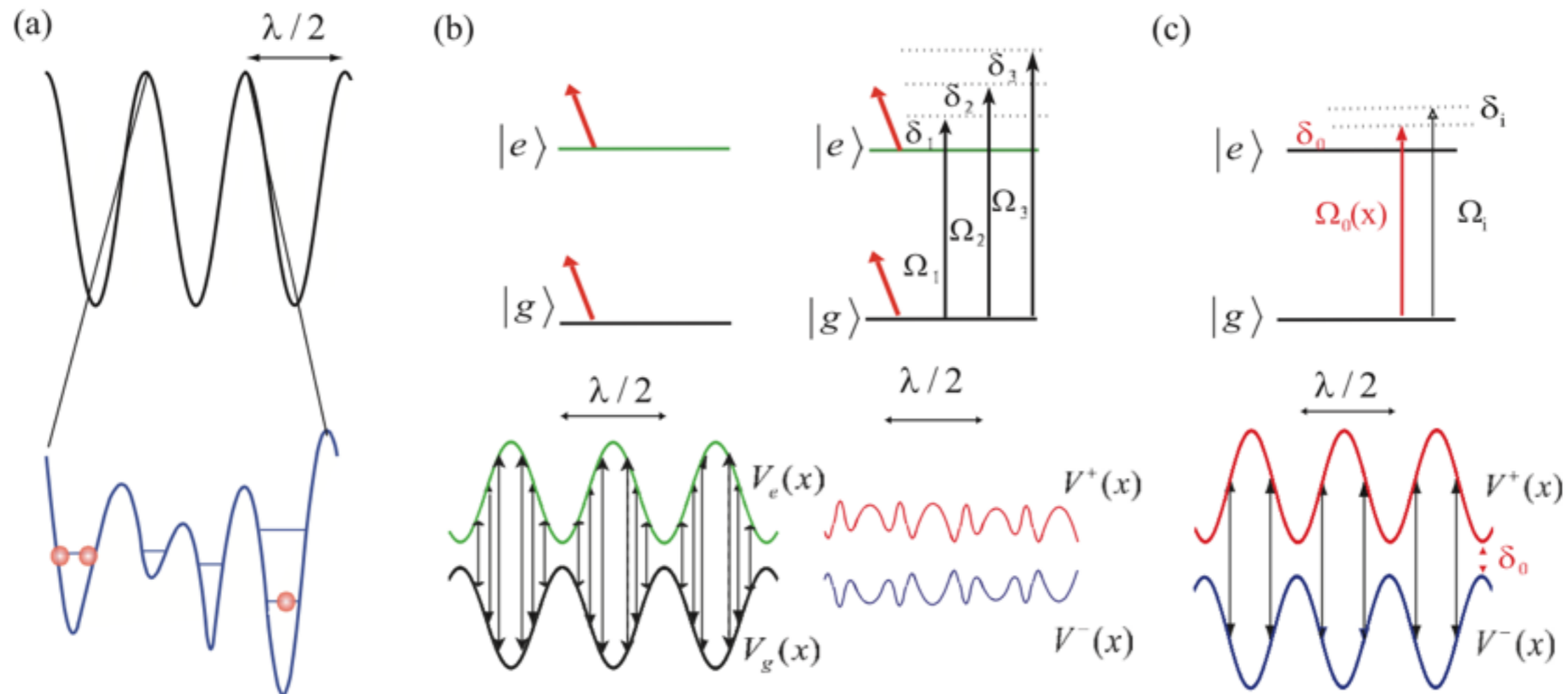

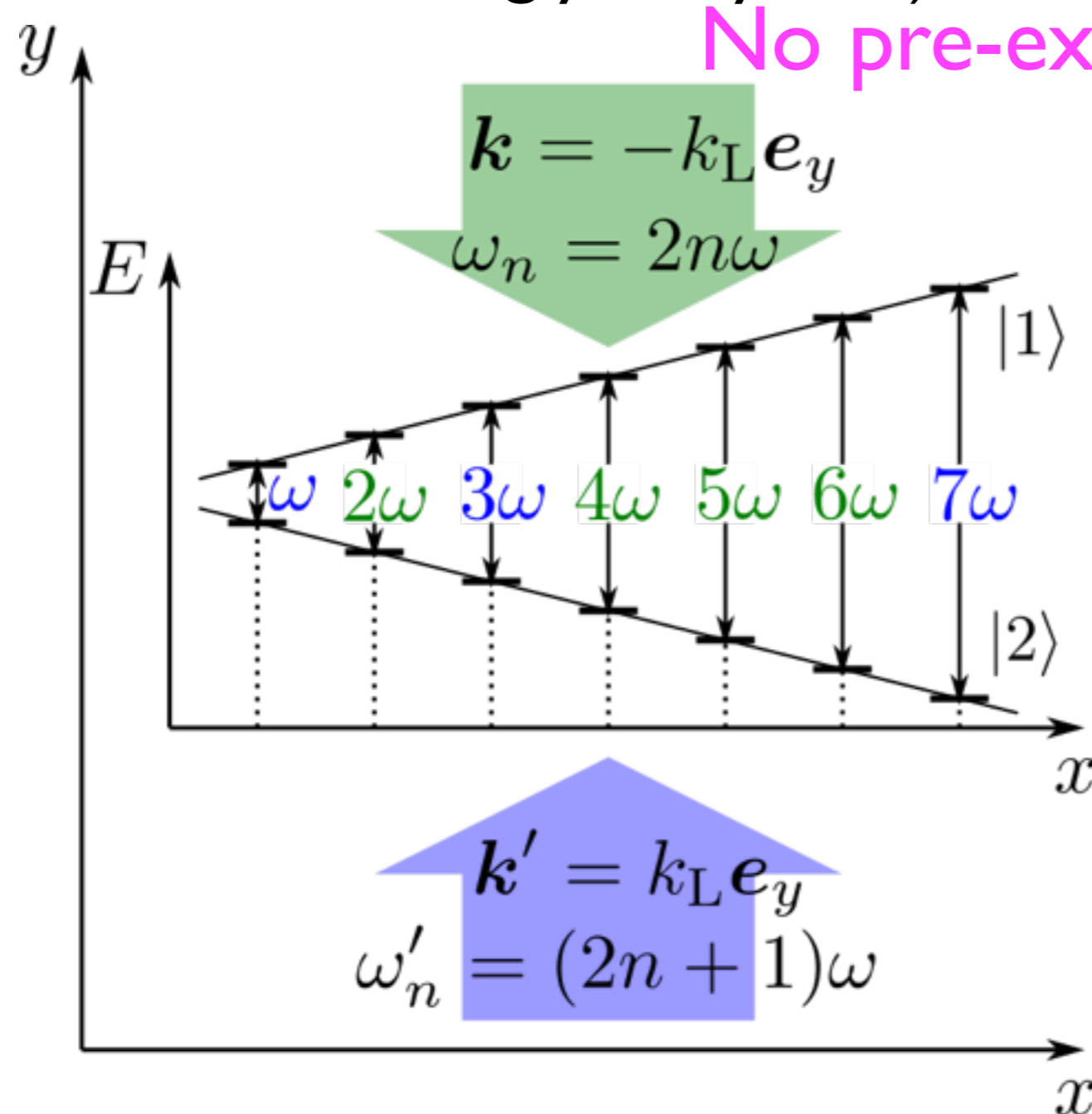


Figure 1. Subwavelength lattice with off-resonant and near-resonant $\lambda/2$ background potentials (see text). (a) Schematic of a controllable subwavelength lattice, where each lattice period of an initial $\lambda/2$ is subdivided into several potential wells whose position and well depth can be controlled by changing the laser parameters. Note that addressability in the subwavelength lattice is modulo

CONCLUSIONS (For part I):

- Magnetic field gradient along the x axis: 
- *Position-dependent detuning between the spin up and down states*
- Frequency-comb Raman coupling between the spin states (with a recoil along y or -y axis)



Optical lattice & **non-staggered magnetic flux** for adiabatic atomic motion

Topological bands **with unit Chern numbers** can be formed

Part II: Optical lattices using synthetic dimensions

Optical lattices in extra dimensions

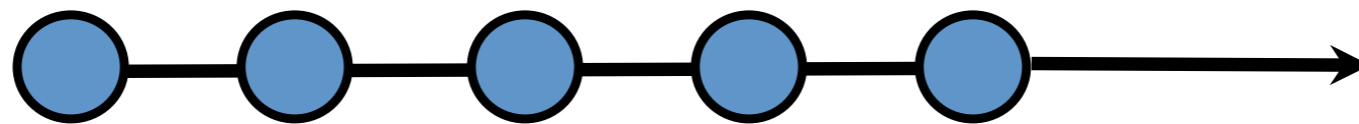
PRL 112, 043001 (2014)

PHYSICAL REVIEW LETTERS

week ending
31 JANUARY 2014

Synthetic Gauge Fields in Synthetic Dimensions

A. Celi,¹ P. Massignan,¹ J. Ruseckas,² N. Goldman,³ I. B. Spielman,^{4,5} G. Juzeliūnas,² and M. Lewenstein^{1,6}



1D atomic chain (real dimension)

Tunneling in real dimension and laser-assisted transitions in the extra dimensions:

➡ 2D semi-synthetic lattice involving real and extra dimensions.

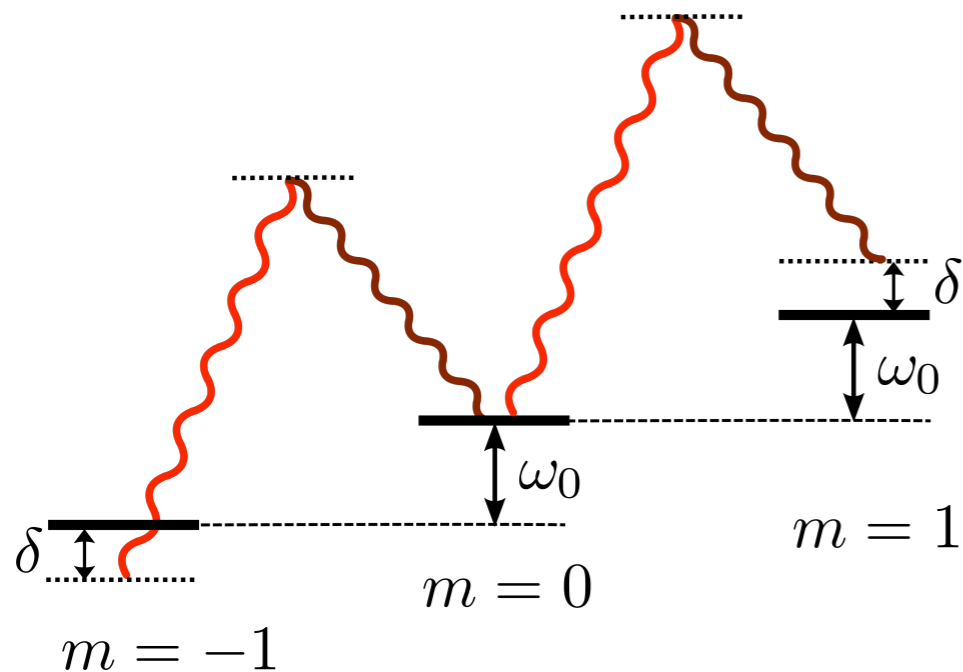
The 2D **semi-synthetic** lattice can be affected by a **non-staggered magnetic flux**

Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=1, \quad m = -1, 0, 1$$

$\Omega_0 e^{ikx}$ - Raman Rabi frequency
(recoil in x direction)



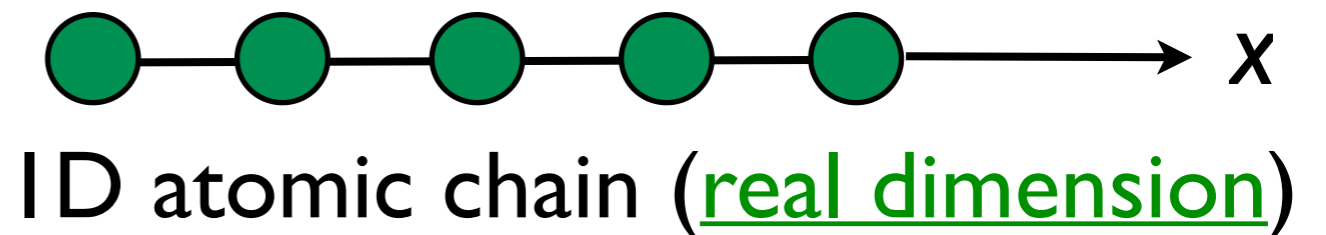
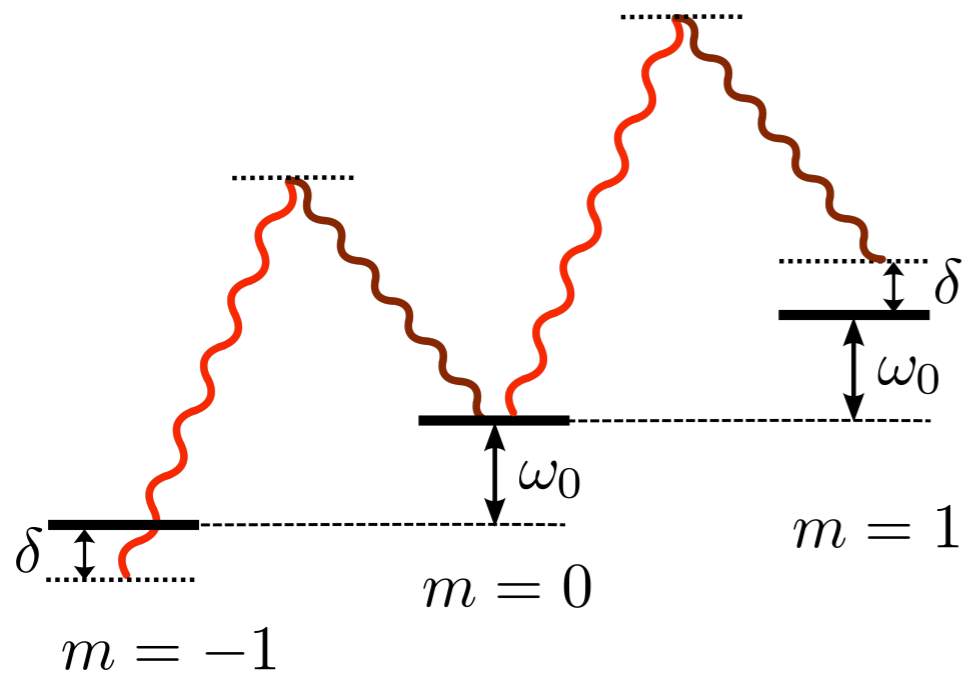
Extra dimension - a set of magnetic sublevels m

Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=I, \quad m = -I, 0, I$$

$\Omega_0 e^{ikx}$ - Raman Rabi frequency
(recoil in x direction)

