Light-induced fractional quantum Hall physics in 2D materials and more

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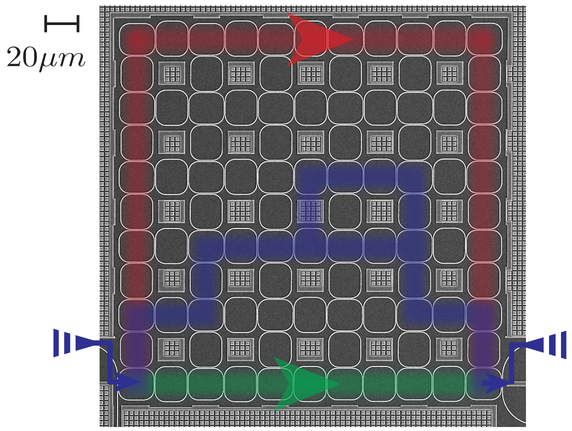
Synthetic dimensions in quantum engineered systems

ETHZ November 2017


VOODOO
"The Magic is in the Hole"
VOODOO
DOUGHNUT
SKIDMORE

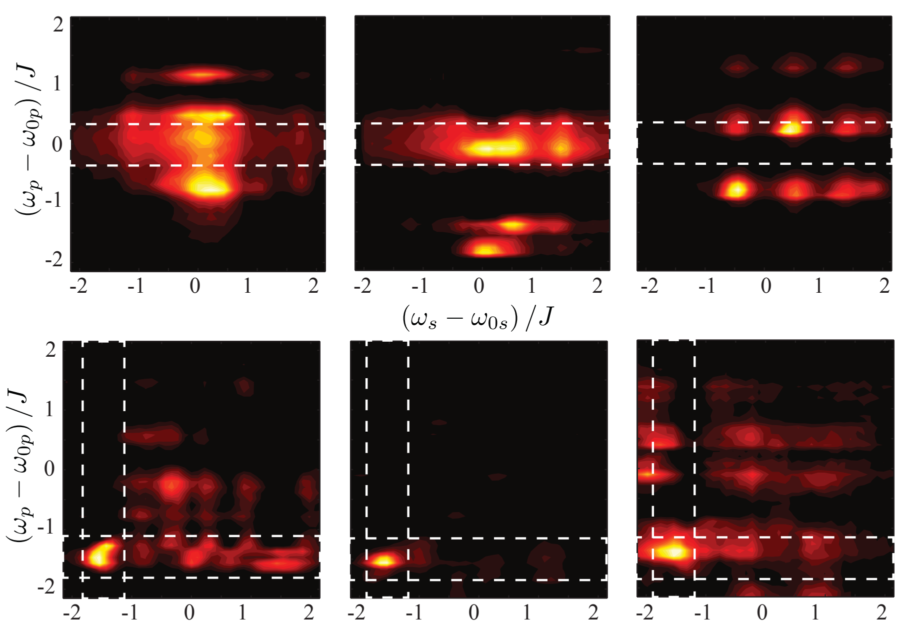
Quantum topological photonics

photon pair generation



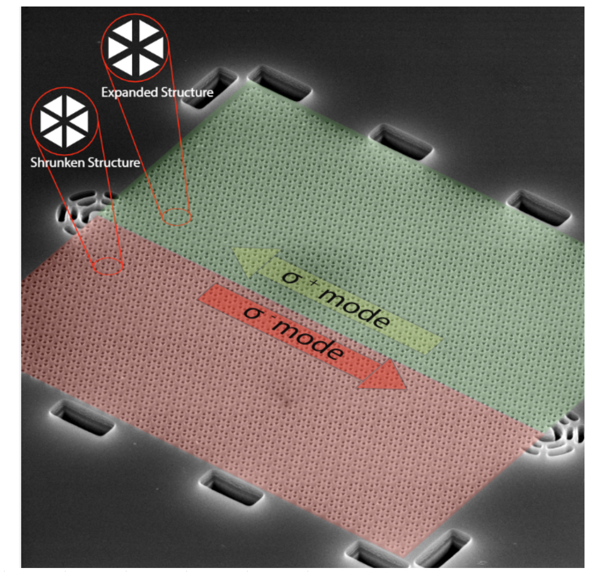
1D

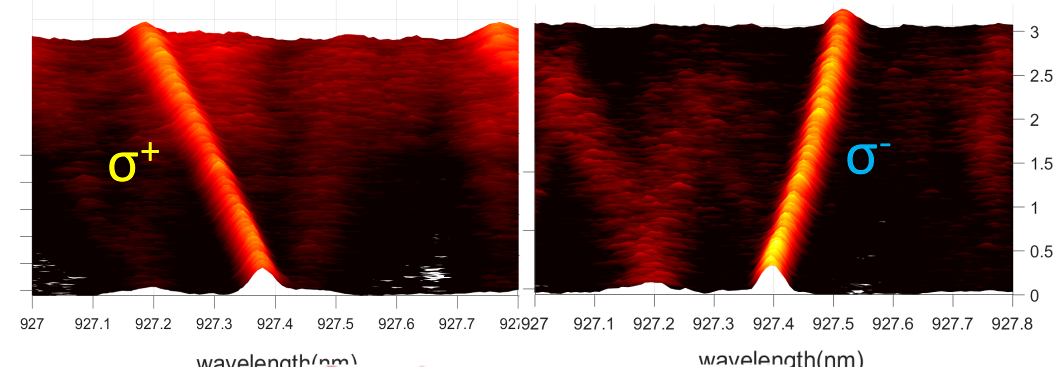
2D

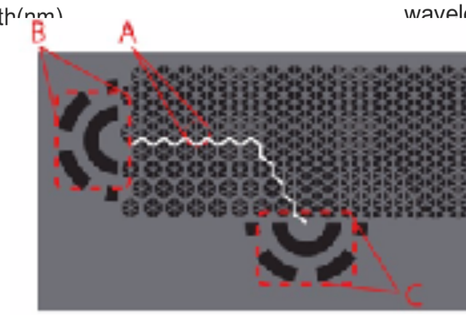


S. Mittal
arXiv:1709.09984

interface with quantum emitter





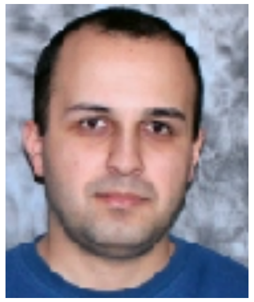


S. Barik, A. Karasahin, C. Flower, T. Cai,
H. Miyake, W. DeGottardi, E. Waks
arXiv:1711.00478

Outline

- Photons and electronic quantum Hall states

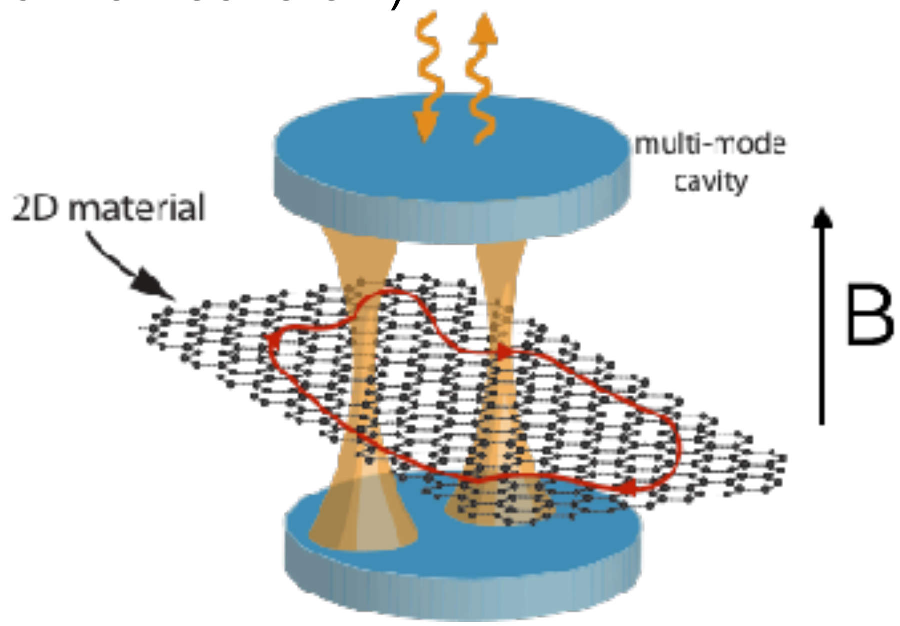
  - Optical probe of IQHE states
    M. Gullans, J. Taylor, A Imamoglu, P. Ghaemi, MH arXiv:1701.03464 (PRB)
  - Driven FQH states and bilayer physics
    A. Ghazaryan, T. Grass, M. J. Gullans, P. Ghaemi, MH arXiv:1612.08748 (PRL)



- *Quantum Origami*: Applying Transversal Gates and Measuring Topological Order (Modular transformation)
  G. Zhu, MH, M. Barkeshli arXiv:1701.03464

Optical probing of quantum Hall states

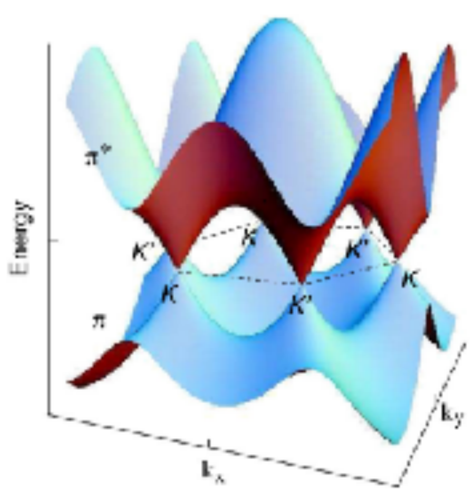
Current approaches:
(1) Transport (coherent but global)
(2) STM, SET (local but non-coherent)



$$H_{\mathrm{quad}} \propto (p + eA)^2 \to \hbar\omega_c a^\dagger a$$
$$\propto \Pi_x^2 + \Pi_y^2 \qquad \Pi_i = p_i + eA_i$$

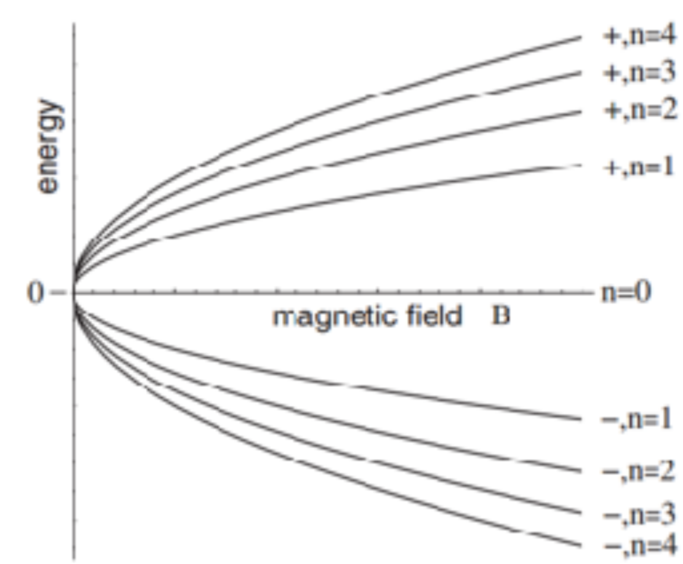
$$[\Pi_x, \Pi_y] \propto i$$ for uniform magnetic field

$$\Pi_x = \frac{\hbar}{\sqrt{2} l_{\mathrm{B}}}(a^\dagger + a) \quad \text{and} \quad \Pi_y = \frac{\hbar}{i\sqrt{2} l_{\mathrm{B}}}(a^\dagger - a)$$



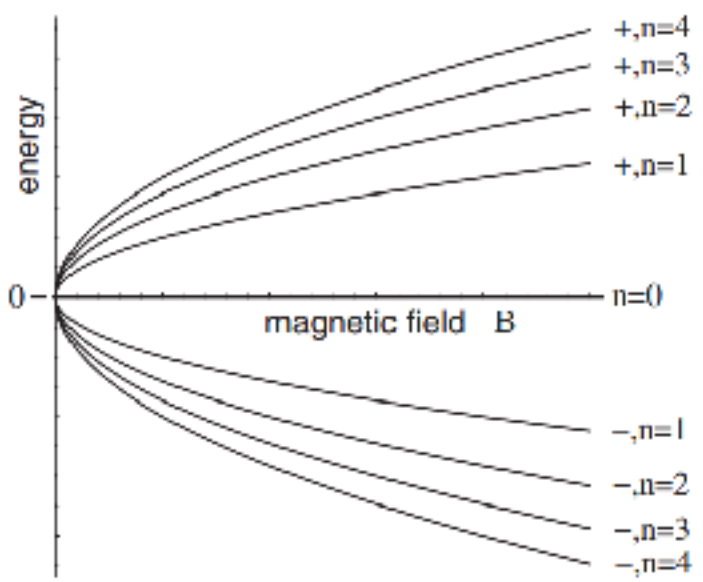
$$H_\xi = \xi v_{\mathrm{F}}(p_x\sigma_x + p_y\sigma_y)$$
$$\xi = \pm \text{ for } K, K'$$

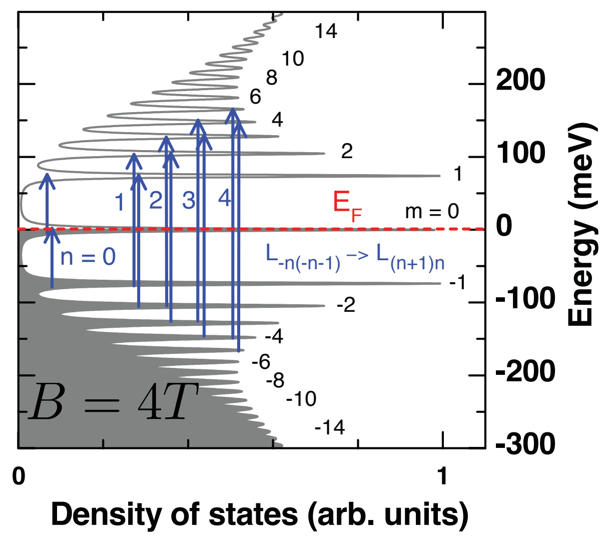
Pauli matrices represent sublattice structure

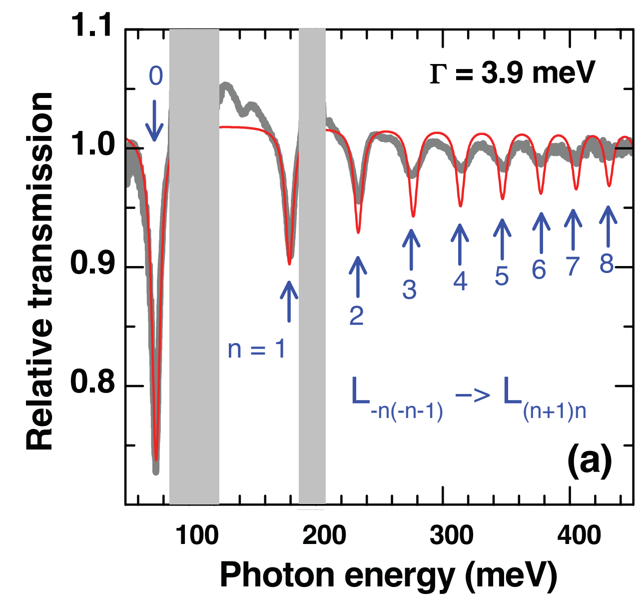


$$H_{\mathrm{lin}} \propto \begin{pmatrix} 0 & v_f(p_x - ip_y + eA_x - ieA_y) \\ v_f(p_x + ip_y + eA_x + ieA_y) & 0 \end{pmatrix} \to \frac{v_f}{l_b}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

Optical spectrum of quantum Hall states







Orlita, Potemski,

Stromer, Kim

$$H_{\text{int}} = (-1)^s \frac{ev}{\sqrt{2}c}[\tau_+ A_+^*(x,y) + \tau_- A_-^*(x,y)]e^{-i\omega t} + h.c.$$

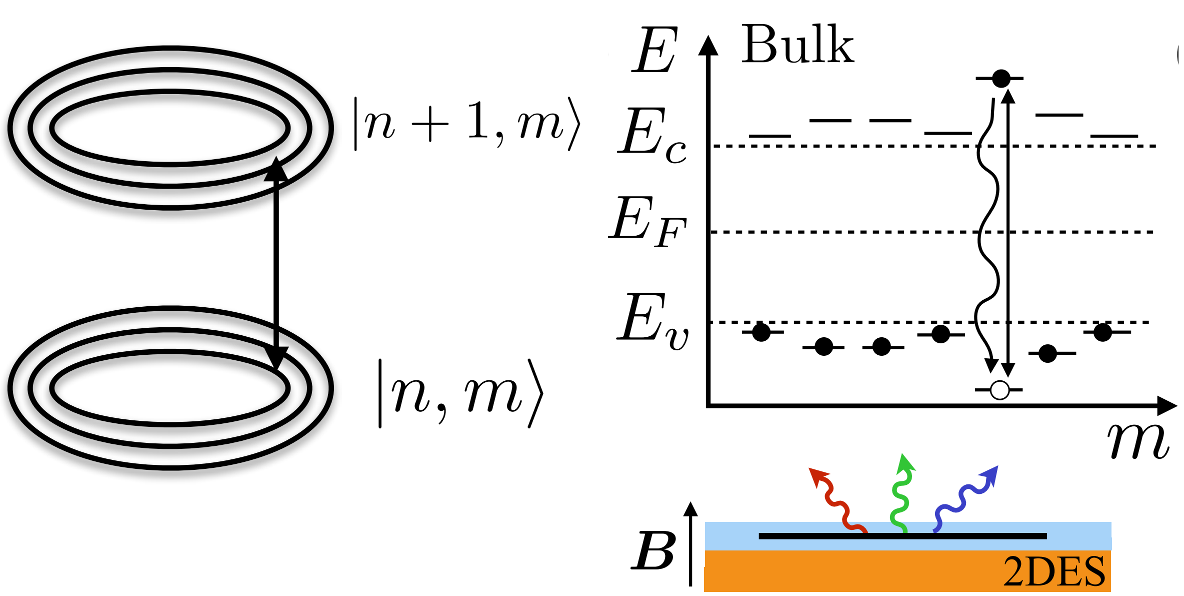
$$\tilde{n} = |n| \pm 1 \quad \text{and} \quad m = \tilde{m}$$