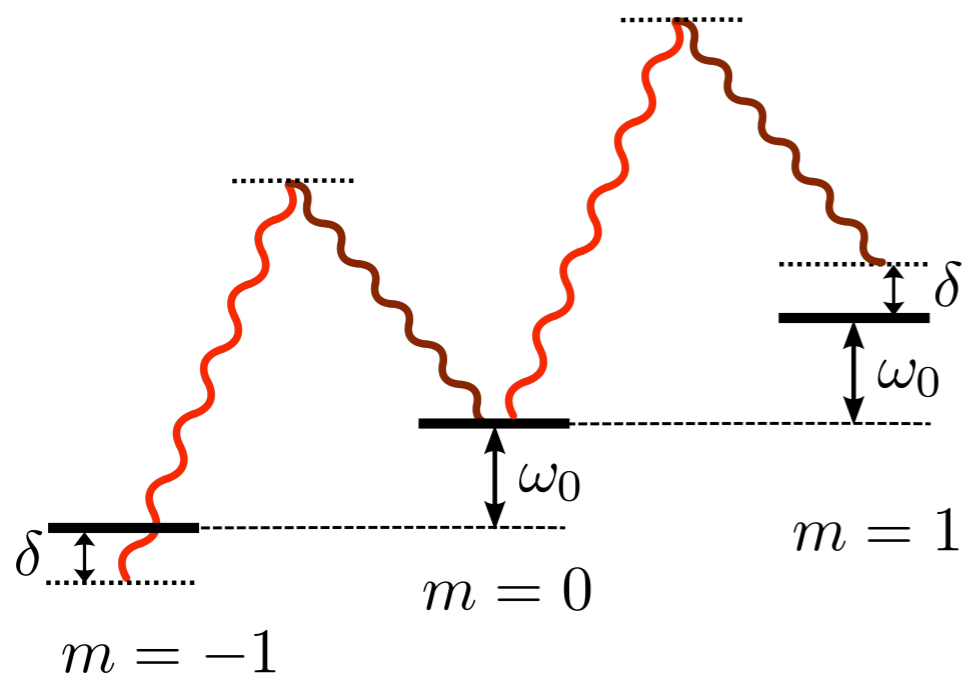


Extra dimension - a set of magnetic sublevels m

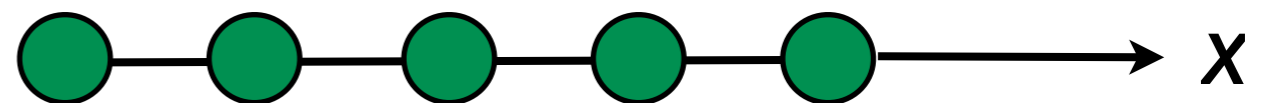
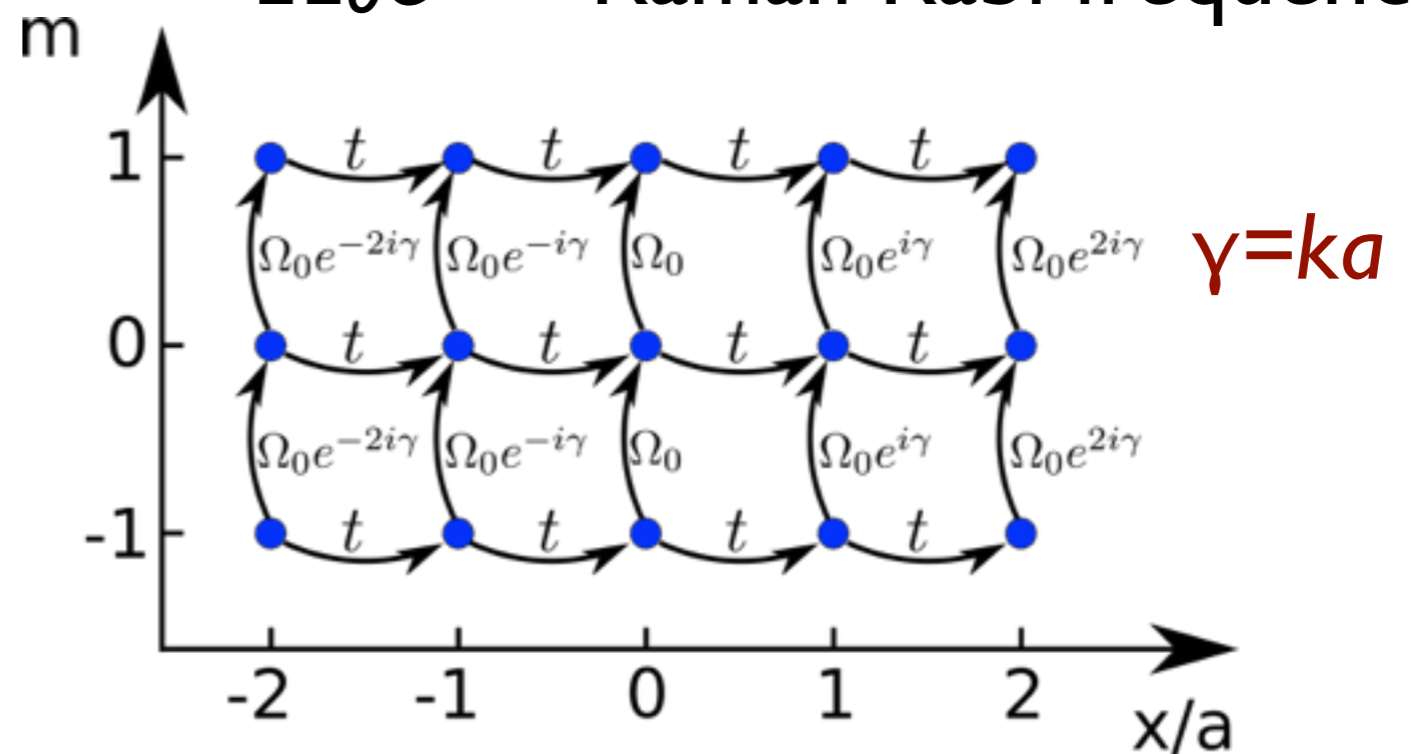
Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=I, \quad m = -I, 0, I$$



$\Omega_0 e^{ikx}$ - Raman Rabi frequency



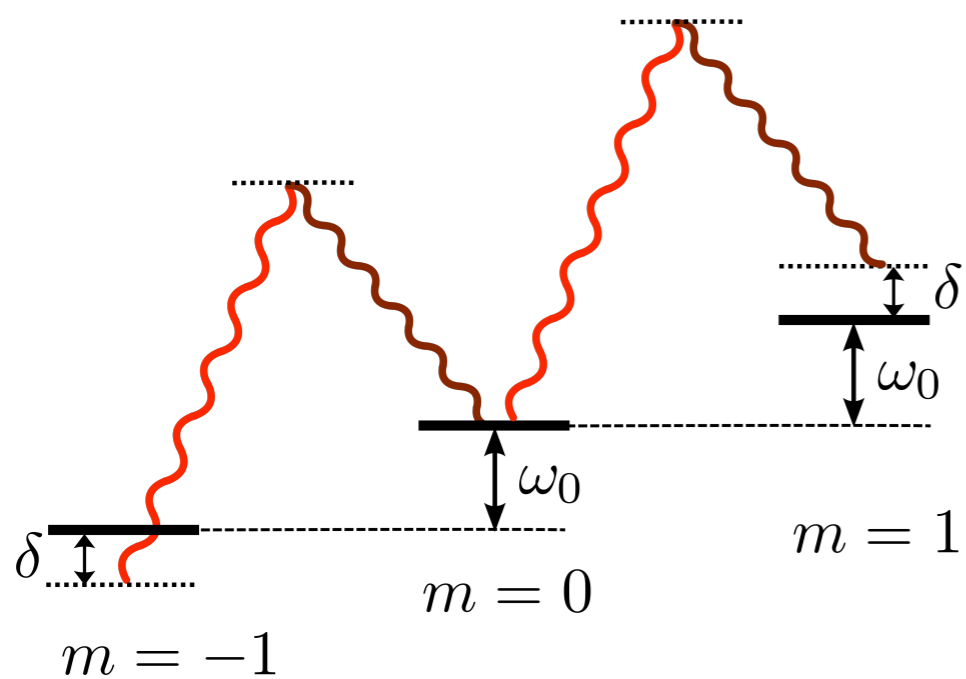
1D atomic chain (real dimension)

Tunneling in real dimension and Raman transitions in the extra dimensions yield a 2D lattice involving real and extra dimensions

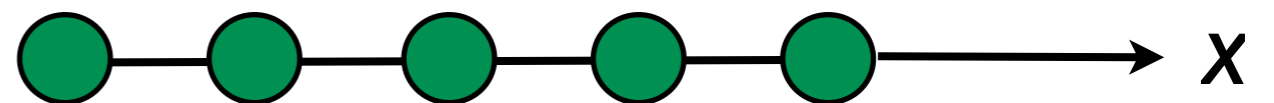
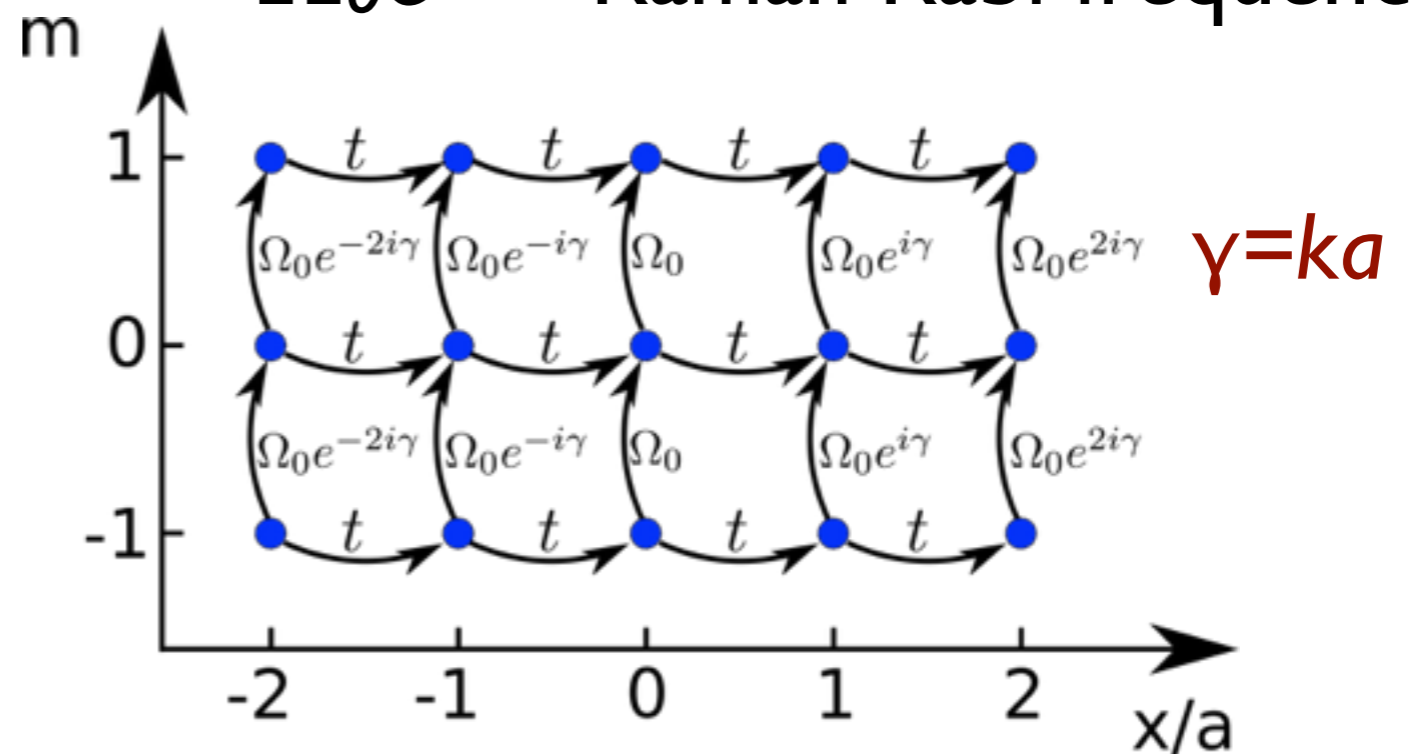
Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=I, \quad m = -I, 0, I$$



$\Omega_0 e^{ikx}$ - Raman Rabi frequency



1D atomic chain (real dimension)

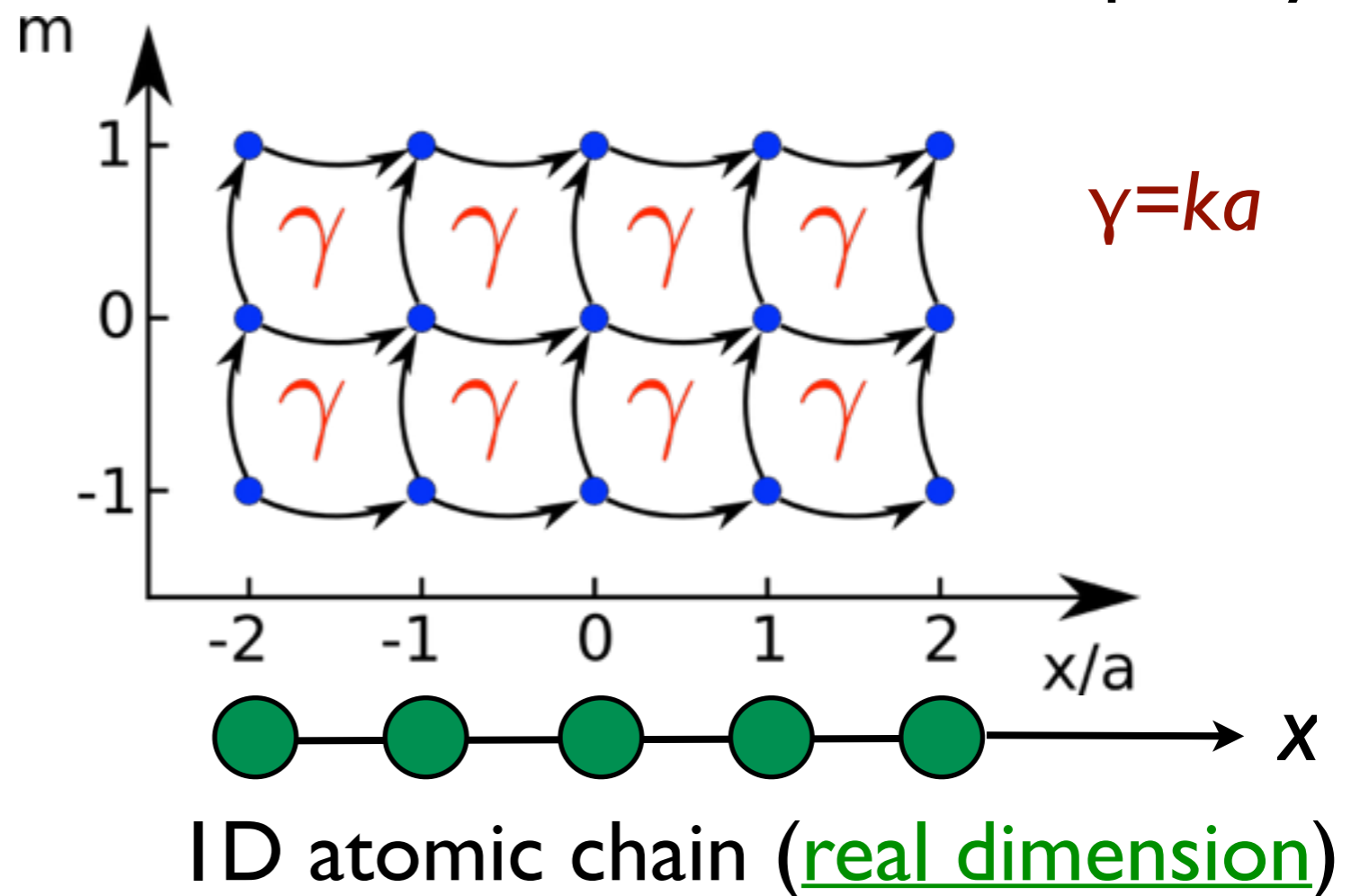
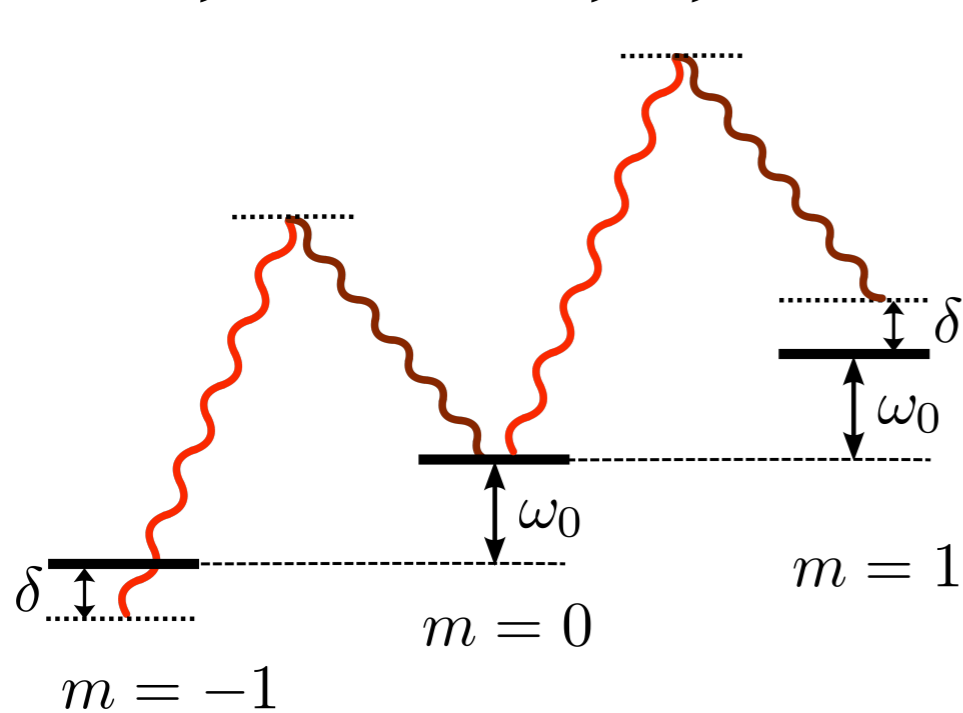
Combination of real and extra dimensions yields strong & non-staggered magnetic flux $\gamma = ka$ per 2D plaquette (due to Raman recoil k)

Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$\Omega_0 e^{ikx}$ - Raman Rabi frequency

$$F=I, \quad m = -I, 0, I$$

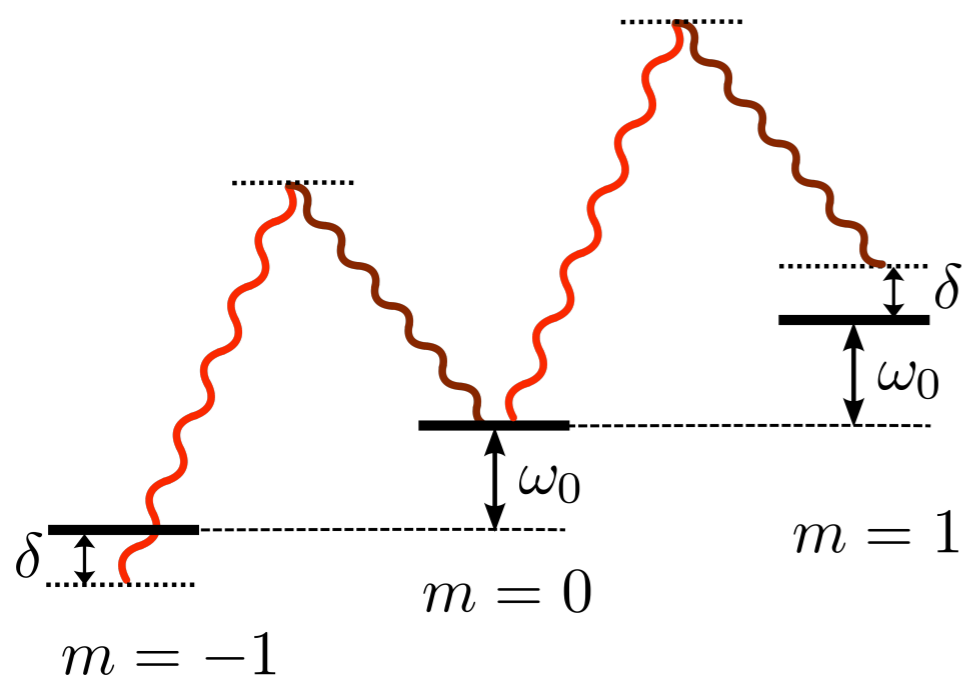


Combination of real and extra dimensions yields strong & non-staggered magnetic flux $\gamma = ka$ per 2D plaquette (due to Raman recoil k)

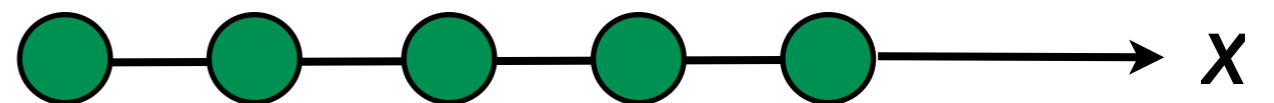
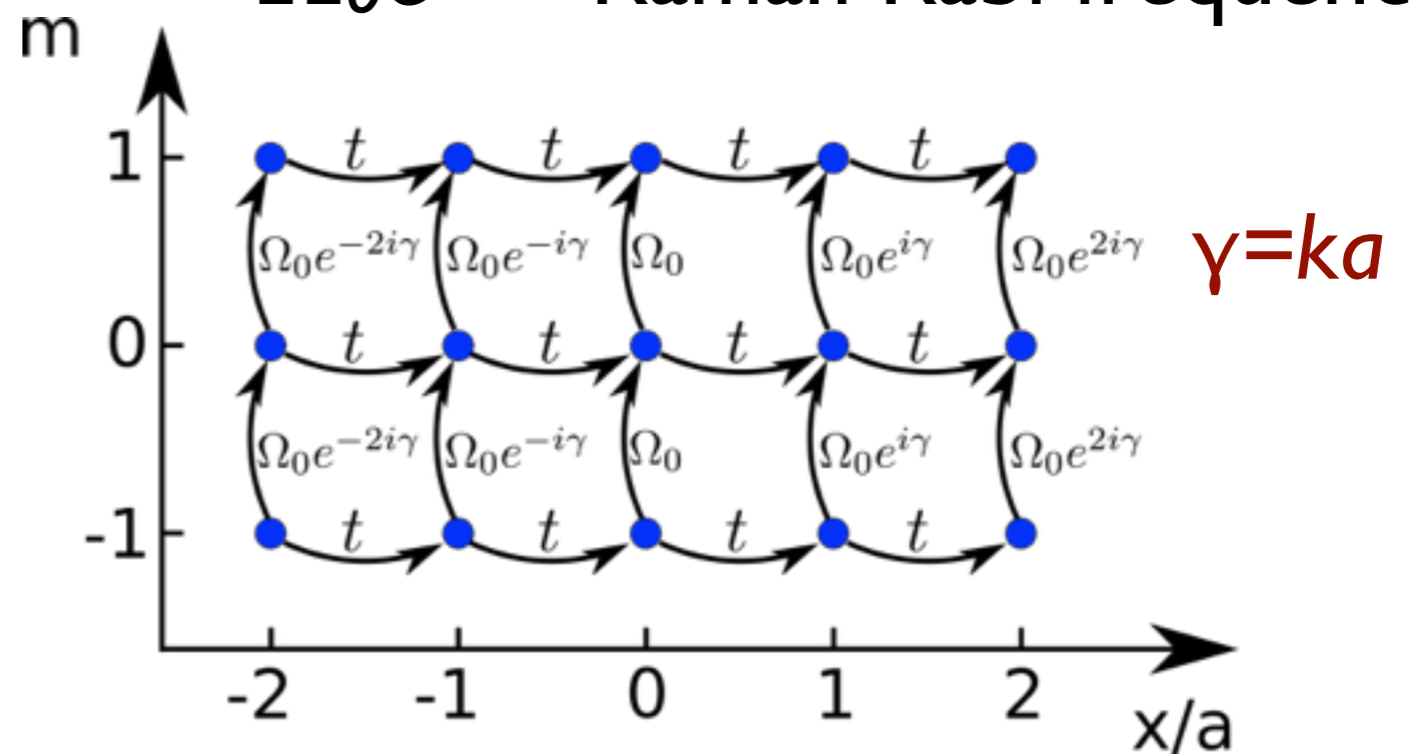
Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=I, \quad m = -I, 0, I$$



$\Omega_0 e^{ikx}$ - Raman Rabi frequency



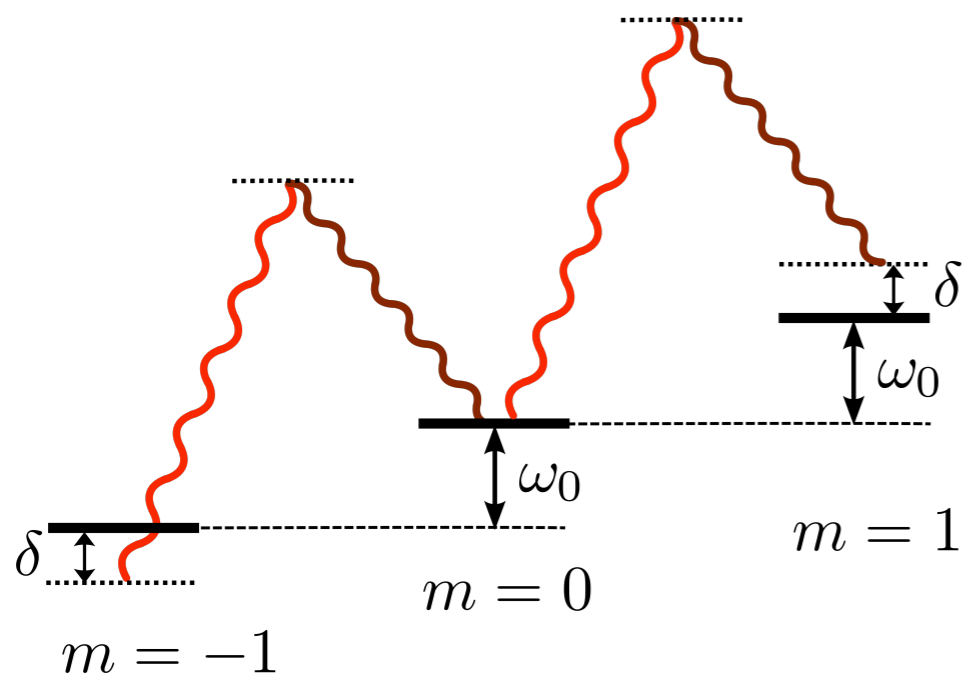
1D atomic chain (real dimension)

Combination of real and extra dimensions yields strong & non-staggered magnetic flux $\gamma = ka$ per 2D plaquette (due to Raman recoil k)

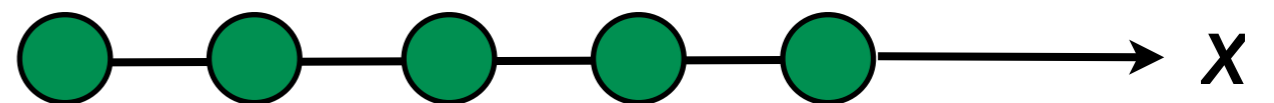
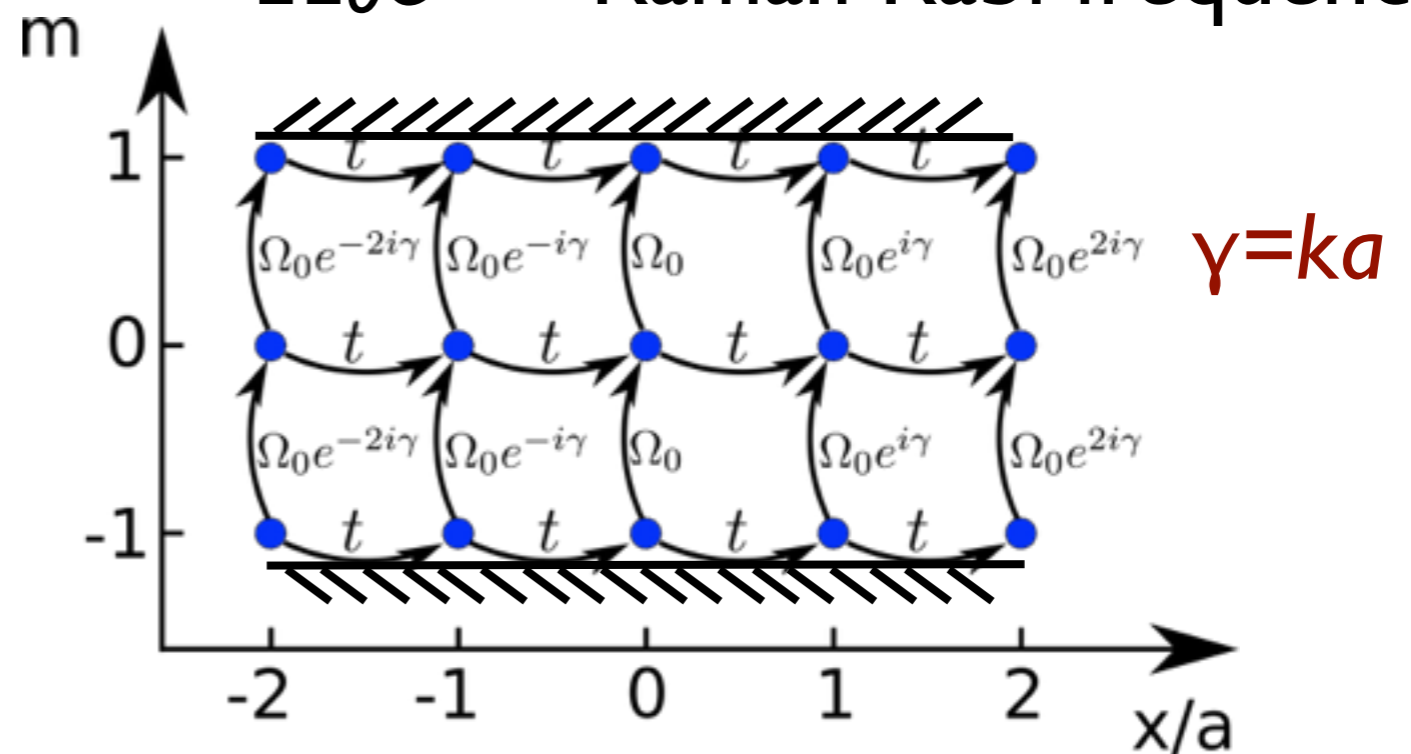
Optical lattices in extra dimensions

Raman transitions between magnetic sublevels m (extra dimension)

$$F=I, \quad m = -I, 0, I$$



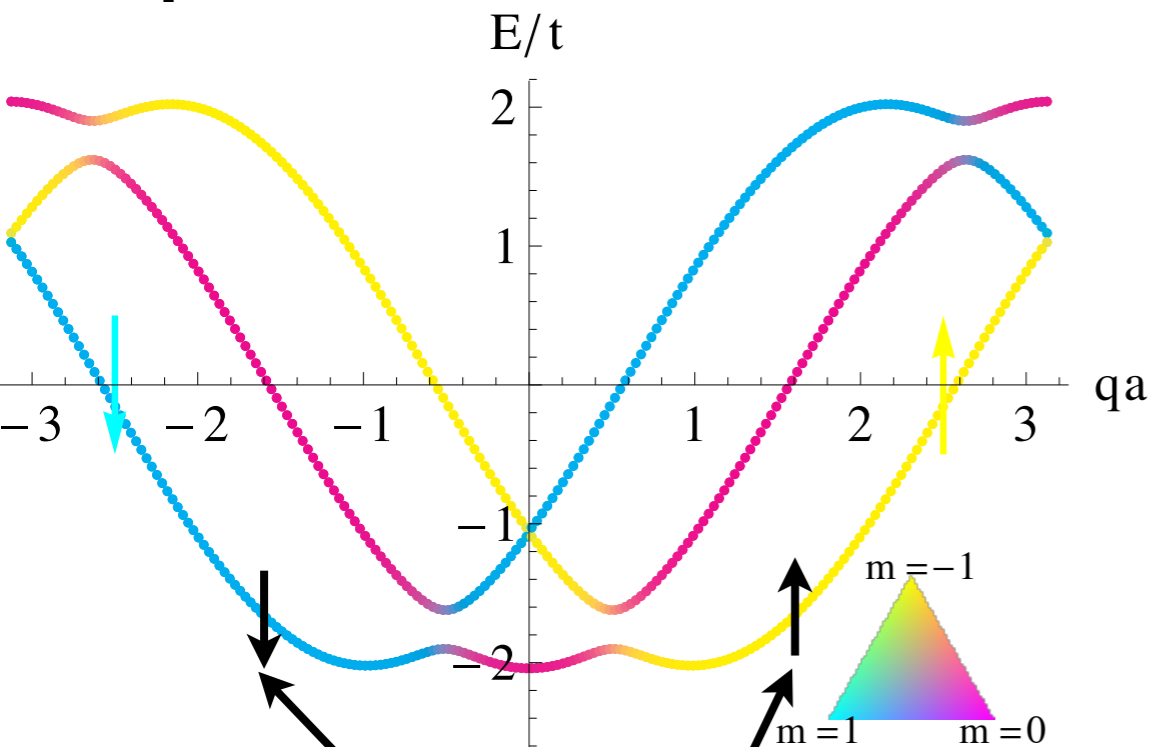
$\Omega_0 e^{ikx}$ - Raman Rabi frequency



Sharp boundaries in extra dimension:
 \Rightarrow Conducting edge states in extra dimension

Optical lattices in extra dimensions

Dispersion branches:

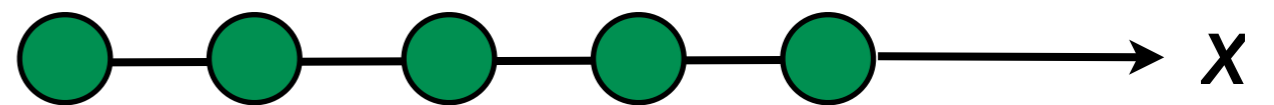
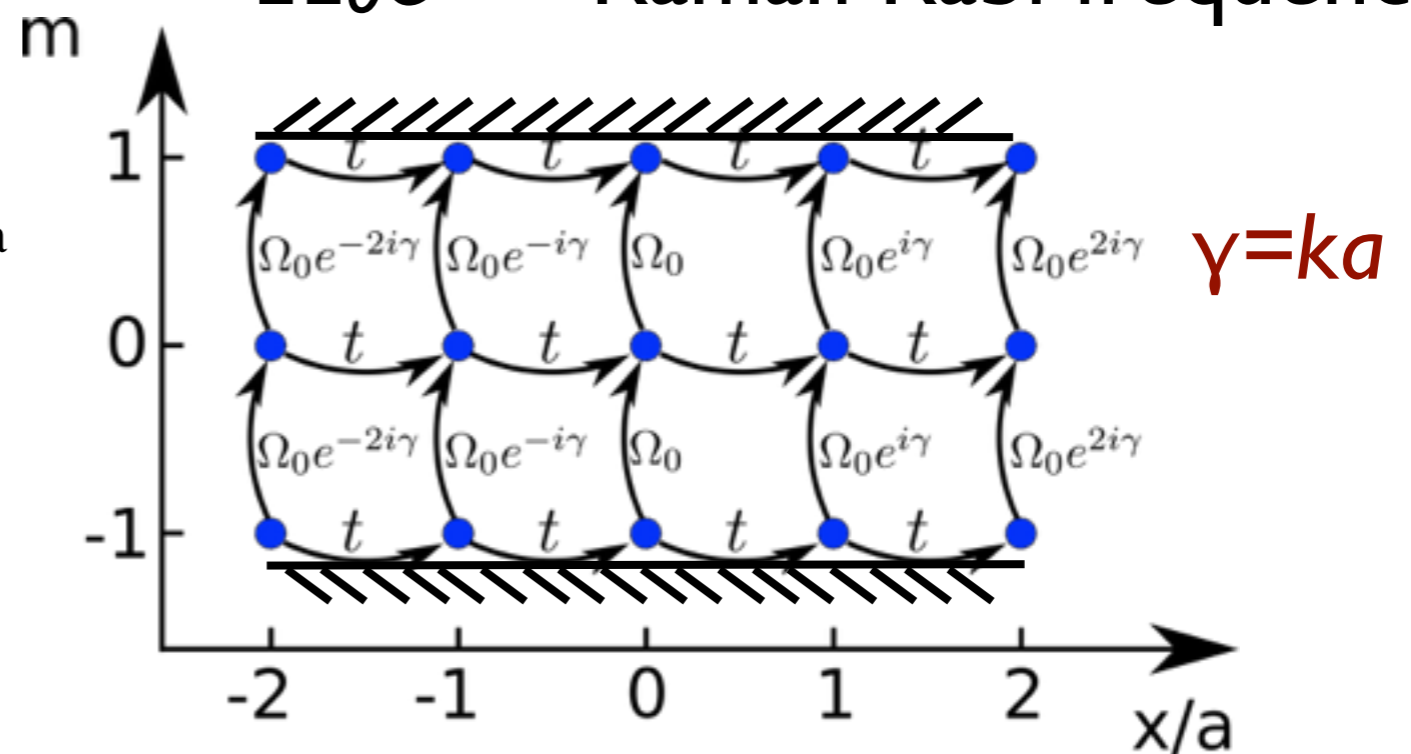