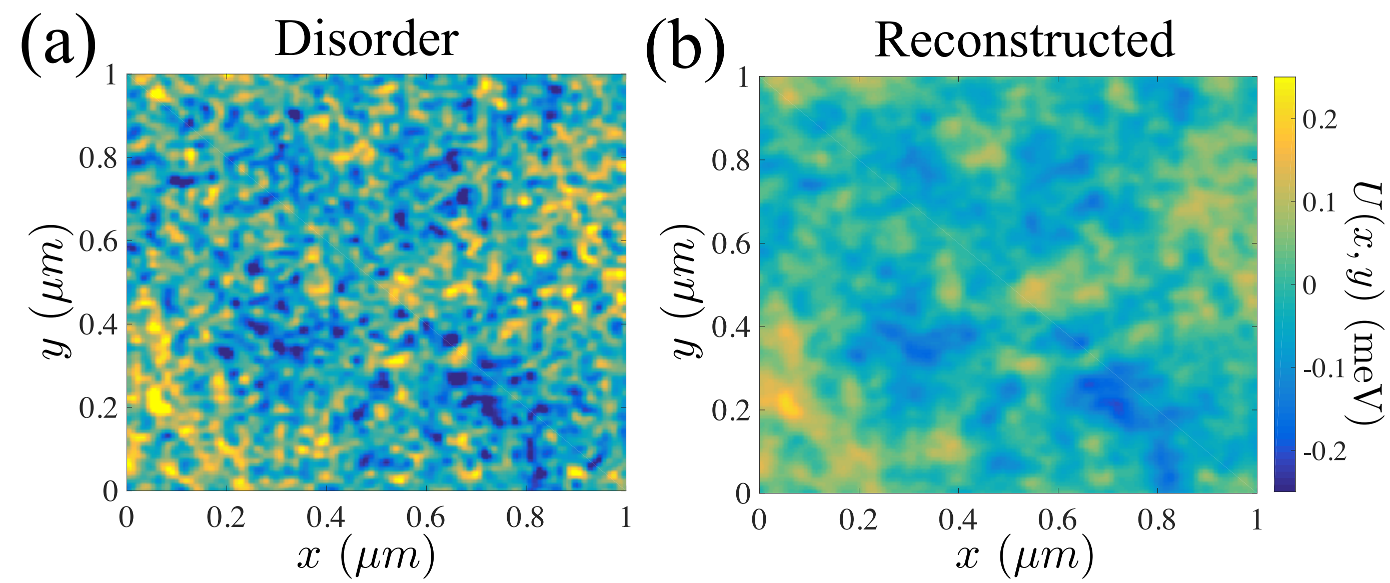
GaAs works by Pinczuk,….
Recently in cavity: Imamoglu (2016)
Dicke-superradiance: Ciuti, Fazio, MacDonald ..

Spatial probe of quantum Hall states

Mapping disorder landscape

$$H_{\text{dis}} = u_0(\boldsymbol{r})I + \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{\tau}$$





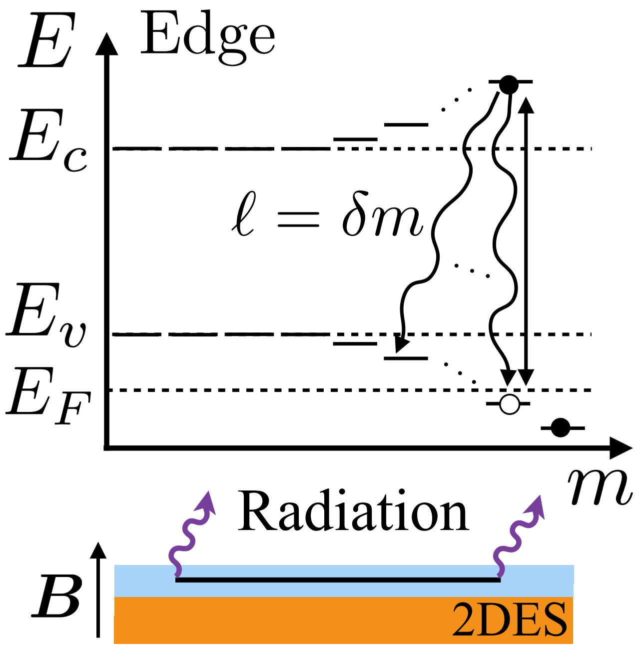
If Fermi level is in between two transitions

$$H_{\text{int}} = (-1)^s \frac{ev}{\sqrt{2}c}[\tau_+ A_+^*(x, y) + \tau_- A_-^*(x, y)]e^{-i\omega t} + h.c.$$

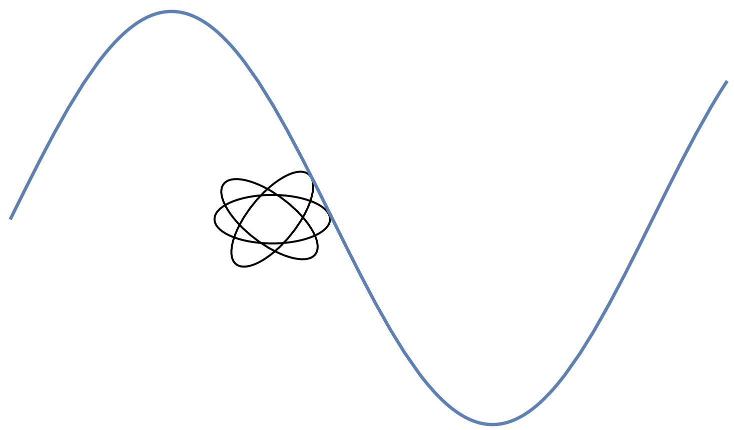
$$\tilde{n} = |n| \pm 1 \quad \text{and} \quad m = \tilde{m}$$

M. Gullans et al. PRB (2017)

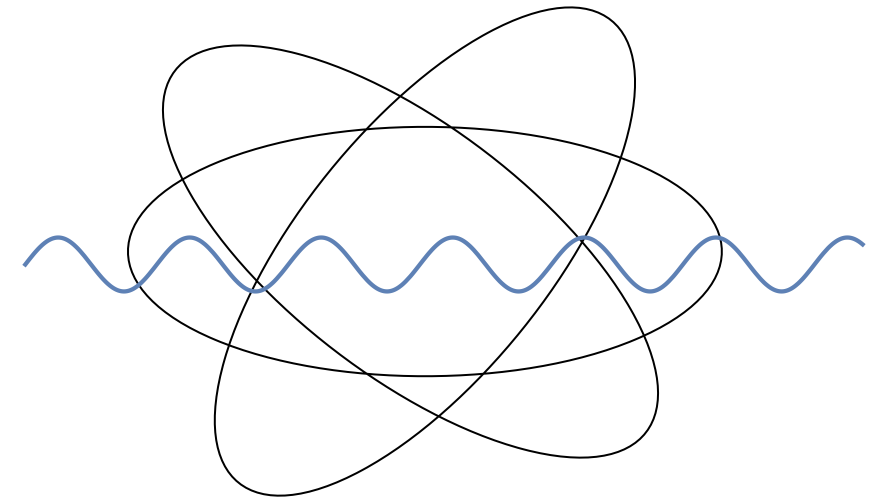
What if the Fermi level is inside a band?