$\gamma = ka = 1.0$ Edge states

$\Omega_0/t = 0.1$

Raman transitions between magnetic sublevels m (extra dimension)

$\Omega_0 e^{ikx}$ - Raman Rabi frequency



1D atomic chain (real dimension)

Sharp boundaries in extra dimension:

\Rightarrow Conducting edge states in extra dimension

\Rightarrow Atoms with opposite spins move in opposite directions

Optical lattices in extra dimensions

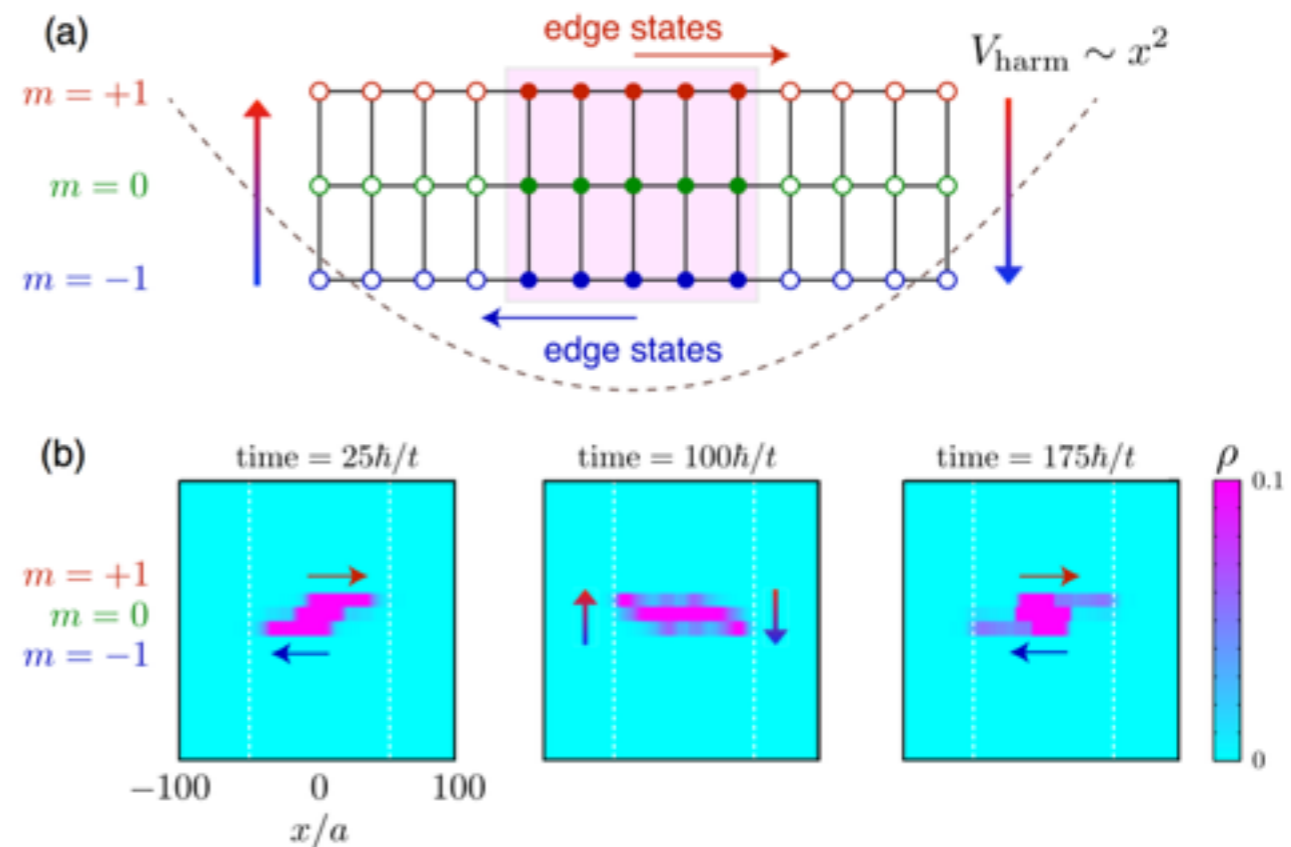
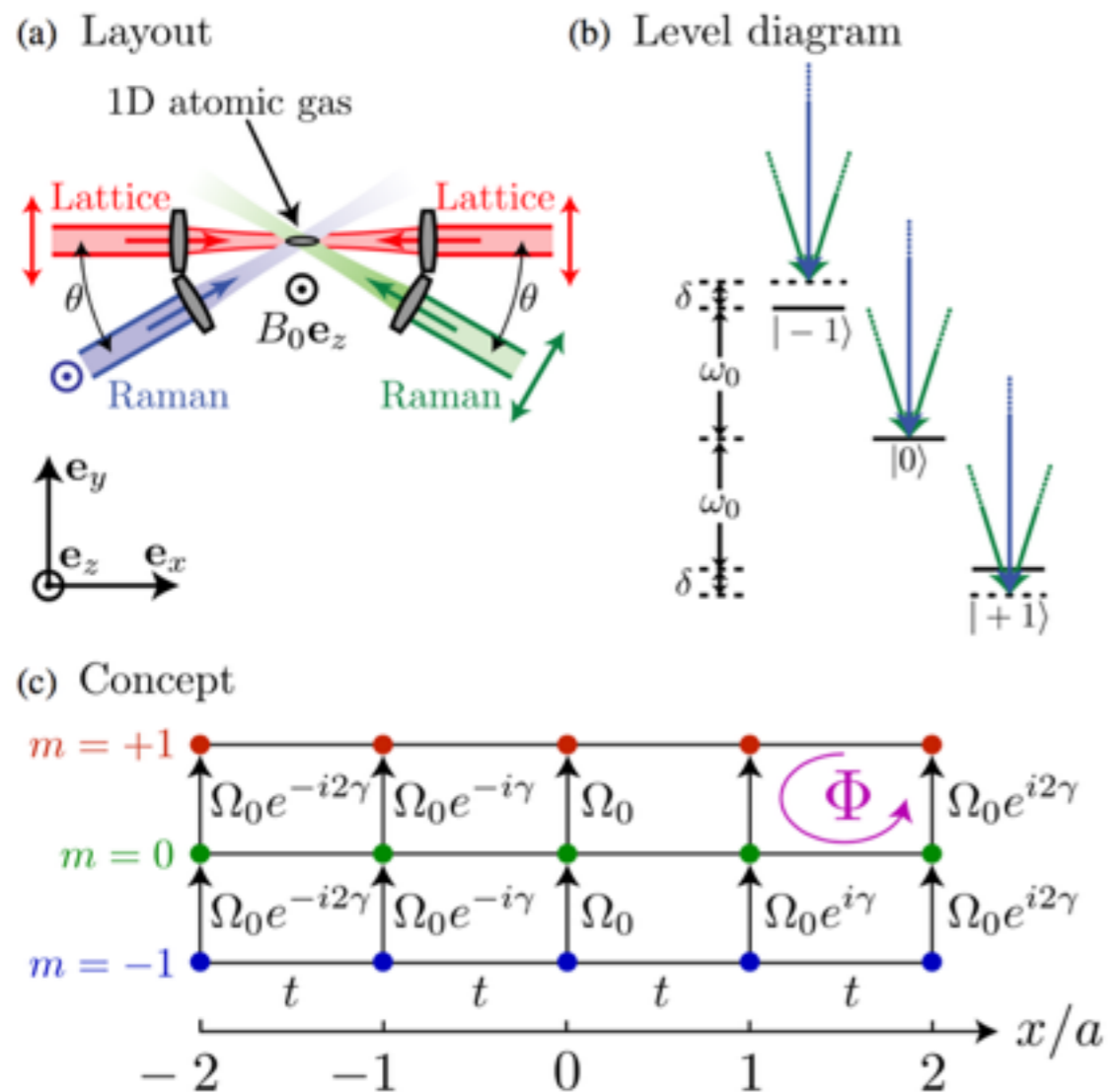
PRL 112, 043001 (2014)

PHYSICAL REVIEW LETTERS

week ending
31 JANUARY 2014

Synthetic Gauge Fields in Synthetic Dimensions (Proposal)

A. Celi,¹ P. Massignan,¹ J. Ruseckas,² N. Goldman,³ I. B. Spielman,^{4,5} G. Juzeliūnas,² and M. Lewenstein^{1,6}



Chiral edge states in semi-synthetic Hall ribbons

EDITORS' CHOICE

EDITED BY KRISTEN MUELLER AND JESSE SMITH

PHYSICS

A Semisynthetic Lattice

Atomic vapors at very low temperatures are useful for the quantum simulation of solid-state systems, because their properties can be finely controlled and tuned. These neutral atoms are not, however, completely analogous to the charged carriers in solids; for instance, an external magnetic field causes electrons to move in circular orbits but has no such effects on neutral atoms. Celi *et al.* propose a simple method for creating a uniform magnetic flux in a one-dimensional (1D) optical lattice that, if realized, might be used to observe exotic phenomena such as Hofstadter-butterfly-like fractal spectra or the dynamics of topological edge states. The method is based on synthetically extending the 1D lattice into the second dimension of internal atomic states (spin) by coupling those states using a pair of Raman laser beams that are directed at an angle with respect to the optical lattice; the required amount of the Raman laser light is substantially smaller than in existing schemes. The resulting band structure supports edge states in the spin variable whose dynamics should be observable through spin-sensitive density measurements. — JS

Phys. Rev. Lett. **112**, 043001 (2014).

Optical lattices in extra dimensions

Experimental realization

B. K. Stuhl, H.-I. Lu, L. M. Ayccock, D. Genkina, and I. B. Spielman, Visualizing edge states with an atomic Bose gas in the quantum Hall regime, [Science](#) **349**, 1514 (2015).

M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, and L. Fallani, Observation of chiral edge states with neutral fermions in synthetic Hall ribbons, [Science](#) **349**, 1510 (2015).

L. F. Livi, G. Cappellini, M. Diem, L. Franchi, C. Clivati, M. Frittelli, F. Levi, D. Calonico, J. Catani, M. Inguscio, and L. Fallani, Synthetic Dimensions and Spin-Orbit Coupling with an Optical Clock Transition, [Phys. Rev. Lett.](#) **117**, 220401 (2016).

S. Kolkowitz, S.L. Bromley, T. Bothwell, M.L. Wall, G.E. Marti, A.P. Koller, X. Zhang, A.M. Rey, J. Ye, Spin-orbit-coupled fermions in an optical lattice clock, [Nature](#) **542**, 66 (2017).

Optical lattices in extra dimensions

Experimental realization

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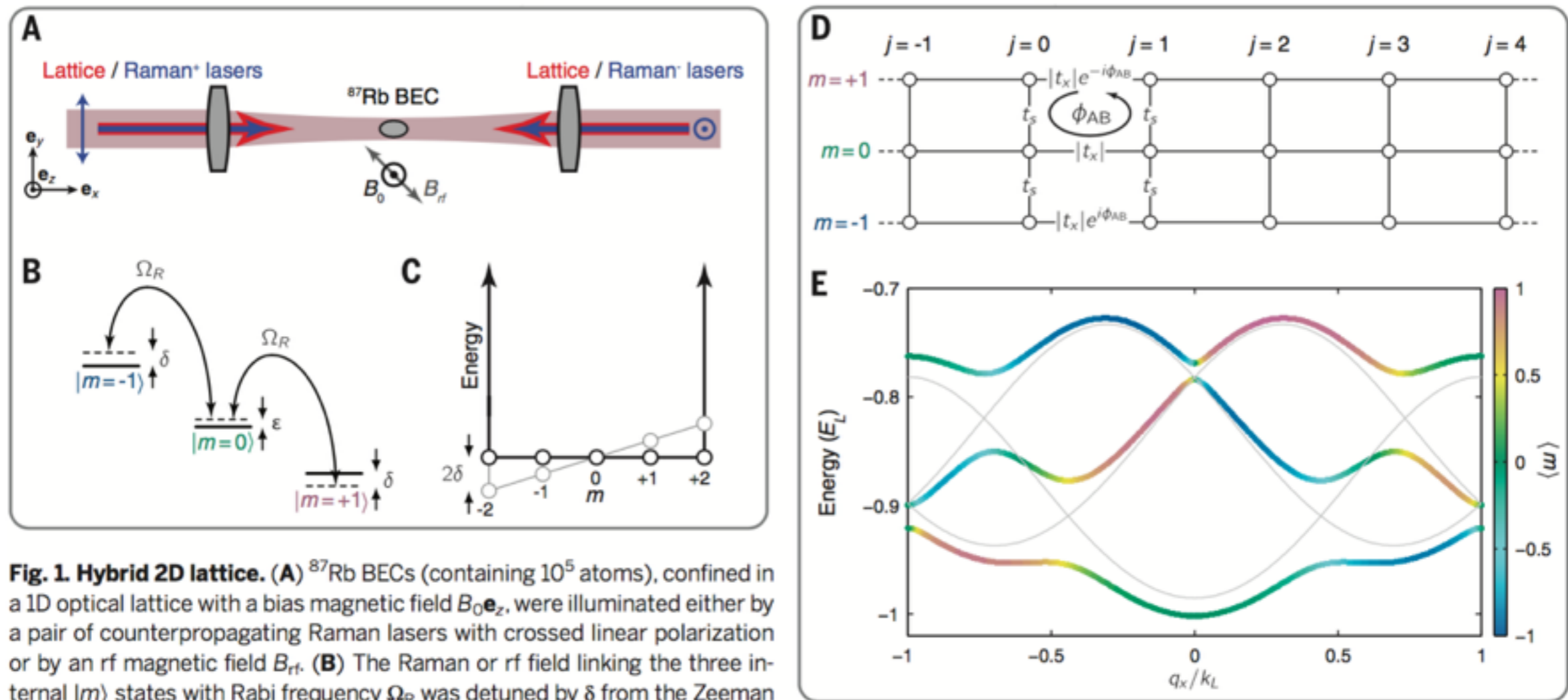


Fig. 1. Hybrid 2D lattice. (A) ^{87}Rb BECs (containing 10^5 atoms), confined in a 1D optical lattice with a bias magnetic field $B_0 \mathbf{e}_z$, were illuminated either by a pair of counterpropagating Raman lasers with crossed linear polarization or by an rf magnetic field B_{rf} . (B) The Raman or rf field linking the three internal $|m\rangle$ states with Rabi frequency Ω_R was detuned by δ from the Zeeman splitting ($g\mu_B B_0/h \approx 0.817$ MHz or 1.35 MHz; g , the Landé g -factor; h , Planck's constant). The corresponding quadratic Zeeman shift lowered $|m=0\rangle$ by $\epsilon = 0.05E_L$ (for the data in Fig. 3) or $0.13E_L$ (for the data in Fig. 4). The Raman lasers' relative phases were actively stabilized at a beam combiner adjacent to the optical-lattice retroreflection mirror (24). (C) The lattice along \mathbf{e}_x can be considered as a square well with hard walls at $m = \pm 2$, for which $\delta \neq 0$ tilts the potential. (D) The 2D hybrid lattice, where the nonspatial dimension is built from the internal states $|m\rangle$, with an effective magnetic flux per plaquette of $\Phi/\Phi_0 = \phi_{AB}/2\pi$. (E) The three lowest magnetic bands (rainbow colors), computed for our full lattice without making the tight-binding approximation, with parameters $(\hbar\Omega_R, V, \delta, \epsilon) = (0.141, 6, 0, 0.05)E_L$. The pale curves represent computations for $\hbar\Omega_R = 0$.

Optical lattices in extra dimensions

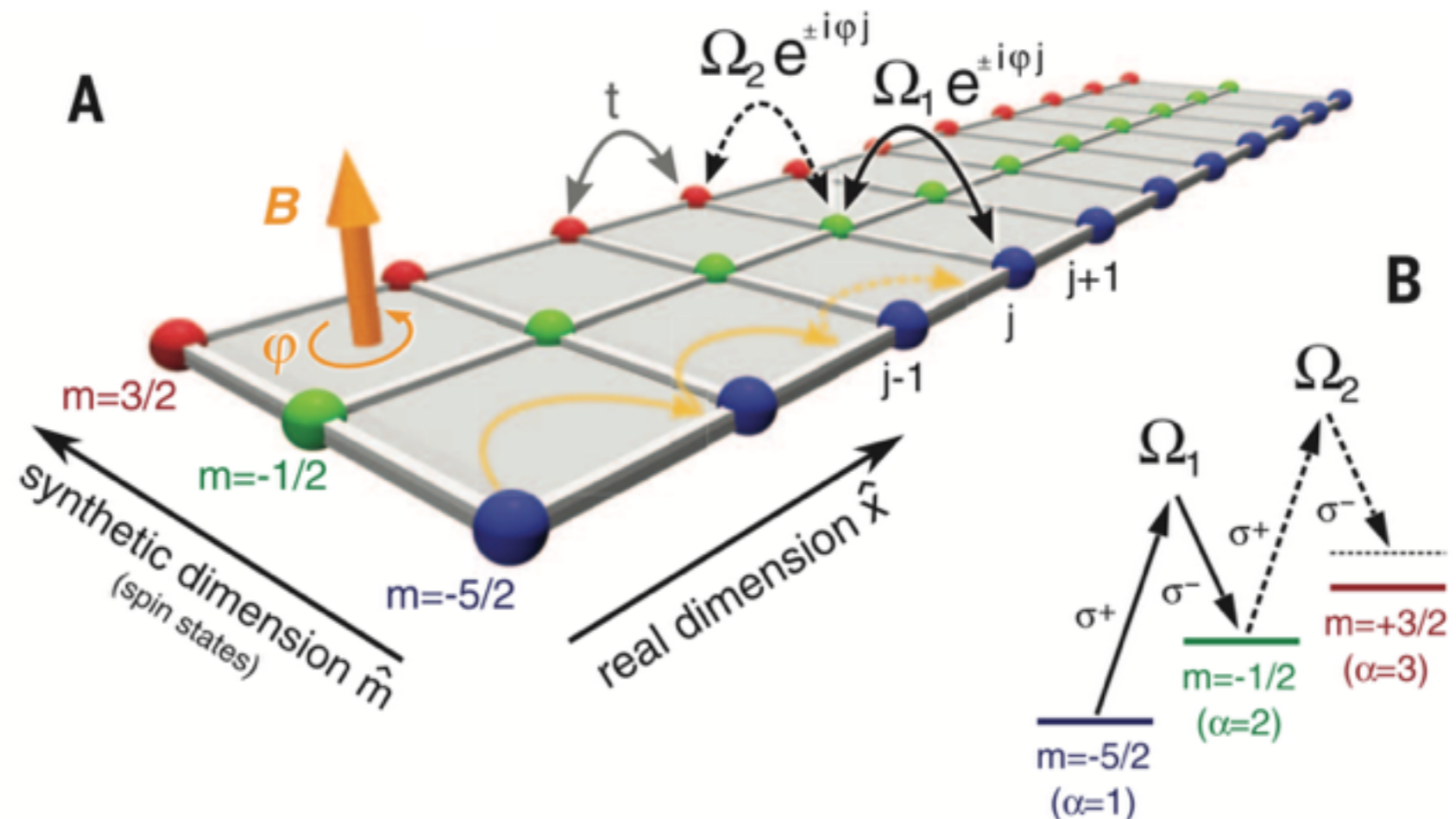
Experimental realization

M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, and L. Fallani, Observation of chiral edge states with neutral fermions in synthetic Hall ribbons, *Science* **349**, 1510 (2015).

Fig. 1. A synthetic gauge field in a synthetic dimension.

(A) We confine the motion of fermionic ultracold atoms in a hybrid lattice, generated by an optical lattice along a real direction \hat{x} with tunneling t , and by laser-induced hopping between spin states along a synthetic direction \hat{m} . By inducing a complex tunneling

$\Omega_{1,2}e^{i\varphi j}$ along \hat{m} , the atom wave function acquires a phase φ per plaquette, mimicking the effect of a transverse magnetic field **B** on effectively charged particles. (B) Scheme of the ^{173}Yb nuclear spin states and Raman transitions used in the experiment.

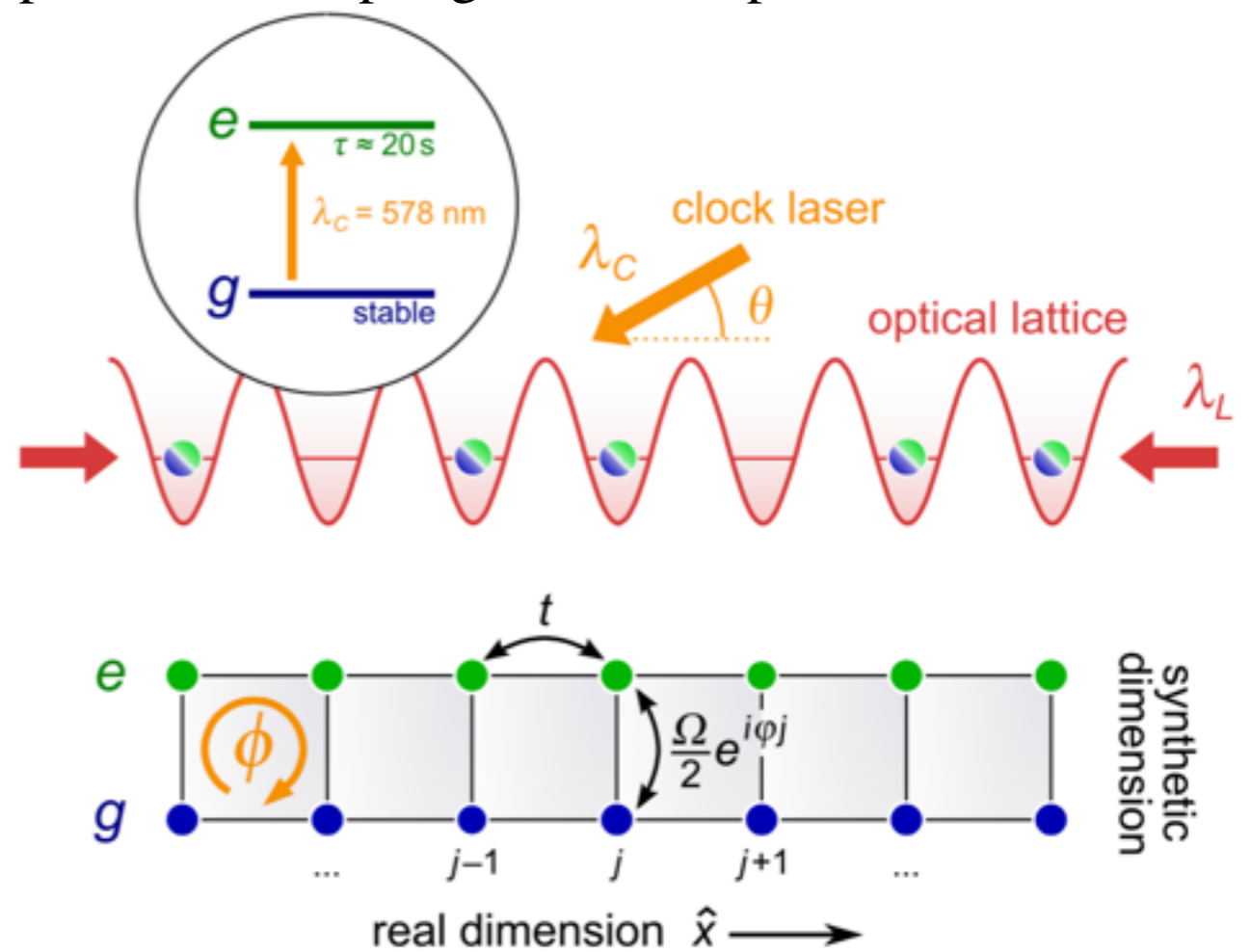


Optical lattices in extra dimensions

Experimental realization

L. F. Livi, G. Cappellini, M. Diem, L. Franchi, C. Clivati, M. Frittelli, F. Levi, D. Calonico, J. Catani, M. Inguscio, and L. Fallani, Synthetic Dimensions and Spin-Orbit Coupling with an Optical Clock Transition, *Phys. Rev. Lett.* **117**, 220401 (2016).

FIG. 1. Sketch of the setup. Ultracold ^{173}Yb fermions are trapped in 1D chains by an optical lattice at wavelength λ_L . An ultranarrow clock laser with wavelength λ_C drives the single-photon transition between the ground state $g = ^1S_0$ and the long-lived electronic state $e = ^3P_0$. The laser momentum transfer $\delta k = 2\pi \cos \theta / \lambda_C$ results in a locking between internal state (interpreted as an effective pseudospin) and atomic momentum. The electronic state can also be treated as an effective synthetic dimension made by two sites connected with a coherent tunneling, resulting in a two-leg ladder pierced by a synthetic magnetic flux per plaquette $\phi = \pi \delta k / k_L$, which can be tuned by adjusting the angle θ .

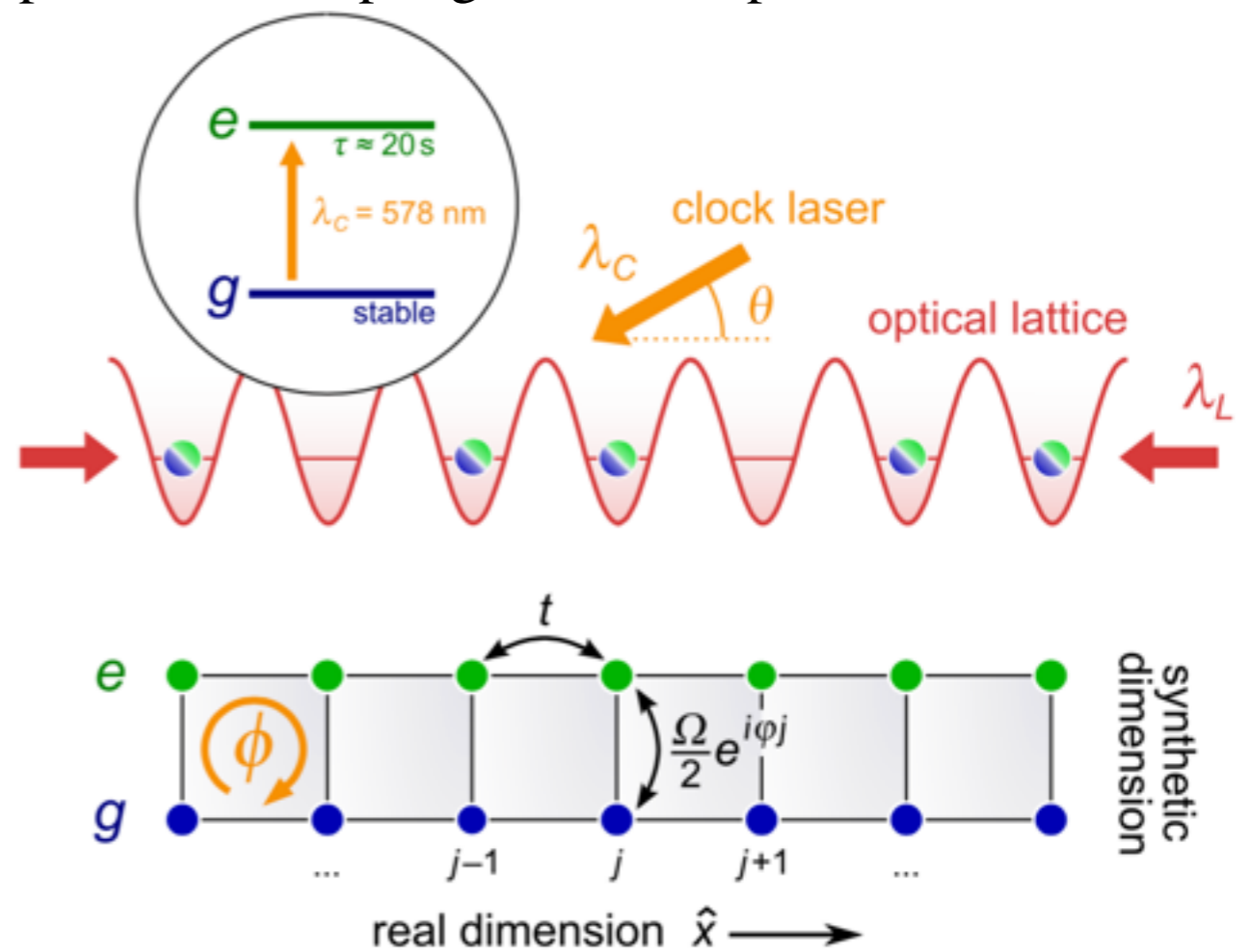


Optical lattices in extra dimensions

Experimental realization

L. F. Livi, G. Cappellini, M. Diem, L. Franchi, C. Clivati, M. Frittelli, F. Levi, D. Calonico, J. Catani, M. Inguscio, and L. Fallani, Synthetic Dimensions and Spin-Orbit Coupling with an Optical Clock Transition, *Phys. Rev. Lett.* **117**, 220401 (2016).

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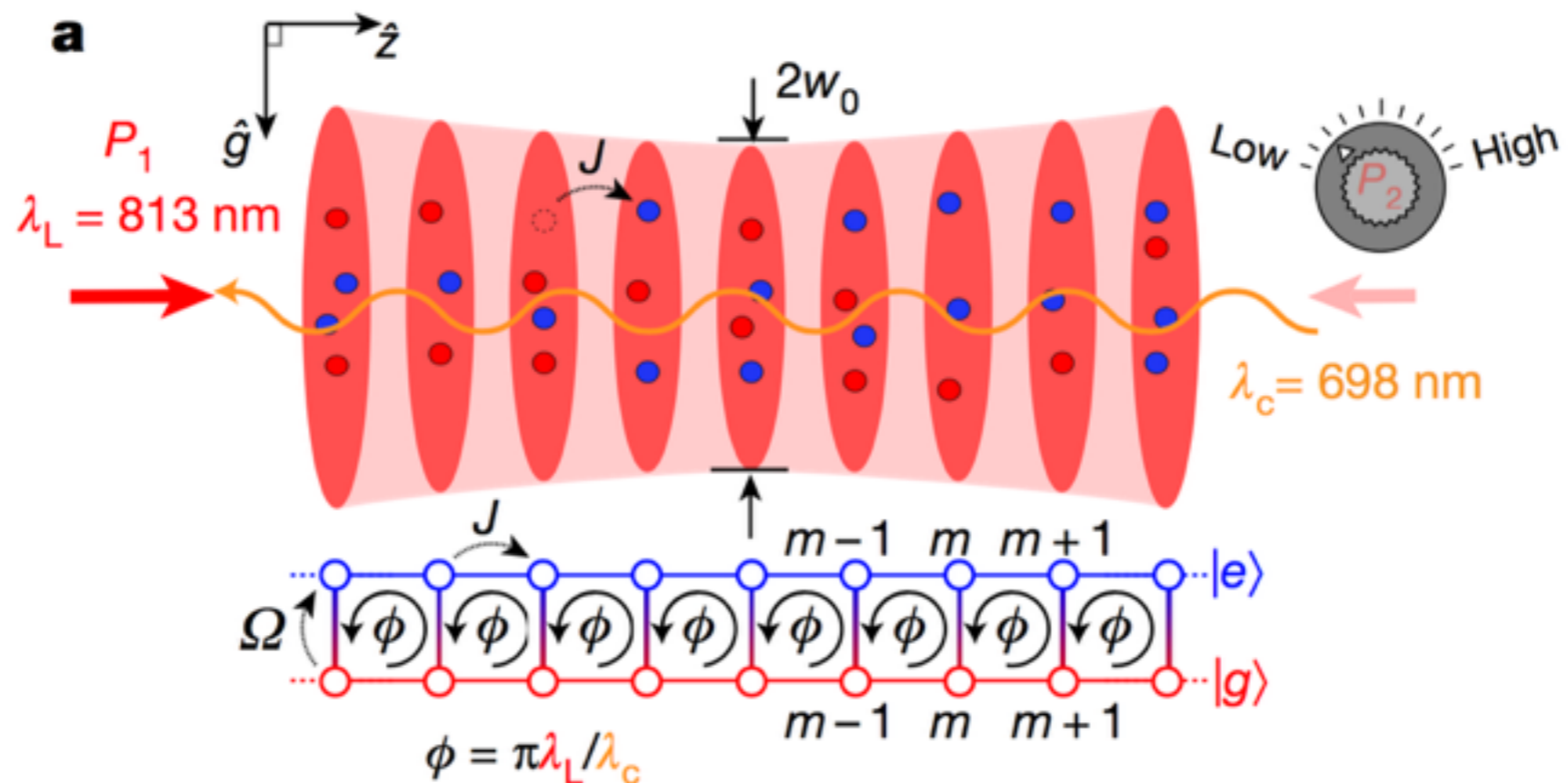


Laser coupling of **ground and excited** electronic orbital states of ^{173}Yb atoms optically trapped at the “magic wave-length”: Atoms feel the same trapping potential for both states:  Semisynthetic square optical lattice.

Optical lattices in extra dimensions

Experimental realization

S. Kolkowitz, S.L. Bromley, T. Bothwell, M.L. Wall, G.E. Marti, A.P. Koller, X. Zhang, A.M. Rey, J. Ye, Spin-orbit-coupled fermions in an optical lattice clock, *Nature* **542**, 66 (2017).



Laser coupling of **ground and excited** electronic orbital states of ^{87}Sr atoms optically trapped at the “magic wave-length”: Atoms feel the same trapping potential for both states:  Semisynthetic square optical lattice.

Optical lattices in extra dimensions

Non-square geometry

Optical lattices in extra dimensions

Non-square geometry

Laser coupled **ground and excited** atomic states should be trapped at the “anti-magic wave-length”: Atoms feel the opposite trapping potential. 
Semisynthetic zigzag optical lattice.