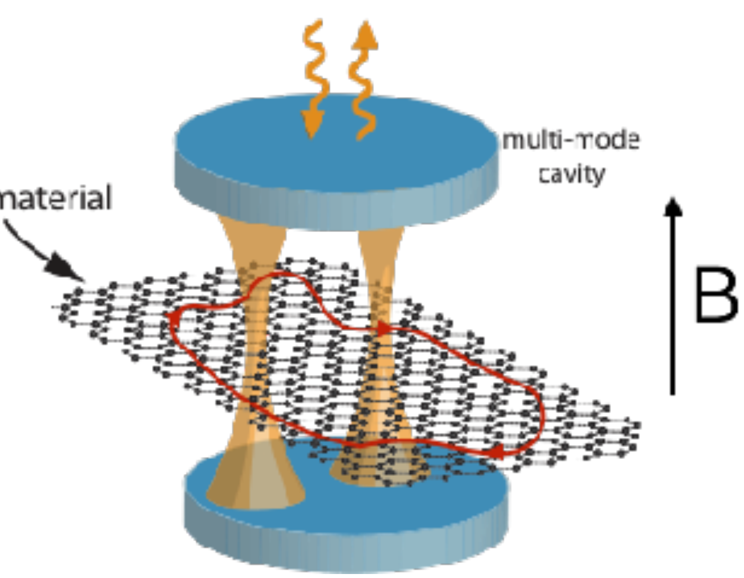
Most cases:
dipole approximation



Multipole emission



Extended states of electrons,
e.g. Quantum Hall, Rydberg excitations

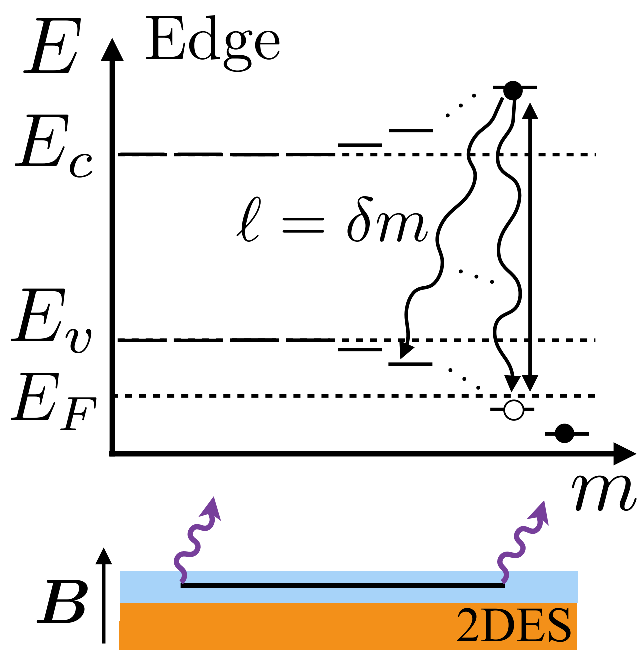


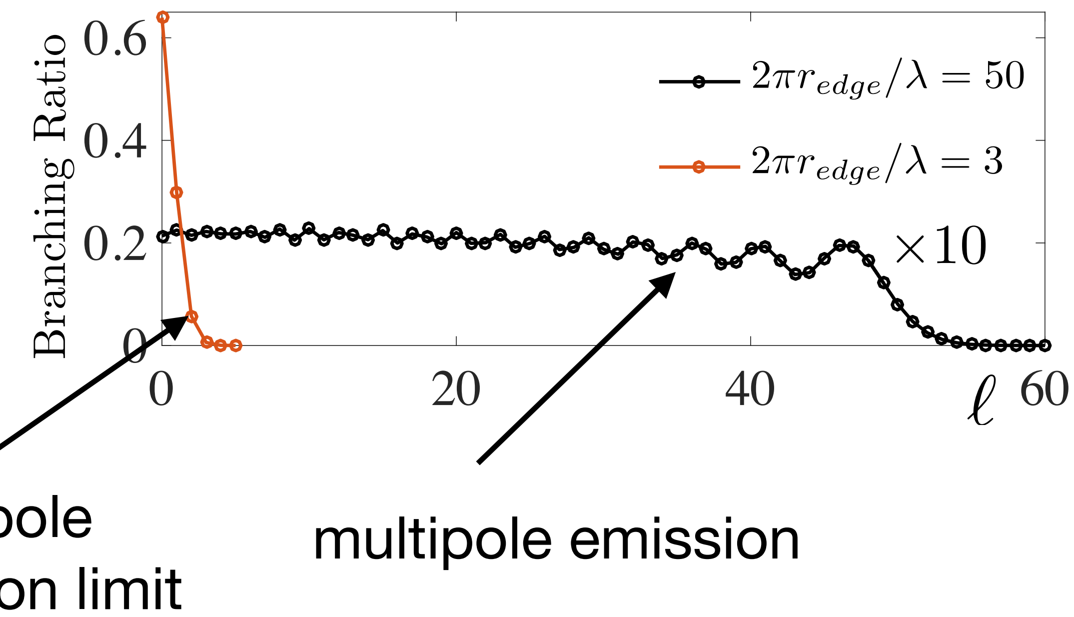
$$(re^{-i\theta})^{m'} \qquad (re^{i\theta})^{m}$$

$$ev_f \langle 1, m' | \tau_+ \mathbf{A} \quad |0, m\rangle \rightarrow \delta_{m',m+l}$$

$$A \propto e^{ikz+il\theta} J_l(k_\perp r)$$

higher orbital angular momentum emission

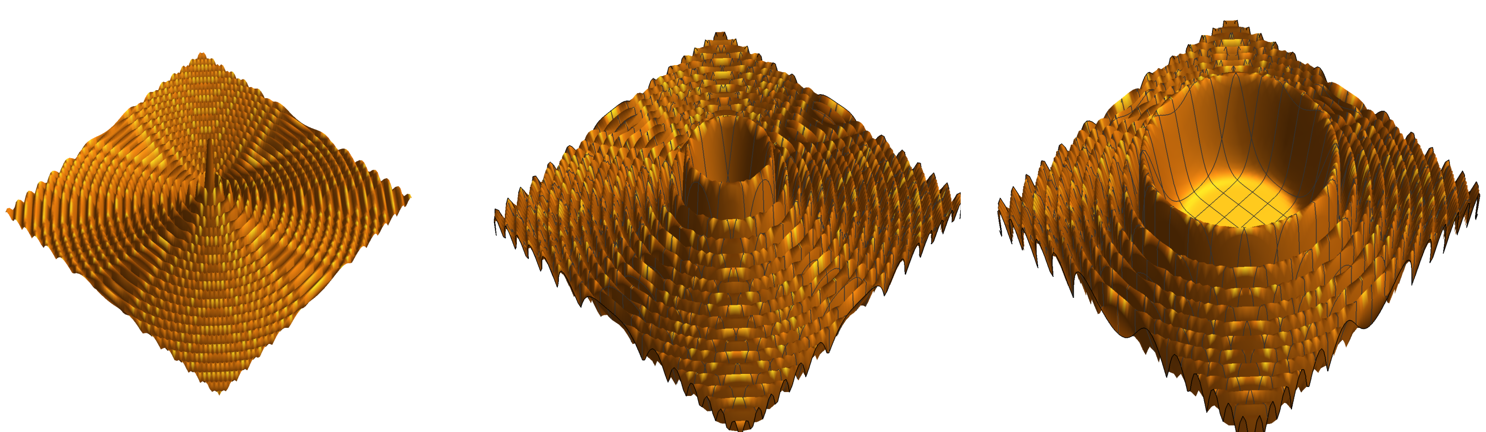




Bessel functions:

$$k_\perp r_{vortex} = l$$

$$k_\perp = \sqrt{(\omega/c)^2 - k_\parallel^2}$$

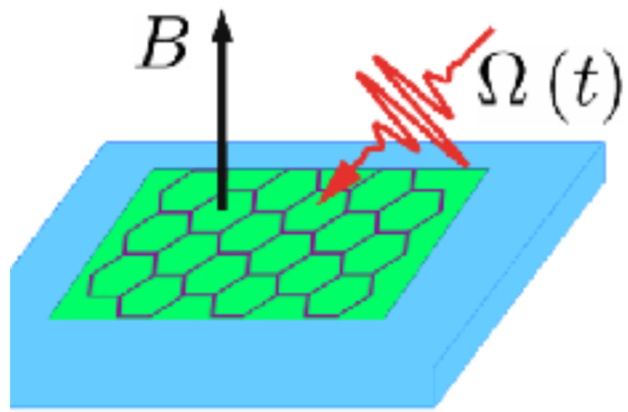


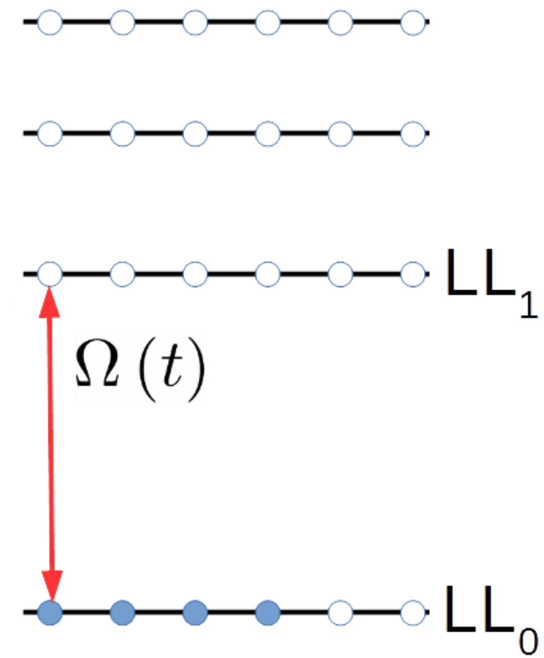
$l = 0$ $l = 10$ $l = 25$

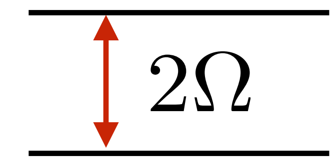
Maximum OAM for largest radius and smallest $k_\perp$:

$$\frac{2\pi}{\lambda} r_{edge} = l_{max}$$

Synthetic bilayer Graphene







Use light to couple two LLs:

(1) Different LL plays the role of layers
(2) Light plays the role of tunneling

LL energy — Coupling

$$H_0(t) = \sum_m \left[\frac{\Delta E}{2} \left(c^\dagger_{n+1,m} c_{n+1,m} - c^\dagger_{n,m} c_{n,m} \right) + \hbar\Omega \left(c^\dagger_{n+1,m} c_{n,m} \mathrm{e}^{-i(\Delta E - \delta)t} + \mathrm{h.c.} \right) \right]$$

In rotating frame after rotating wave approximation

$$H_0 = \sum_m \left[\hbar\delta c^\dagger_{n+1,m} c_{n+1,m} + \hbar\Omega c^\dagger_{n+1,m+\mu} c_{n,m} \right] + \mathrm{h.c.}$$

A. Ghazaryan, T. Grass, M. J. Gullans,
B. P. Ghaemi, and M. H. arXiv:1612.08748

Coulomb interaction in synthetic bilayer

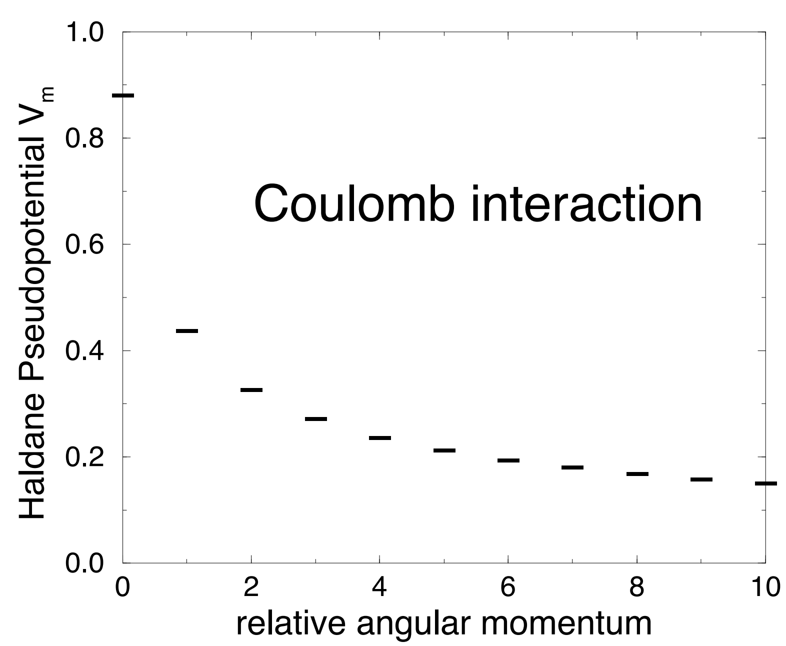
Total Hamiltonian: $H = H_0 + V^{(\mathrm{RWA})}$

$$V_{m_1,m_2,m_3,m_4}^{n_1,n_2,n_3,n_4(\mathrm{RWA})} = \delta_{n_1+n_2-n_3-n_4} V_{m_1,m_2,m_3,m_4}^{n_1,n_2,n_3,n_4}$$

Pseudopotential expansion: $V_{m_1,m_2,m_3,m_4}^{n_1,n_2,n_3,n_4} = \sum_{m,M} V_m^{n_1,n_2,n_3,n_4} \langle m_1, m_2 | m, M \rangle \langle m, M | m_3, m_4 \rangle$

We consider two LLs, so (n_1, n_2, n_3, n_4) can be represented by pseudo-spin 1/2.