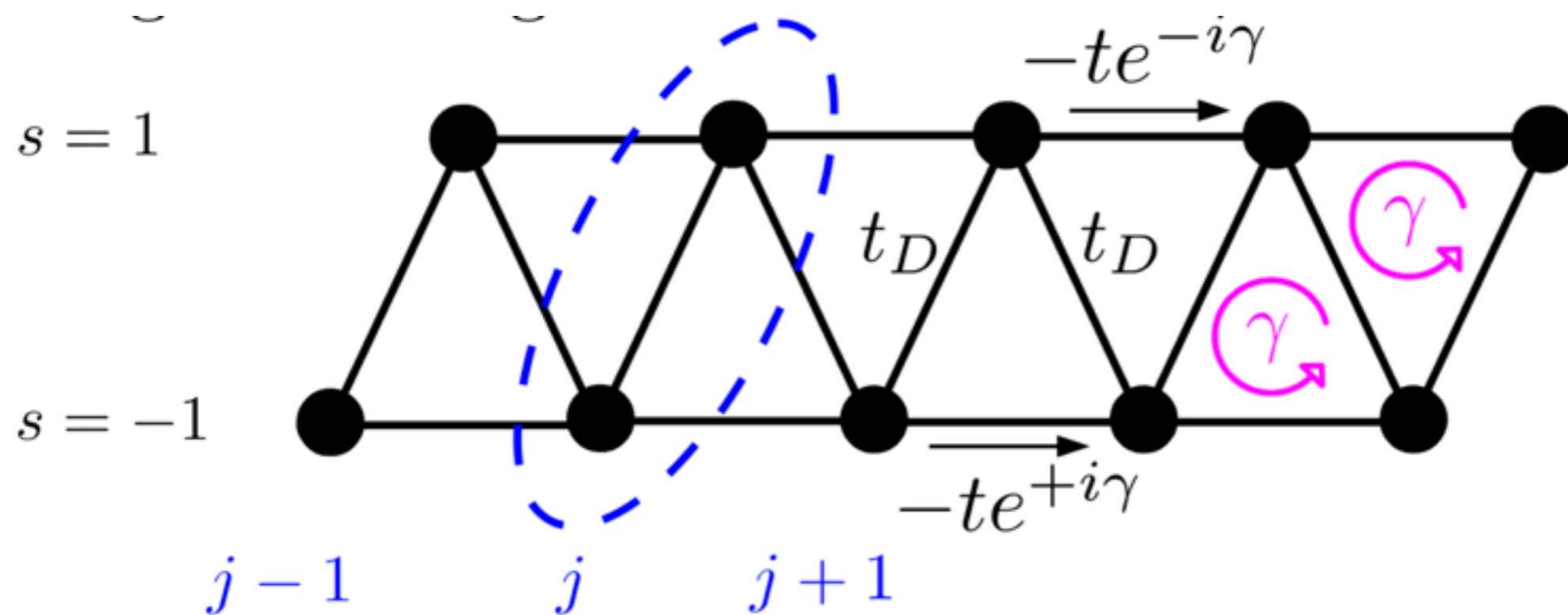
E. Anisimovas, M. Račiūnas, C. Sträter, A. Eckardt, I. B. Spielman, G. Juzeliūnas,
[Semi-synthetic zigzag optical lattice for ultracold bosons](#), Phys. Rev. A 94, 063632 (2016)

NEXT

Optical lattices in extra dimensions

Non-square geometry

Laser coupled **ground and excited** atomic states should be trapped at the “anti-magic wave-length”: Atoms feel the opposite trapping potential. 
Semisynthetic zigzag optical lattice.



E. Anisimovas, M. Račiūnas, C. Sträter, A. Eckardt, I. B. Spielman, G. Juzeliūnas,
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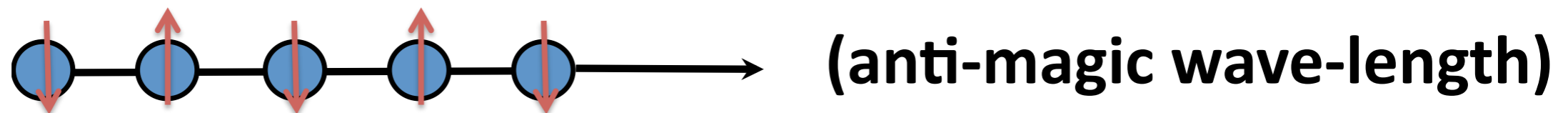
NEXT

Optical lattices in extra dimensions

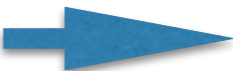
Non-square geometry

E. Anisimovas, M. Račiūnas, C. Sträter, A. Eckardt, I. B. Spielman, G. Juzeliūnas,
[Semi-synthetic zigzag optical lattice for ultracold bosons](#), Phys. Rev. A 94, 063632 (2016)

1D chain of atoms (real dimension)



(anti-magic wave-length)

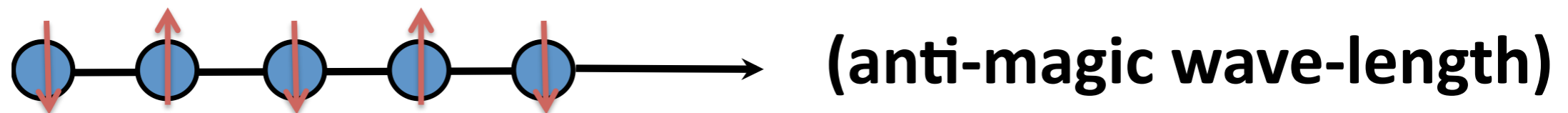
Atoms with different internal states are trapped at different lattice sites of a one-dimensional lattice  **Semisynthetic zigzag lattice**
(& laser-assisted transitions between \uparrow and \downarrow states)

Optical lattices in extra dimensions

Non-square geometry

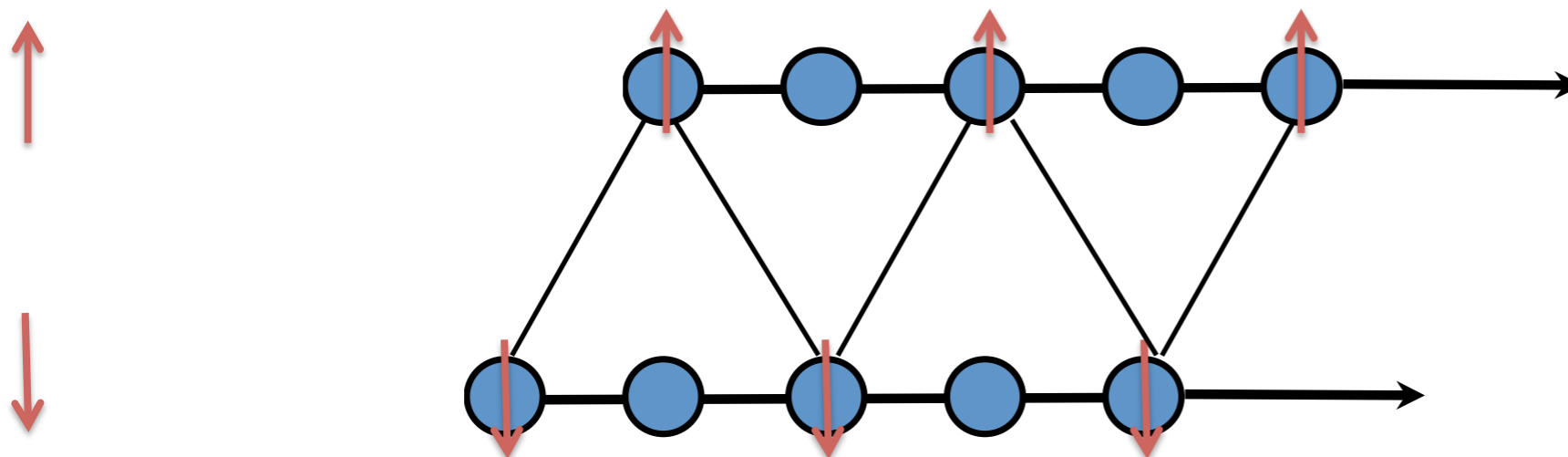
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1D chain of atoms (real dimension)



Atoms with different internal states are trapped at different lattice sites of a one-dimensional lattice  **Semisynthetic zigzag lattice**

(& laser-assisted transitions between \uparrow and \downarrow states)
(accompanied by a recoil)



Magnetic
flux: $\gamma = ka$

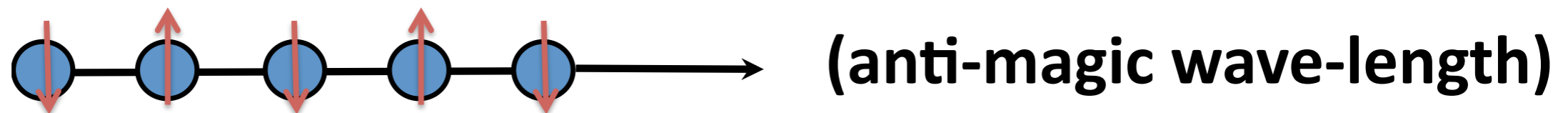
k - laser
wave-vector

Optical lattices in extra dimensions

Non-square geometry

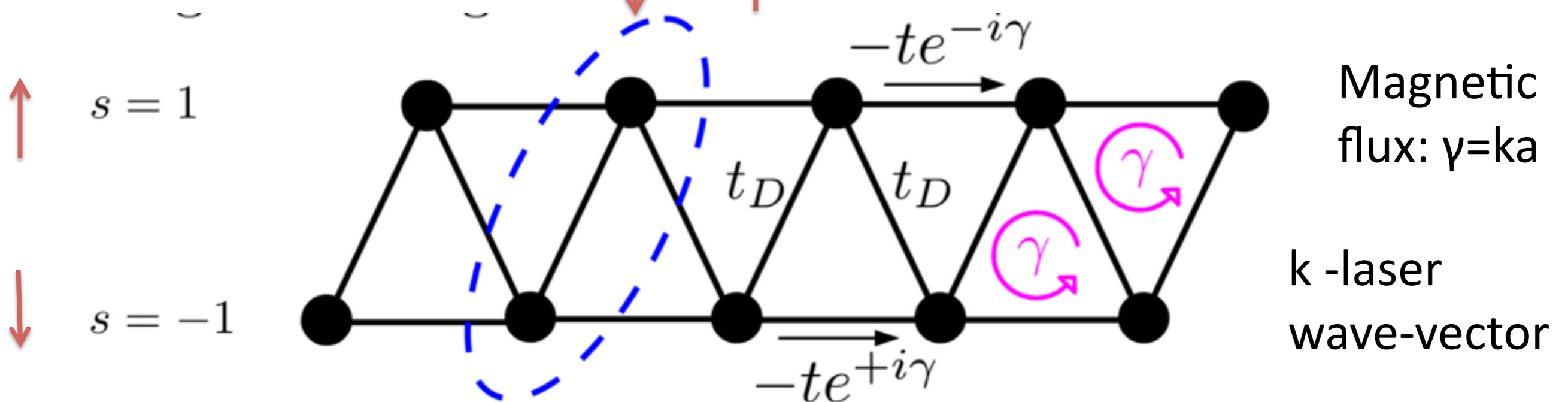
E. Anisimovas, M. Račiūnas, C. Sträter, A. Eckardt, I. B. Spielman, G. Juzeliūnas,
[Semi-synthetic zigzag optical lattice for ultracold bosons](#), Phys. Rev. A 94, 063632 (2016)

1D chain of atoms (real dimension)



Atoms with different internal states are trapped at different lattice sites of a one-dimensional lattice \rightarrow **Semisynthetic zigzag lattice**

Non-local interactions between \downarrow **and** \uparrow **atoms (in the semisynth. zigzag direction)**



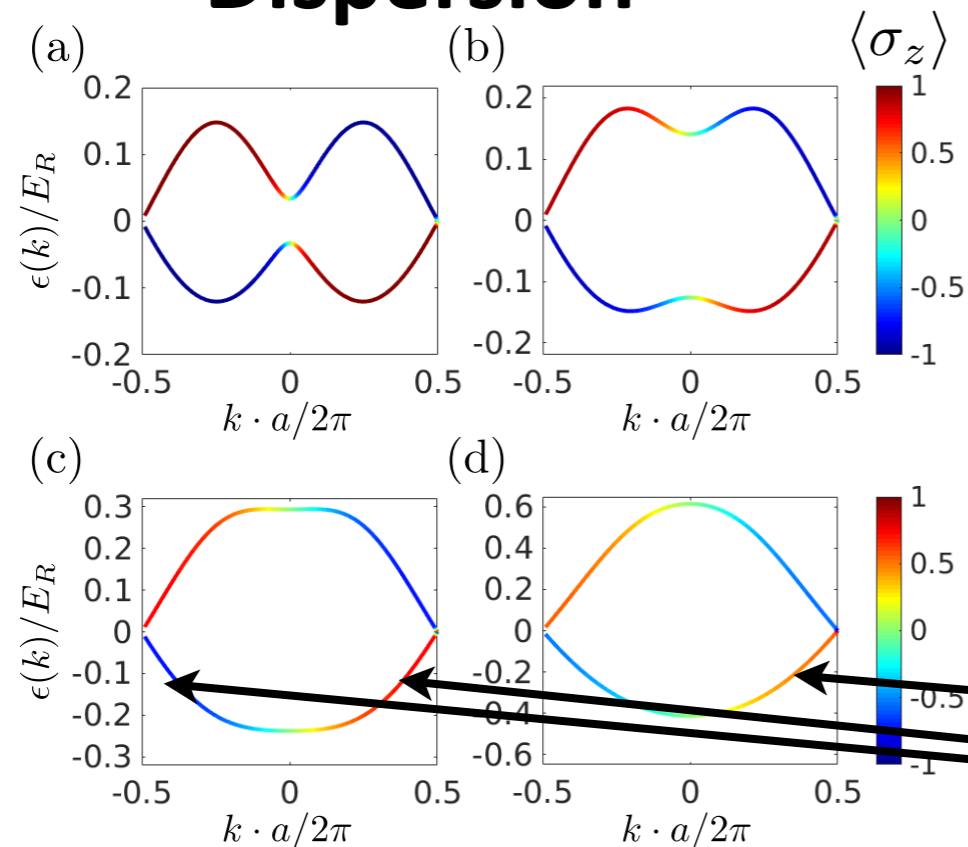
Optical lattices in extra dimensions

Non-square geometry

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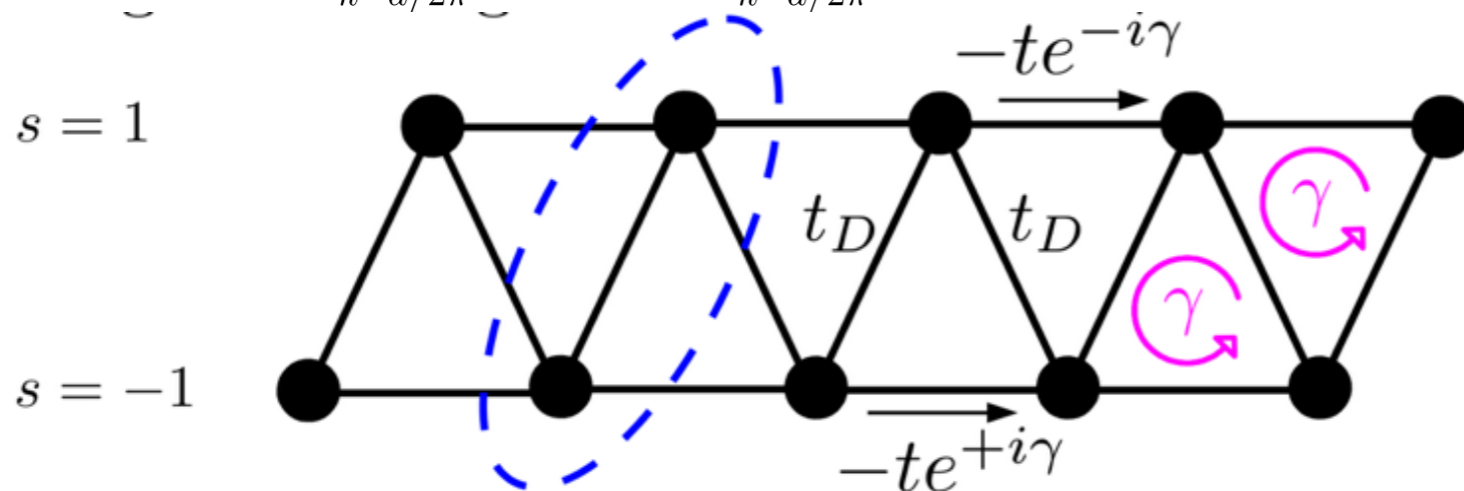
Dispersion

(for different values of t_D)



$$\gamma = ka = \pi/2$$

Edge states



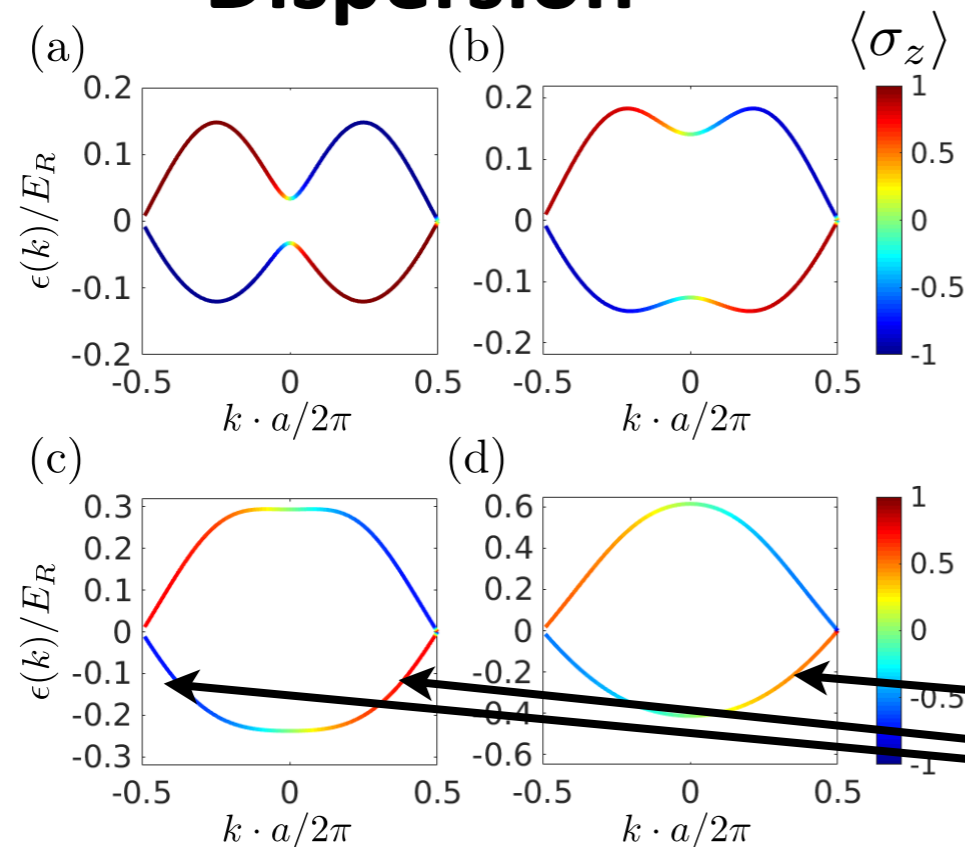
Optical lattices in extra dimensions

Non-square geometry

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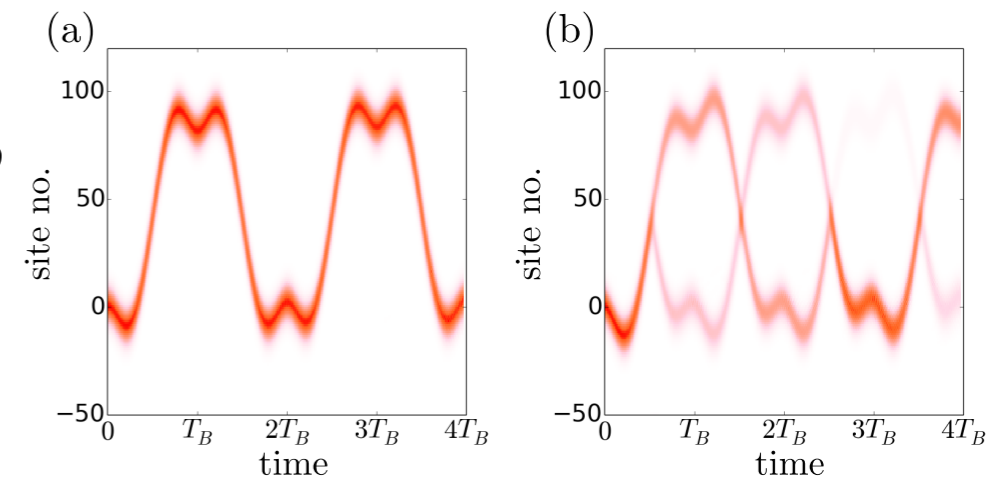
Dispersion

(for different values of t_D)



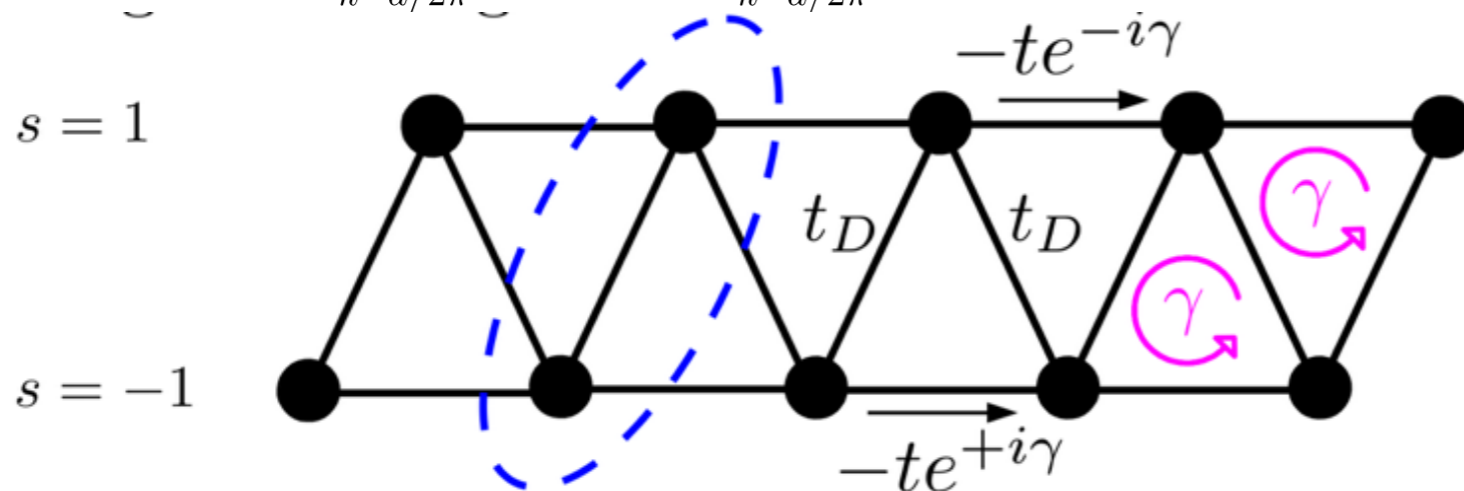
$$\gamma = ka = \pi/2$$

Bloch oscillations:



(a) doubling of the Bloch period

Edge states



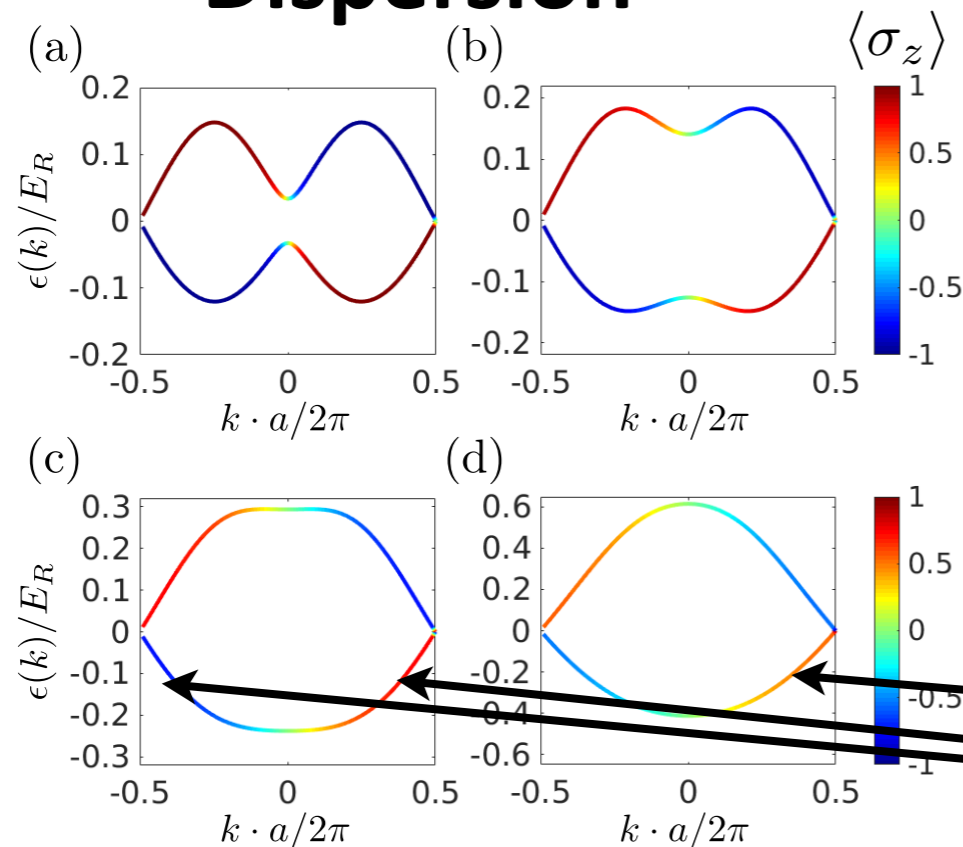
Optical lattices in extra dimensions

Non-square geometry

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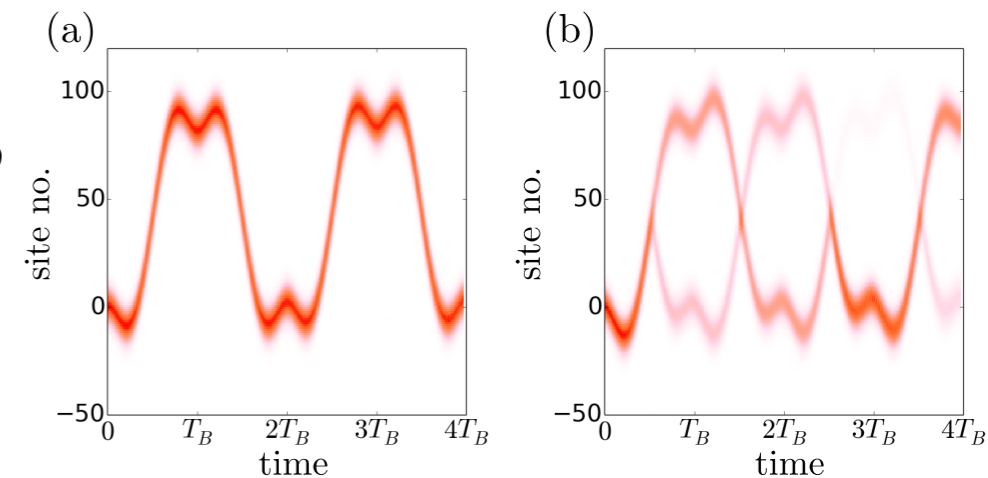
Dispersion

(for different values of t_D)



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Bloch oscillations:

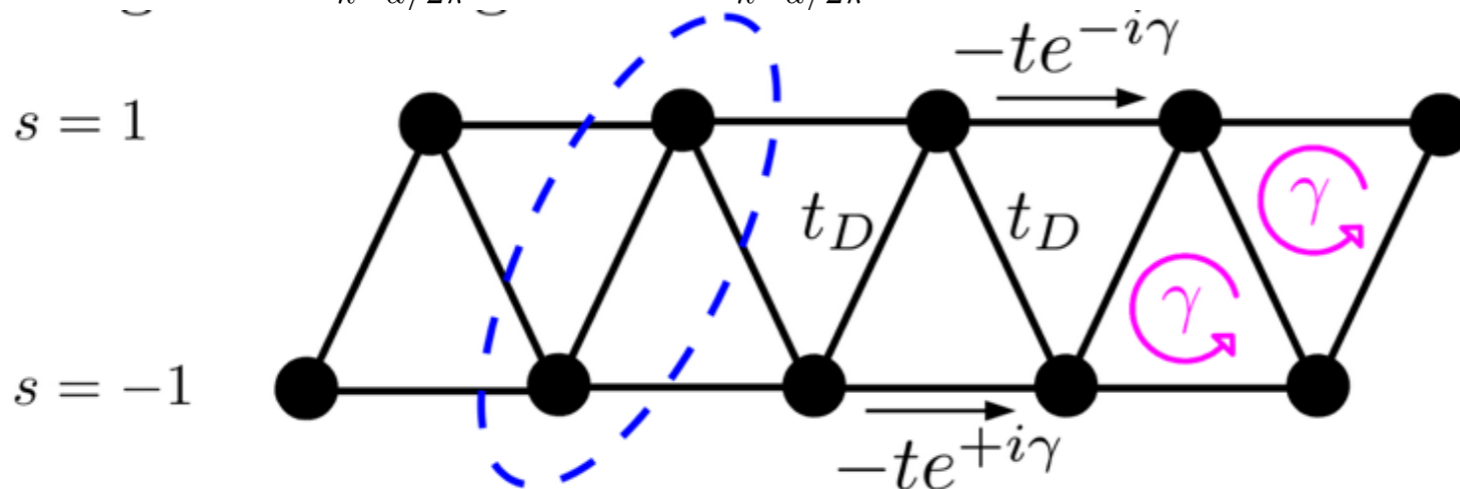


(a) doubling of the Bloch period

(b) - an additional spin-dependent detuning $0.3 \sigma_z$

Edge states

-> Landau-Zener tunneling



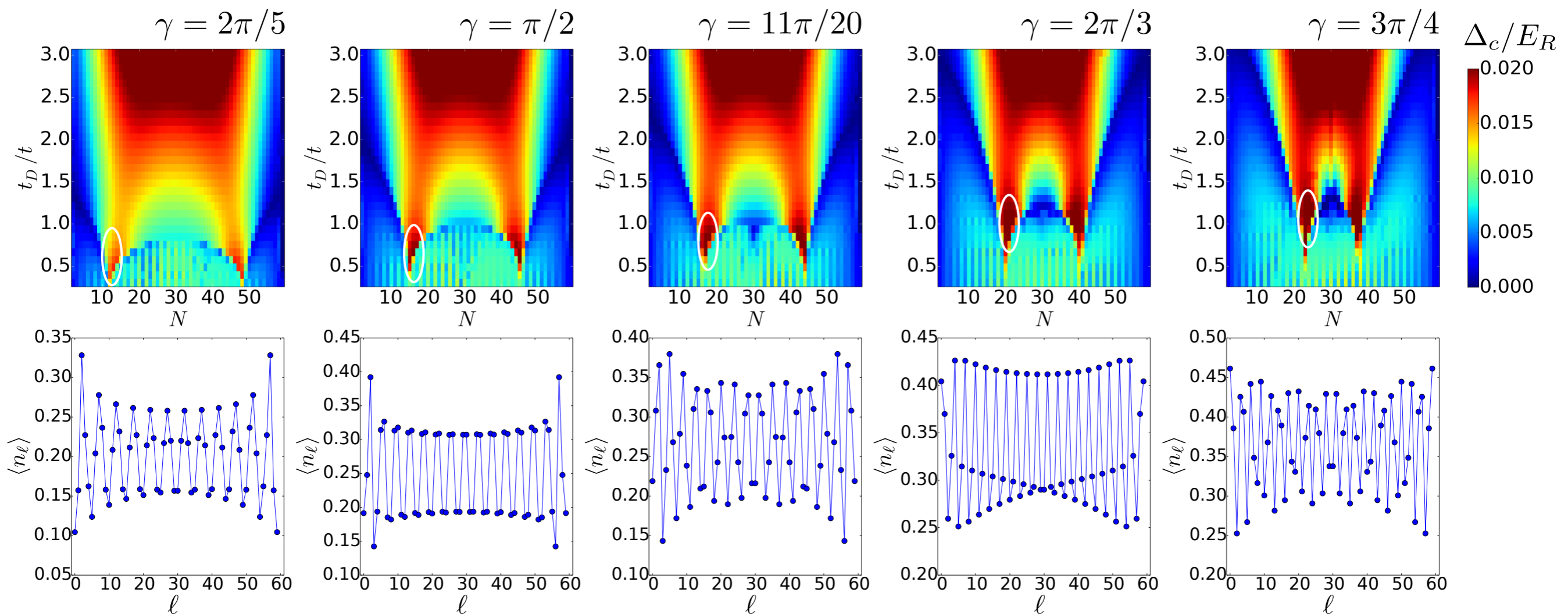
Optical lattices in extra dimensions

Non-square geometry

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Boson phase diagram for different γ