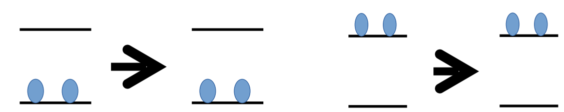
- Haldane pseudo-potential: Any interaction in the presence of rotation symmetry can be simplified in terms of relative momenta



Review: Girvin Les Houches (1999)

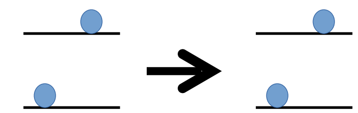
Haldane pseudo potential for synthetic bilayer

$$\hat{V} = \sum_M \Bigg[\sum_{m\ \mathrm{odd}} \left(V_m^{\uparrow} |mM,\uparrow\uparrow\rangle \langle mM,\uparrow\uparrow| + V_m^{\downarrow} |mM,\downarrow\downarrow\rangle \langle mM,\downarrow\downarrow| \right) +$$



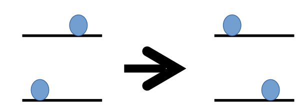
intra-layer: breaks Z_2
(not true in conventional bilayer)

$$\sum_m V_m^{||} \left(|mM,\uparrow\downarrow\rangle \langle mM,\uparrow\downarrow| + |mM,\downarrow\uparrow\rangle \langle mM,\downarrow\uparrow| \right) +$$



inter-layer: direct

$$\sum_m V_m^{\times} \left(|mM,\uparrow\downarrow\rangle \langle mM,\downarrow\uparrow| + |mM,\downarrow\uparrow\rangle \langle mM,\uparrow\downarrow| \right) \Bigg]$$



inter-layer: cross
(absent in conventional bilayer)

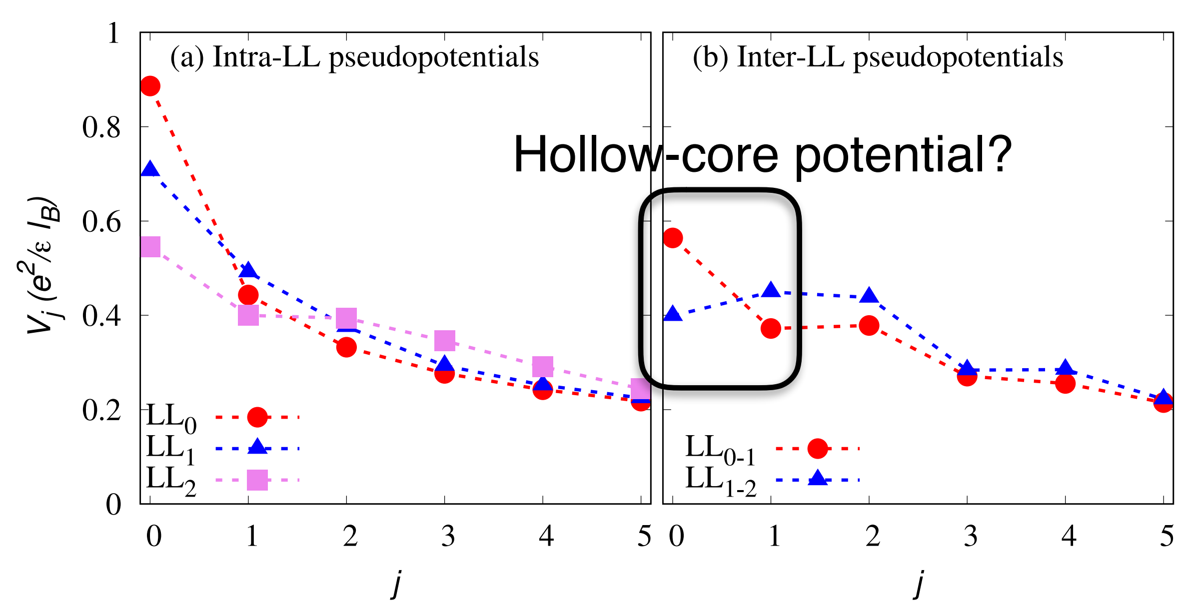
$$|+\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$|-\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

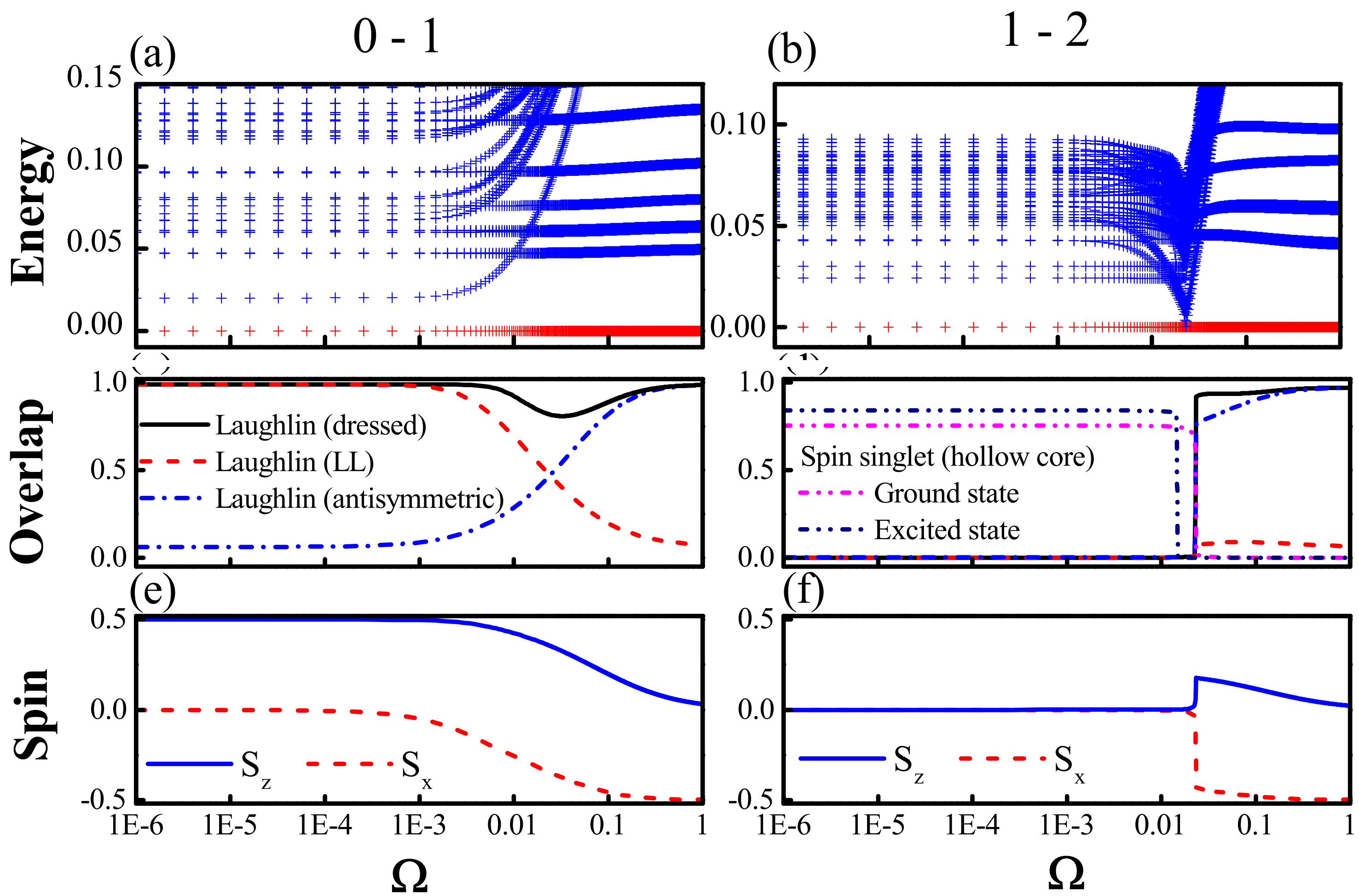
$$\hat{V} = \sum_M \Bigg[\sum_{m\ \mathrm{odd}} V_m^{\uparrow} \left(|mM,\uparrow\uparrow\rangle \langle mM,\uparrow\uparrow| + V_m^{\downarrow} |mM,\downarrow\downarrow\rangle \langle mM,\downarrow\downarrow| \right) +$$

$$\sum_{m\ \mathrm{odd}} \left[V_m^{||} + V_m^{\times} \right] |mM,+\rangle \langle mM,+| + \sum_{m\ \mathrm{even}} \left[V_m^{||} - V_m^{\times} \right] |mM,-\rangle \langle mM,-| \Bigg]$$