Δ_c : charge gap



Blue: superfluid phase

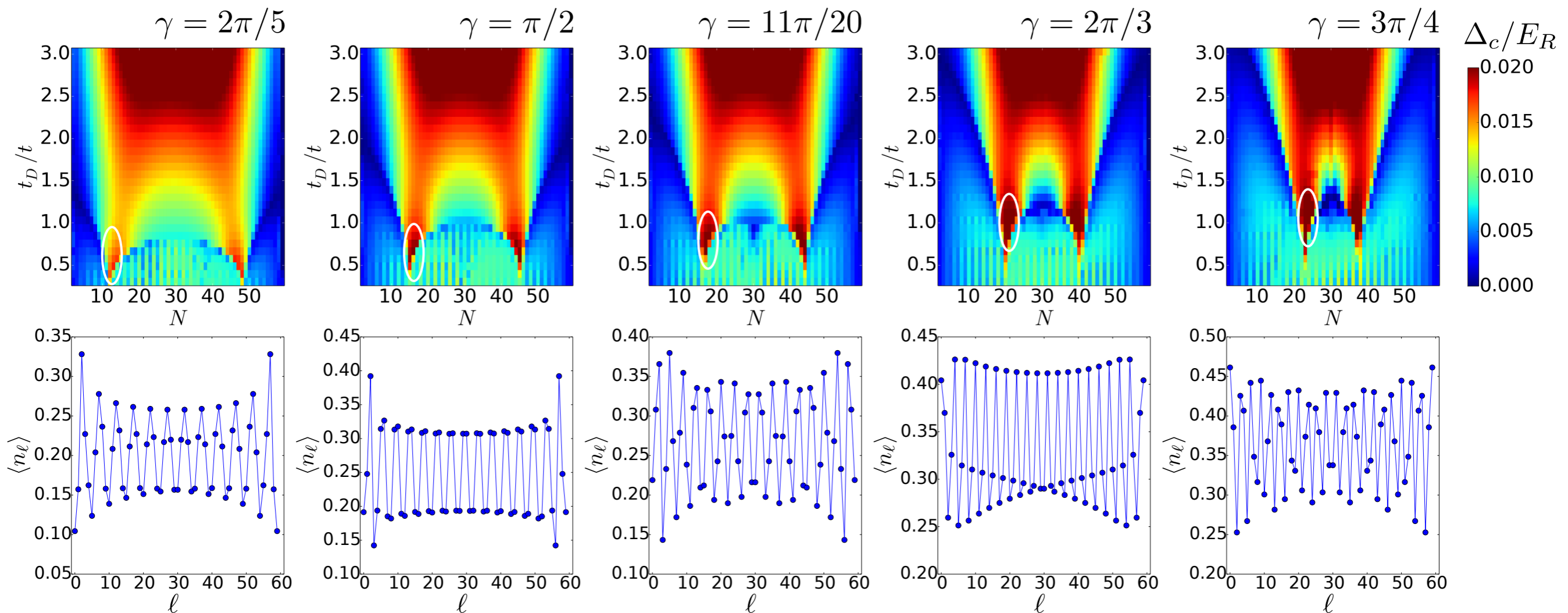
Optical lattices in extra dimensions

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Boson phase diagram for different γ

Δ_c : charge gap



Lower peaks: Charge density wave - one atom per magnetic unit cell

Optical lattices in extra dimensions

Non-square geometry

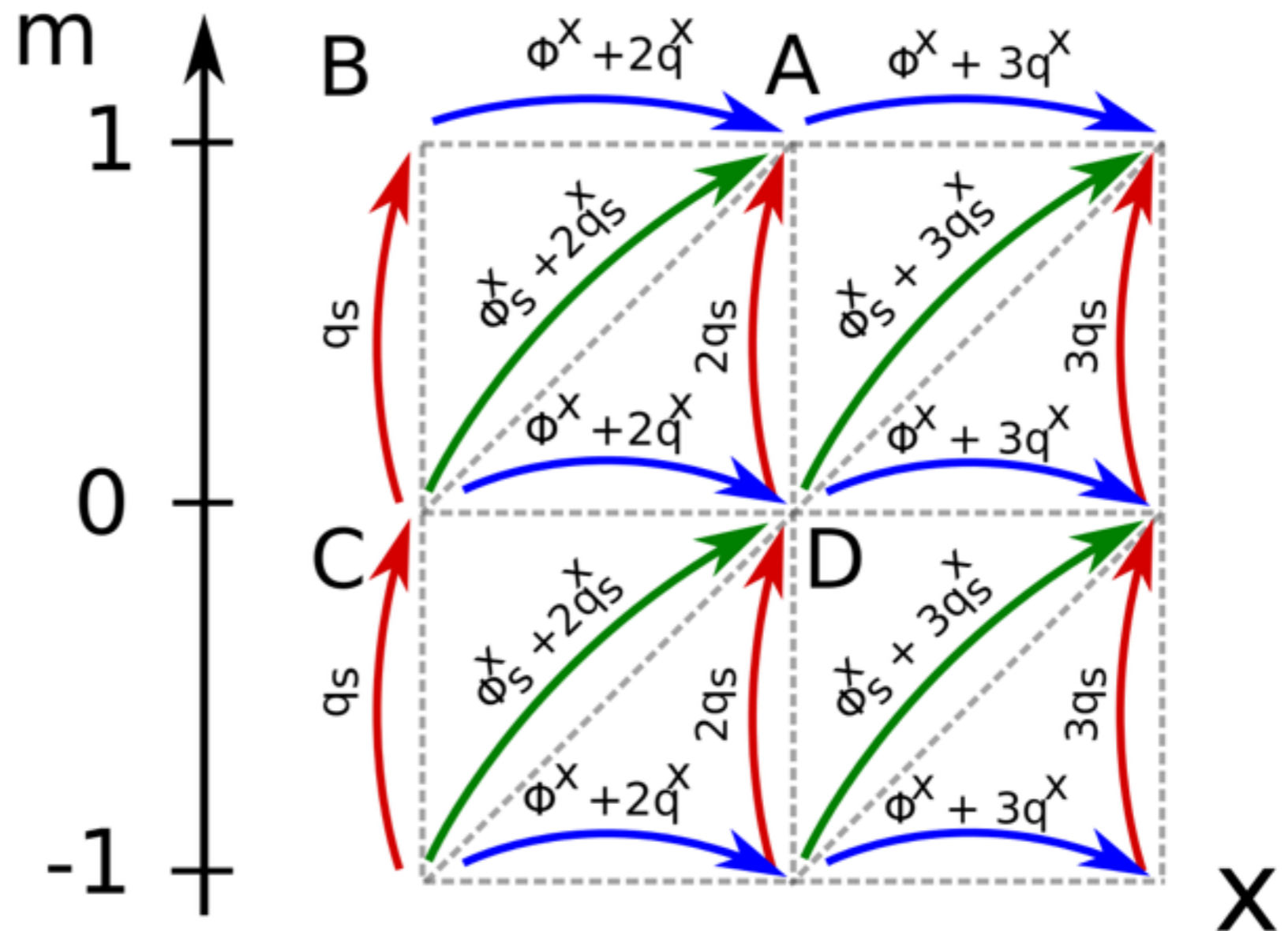
An alternative scheme

Optical lattices in extra dimensions

Non-square geometry

Laser-assisted tunnelling in addition to Raman transitions

D. Suszalski and J. Zakrzewski,
Phys. Rev. A **94**, 033602 (2016)



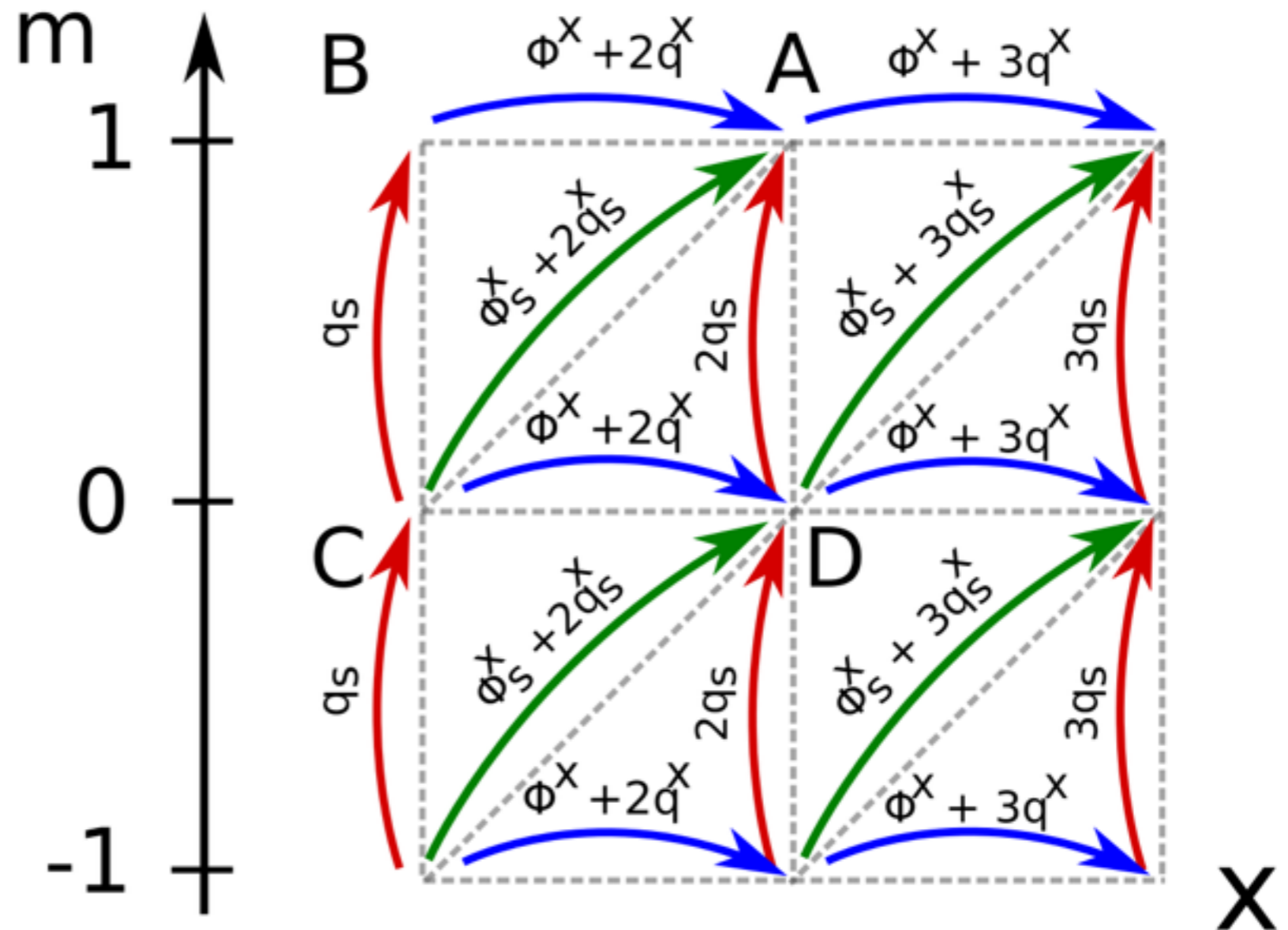
Optical lattices in extra dimensions

Non-square geometry

Laser-assisted tunnelling in addition to Raman transitions

D. Suszalski and J. Zakrzewski,
Phys. Rev. A **94**, 033602 (2016)

Might be complicated

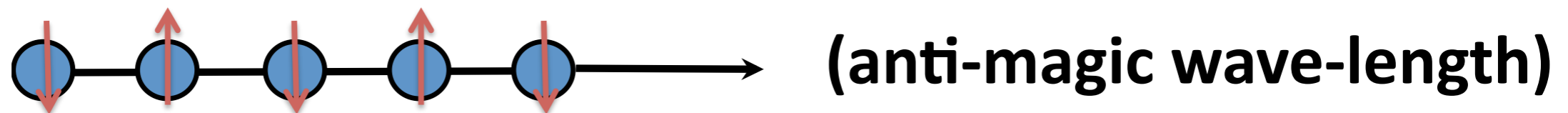


Optical lattices in extra dimensions

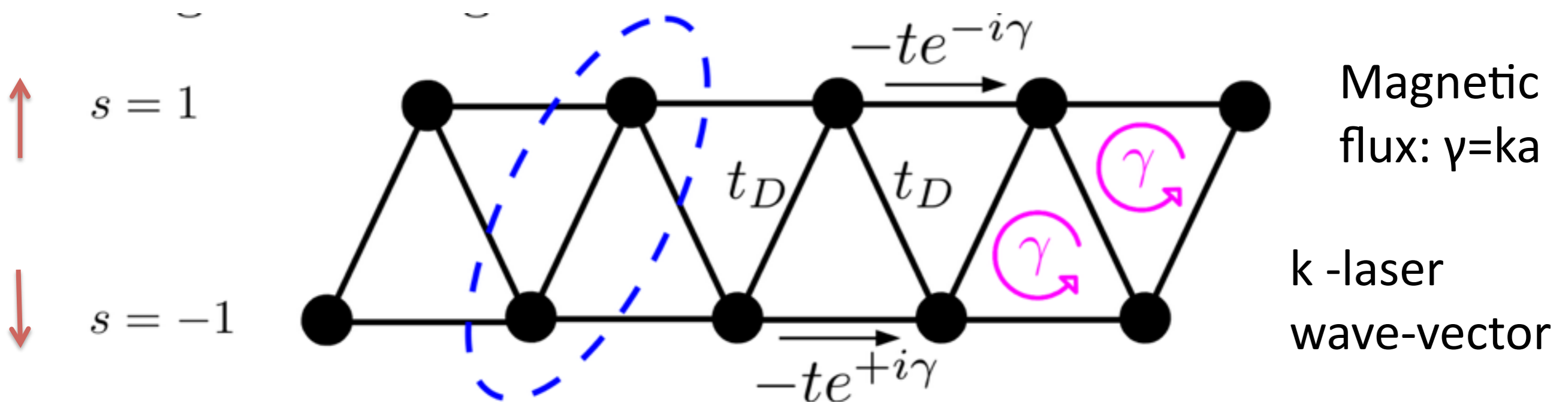
Non-square geometry - easier to implement

E. Anisimovas, M. Račiūnas, C. Sträter, A. Eckardt, I. B. Spielman, G. Juzeliūnas,
[Semi-synthetic zigzag optical lattice for ultracold bosons](#), Phys. Rev. A 94, 063632 (2016)

1D chain of atoms (real dimension)



Atoms with different internal states are trapped at different lattice sites of a one-dimensional lattice \rightarrow Semisynthetic zigzag lattice



Optical lattices in extra dimensions

Non-square geometry

Another related work

PHYSICAL REVIEW A **91**, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,¹ Christine Muschik,^{1,2,3} Alessio Celi,¹ Ravindra W. Chhajlany,^{1,4} and Maciej Lewenstein^{1,5}

¹*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*

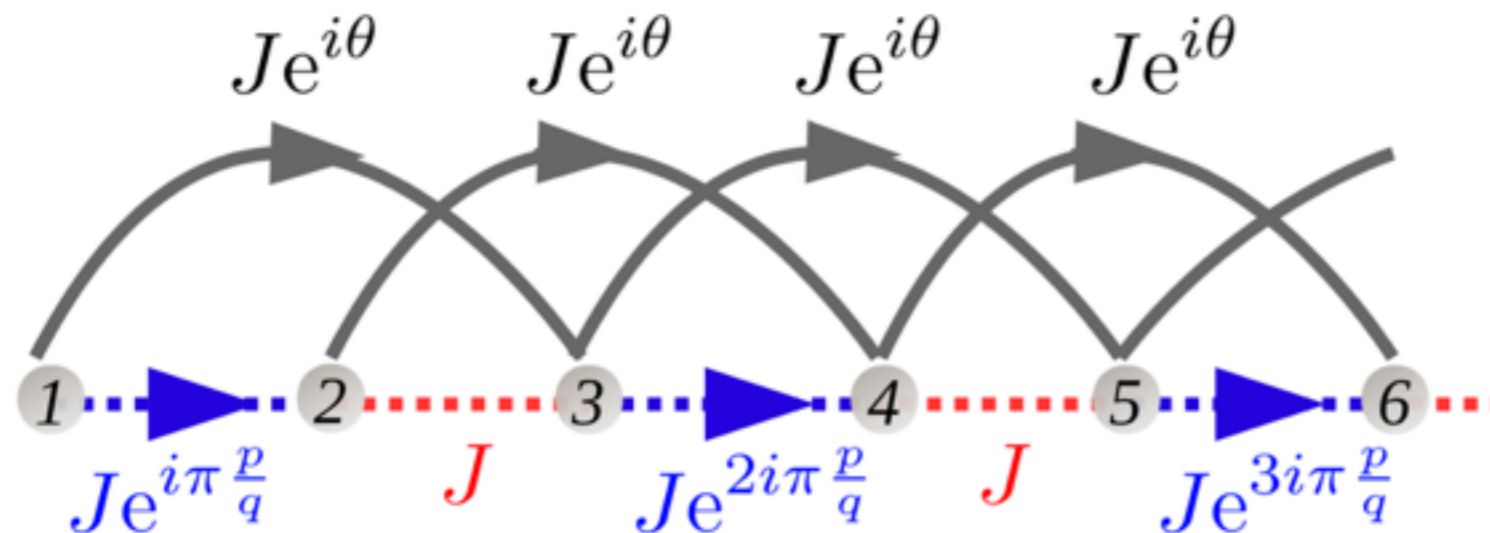
²*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*

³*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*

⁴*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

⁵*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluís Companys 23, 08010 Barcelona, Spain*

(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)



Conclusions (for Part II)

- Artificial magnetic field can be created in 1D optical lattices:
 - The atomic internal states serve as an extra dimension.
 - This makes a semi-synthetic 2D lattice (involving real and extra dimensions) affected by a non-staggered magnetic flux.
- The synthetic dimension has sharp boundaries at which the conducting edge states are formed.
- The edge states are immune to a short range scattering potential (or at least for lower energies).
- By closing the boundaries in the synthetic dimension one can get the Hofstadter butterfly spectrum in a remarkably simple manner.
- Semi-synthetic zigzag lattice can also be created exhibiting non-local atom-atom interaction.

THANK YOU!