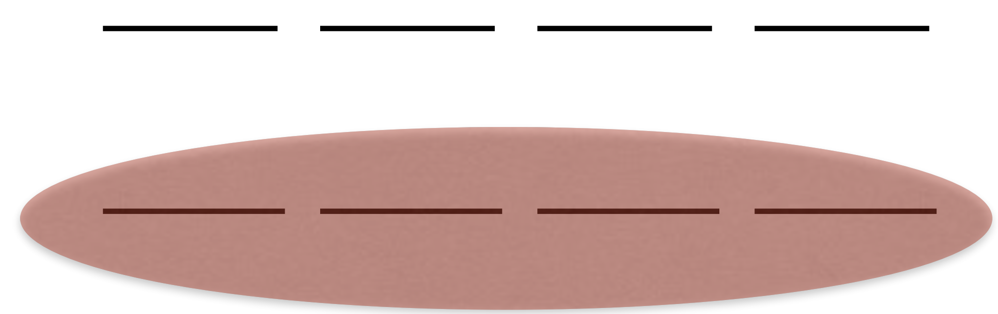
$$V_m^{\mathrm{inter}} = \begin{cases} V_m^{||} + V_m^{\times} & \text{if } m \text{ is odd,} \\ V_m^{||} - V_m^{\times} & \text{if } m \text{ is even} \end{cases}$$



- Filling factor is $\nu = 2/3$



Dressed Laughlin $\quad \Omega \ll e^2/l_B$



$$z_i^m \longrightarrow \prod_{i<j} (z_i - z_j)^m$$

	Sphere	Disk	Torus
$\nu = 1/2$ (HR)	$0.85\ (N=6)$ $0.75\ (N=8)$ $0.72\ (N=10)$	0.97 $(N=6, L=24)$	$0.83\ (\mathbf{K}=\mathbf{0})$ $0.72\ (\mathbf{K}\neq\mathbf{0})$ $(N=8)$
$\nu = 2/3$ (IP)	$0.99\ (N=4)$ $0.55\ (N=8)$ $0.39\ (N=12)$	$0.81\ (N=6, L=18)$ $0.63\ (N=8, L=36)$	

→ overlap with hollow-core potential

For bilayer: McDonald Haldane PRB (1996)
Recently: Peterson, Barkeshli, Wen, Vaezi, …

Outlook:

Thermalization in the driven system: Can phonons cool the system in the rotating frame?

Dehghani, Oka, and Mitra, PRB(2014)
Iadecola and Chamon PRB (2015)

Seetharam, Bardyn, Lindner, Rudner, and Refael, PRX (2015)

Engineering tunneling, interaction, parent Hamiltonian

Constructing twist defects?

Barkeshli, Qi PRX (2014)

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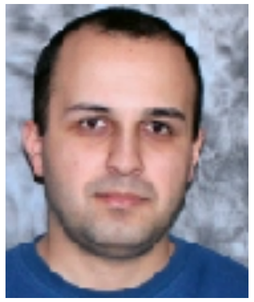

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  G. Zhu, MH, M. Barkeshli arXiv:1701.03464

Topologically protected operations

<table>
<tr><th>Topology</th><th>Topological phase of matter</th><th>Topological quantum error correction code</th></tr>
<tr><td>surface Σ</td><td>ground-state subspace $\mathcal{H}_\Sigma$</td><td>code space $\mathcal{H}_\Sigma$</td></tr>
<tr><td>self-diffeomorphism $h : \Sigma \to \Sigma$</td><td colspan="2">protected logical operation $U(h) : \mathcal{H}_\Sigma \to \mathcal{H}_\Sigma$</td></tr>
<tr><td rowspan="2">insensitive to local deformation of the metric</td><td colspan="2">insensitive to local perturbation</td></tr>
<tr><td>protected by the gap</td><td>active error correction</td></tr>
</table>

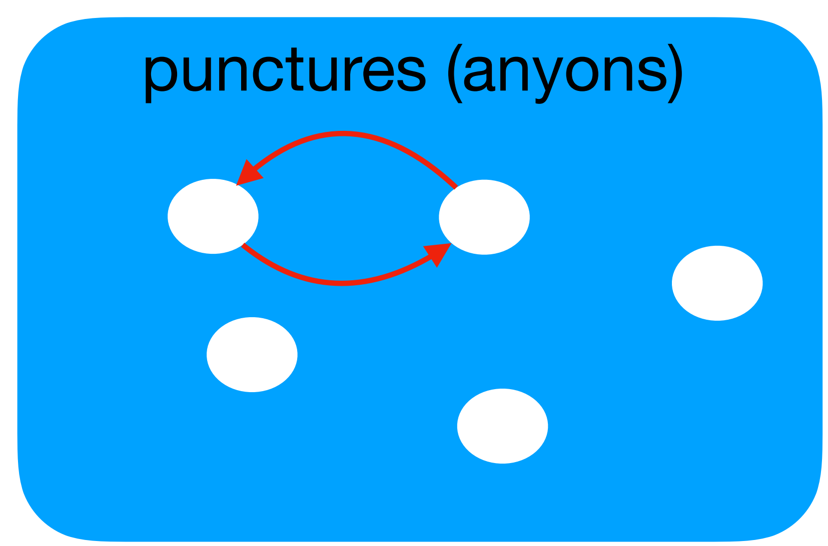
Mapping class group (MCG) of Σ :

Smooth deformations of the geometry that bring the system back to itself (self-diffeomorphism), modulo continuously deformable to the identity map.

Example of surface diffeomorphism:

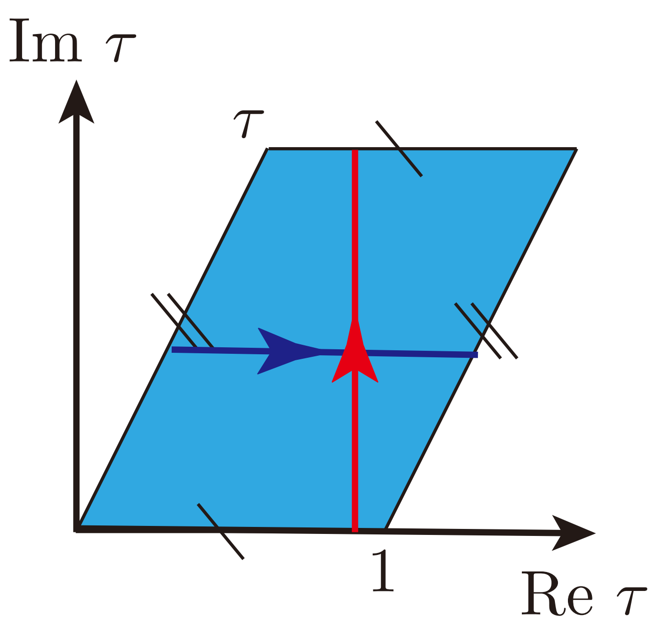
h : braid group of n punctures on Σ

$U(h)$: representation of the braid group on $\mathcal{H}_\Sigma$

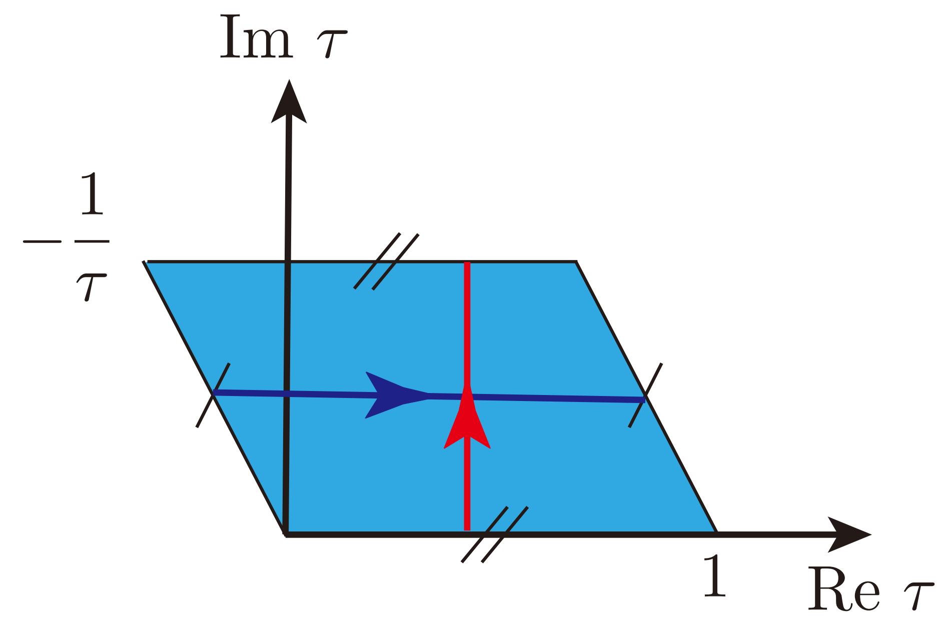


Group elements of MCG: modular transformations

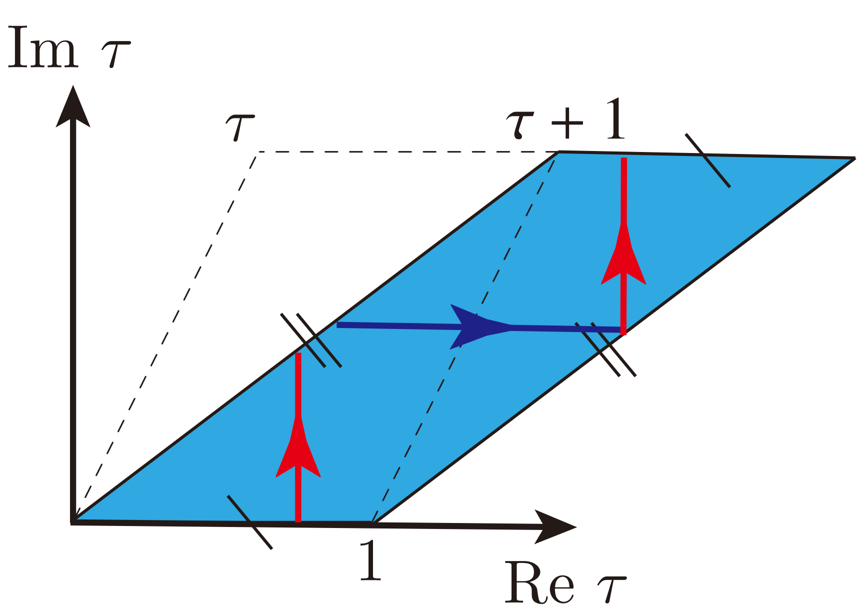
Generators of the MCG on a torus



$$\mathcal{S} : \tau \to -\frac{1}{\tau}$$



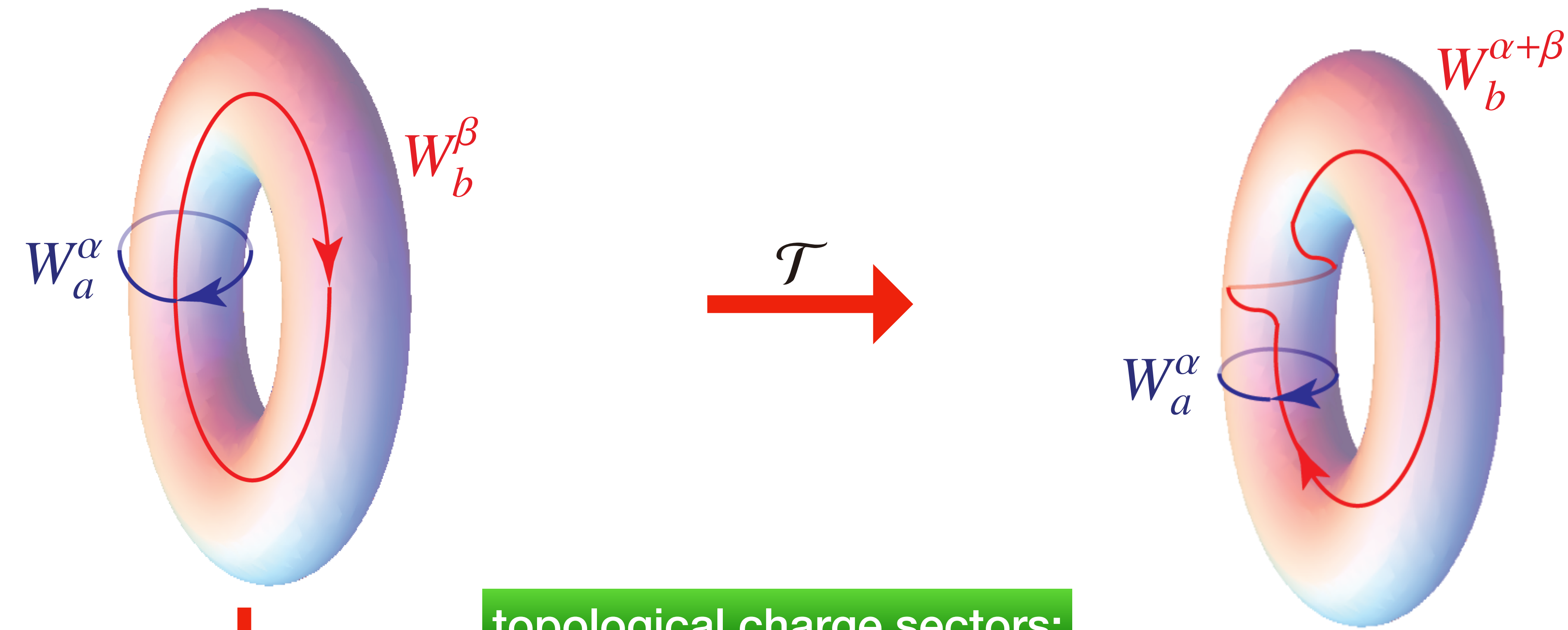
$$\mathcal{T} : \tau \to \tau + 1$$

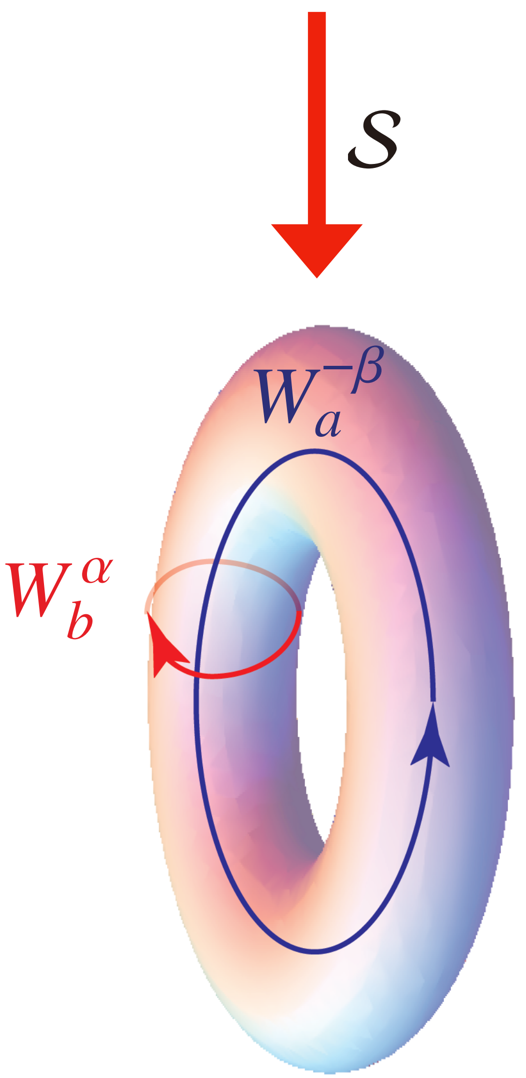


$$\mathcal{S} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\tau \to \tau' = \frac{\omega_1'}{\omega_2'} = \frac{a\omega_1 + b\omega_2}{c\omega_1 + d\omega_2} = \frac{a\tau + b}{c\tau + d}$$

Representation of the modular transformations





topological charge sectors:

$$|\,a\,\rangle_\alpha = W_a^\alpha |\,\mathbb{I}\,\rangle_\alpha \qquad\qquad |\,a\,\rangle_\beta = W_a^\beta |\,\mathbb{I}\,\rangle_\beta$$

Z2 spin liquid (toric code) $\quad |\,\mathbb{I}\,\rangle_\alpha$ (‘vacuum’) $\quad |\,e\,\rangle_\alpha$ (spinon) $\quad |\,m\,\rangle_\alpha$ (vison) $\quad |\,em\,\rangle_\alpha$ (fermion)

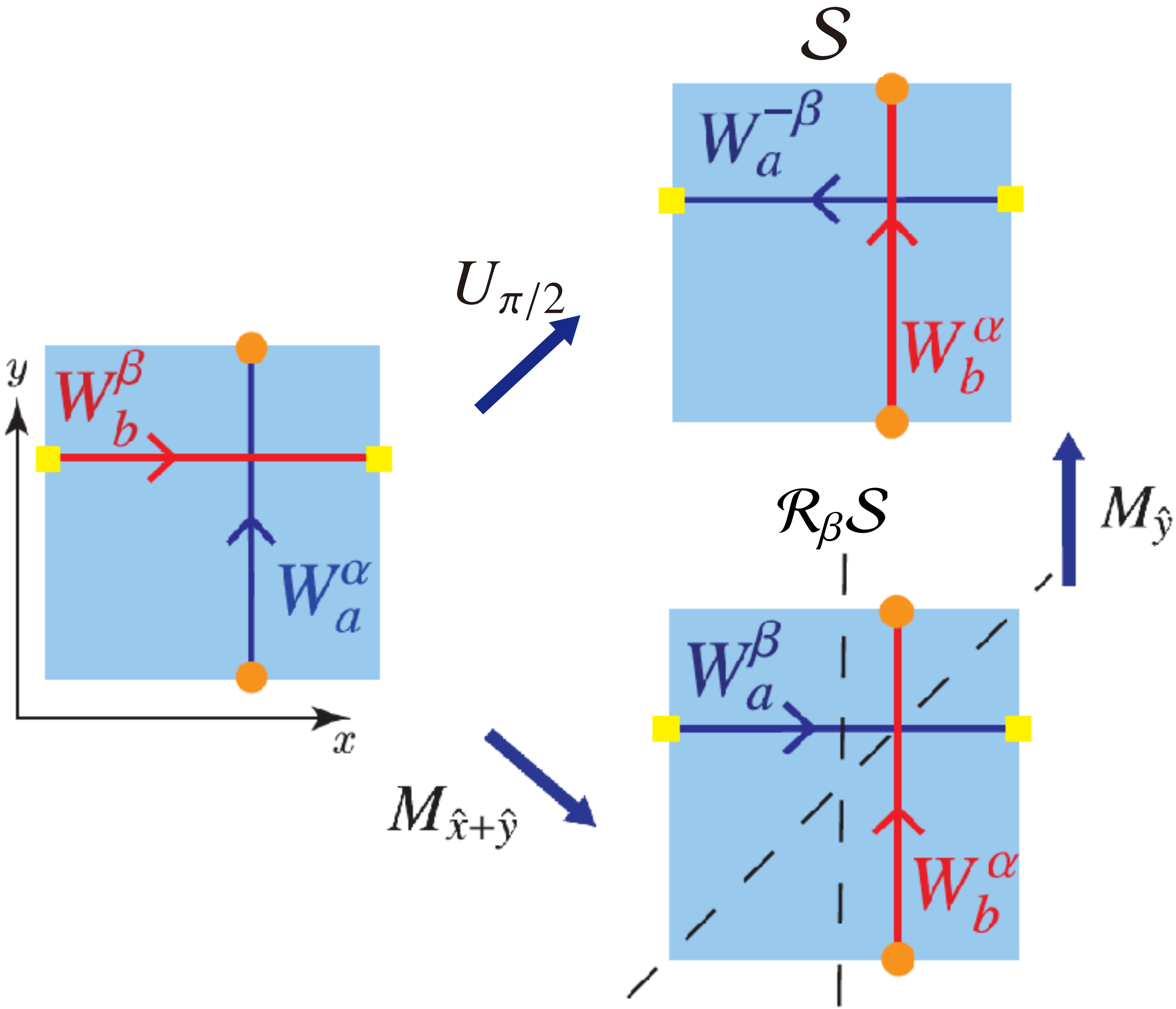
Representation of the MCG on the ground-state (code) subspace $\mathcal{H}_\Sigma$

$$\mathcal{S} : (\alpha,\beta) \to (-\beta,\alpha) \qquad \mathcal{S} W_a^\alpha \mathcal{S}^\dagger = W_a^{-\beta} = (W_a^\beta)^\dagger \qquad \mathcal{S} W_a^\beta \mathcal{S}^\dagger = W_a^\alpha$$

$$\mathcal{T} : (\alpha,\beta) \to (\alpha,\alpha+\beta) \qquad \mathcal{T} W_a^\beta \mathcal{T}^\dagger = W_a^{\alpha+\beta}$$

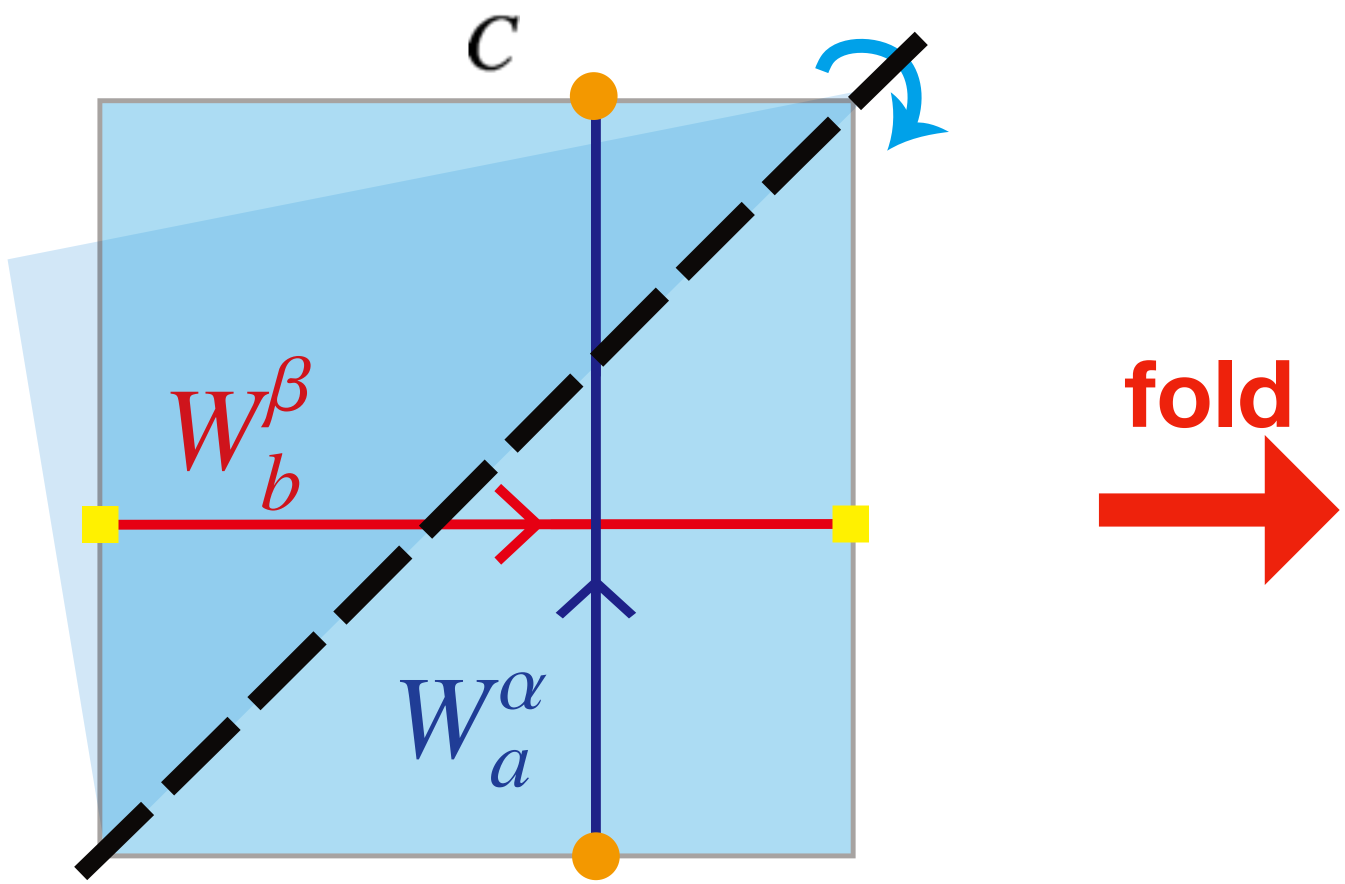
- Topological phase: Modular matrices encodes fractional statistics
- Topological code: Augment the capabilities

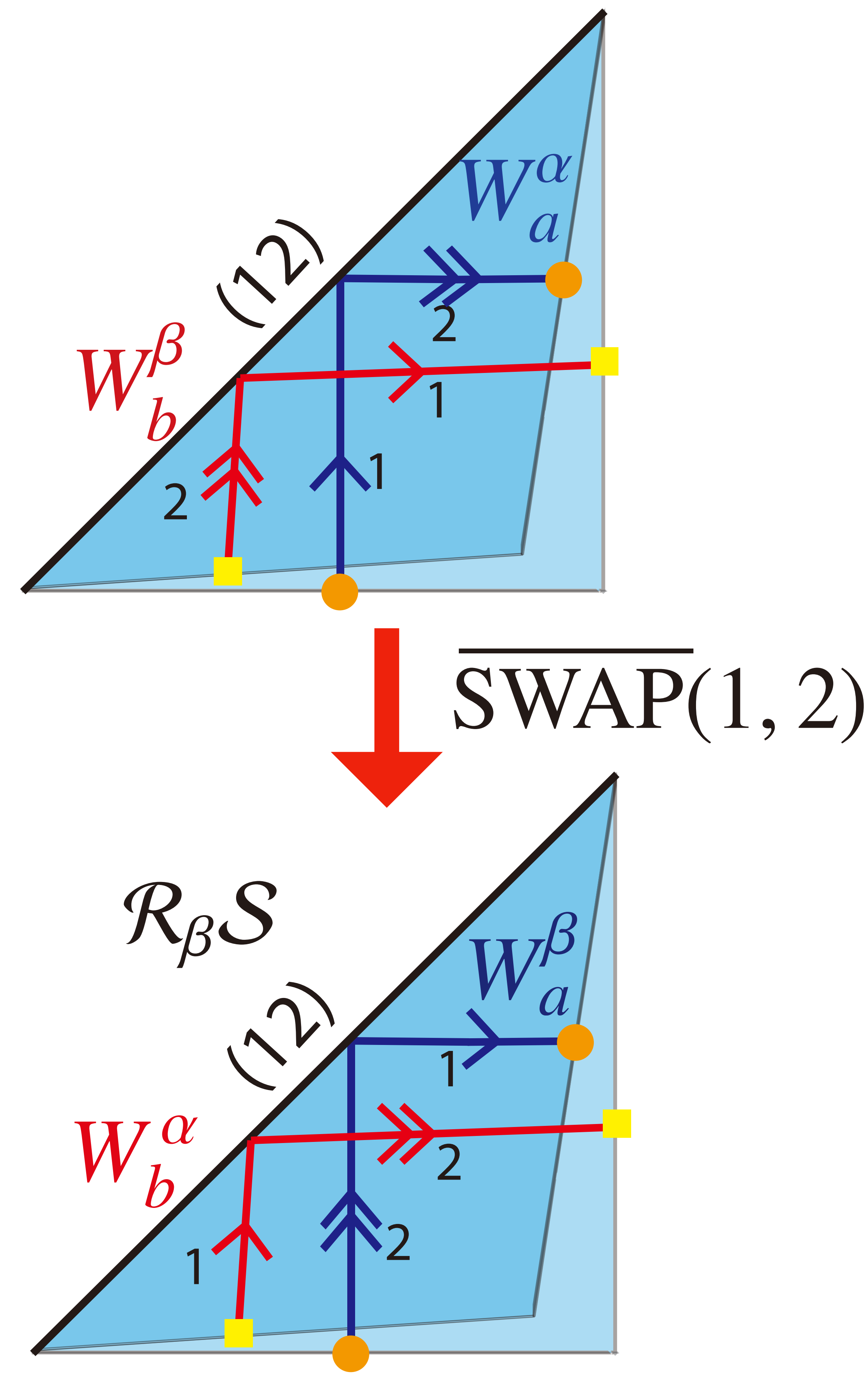
Realize modular S with spatial symmetry transformation



Requires long-range tunneling!

Modular RS on a folded system = local SWAP



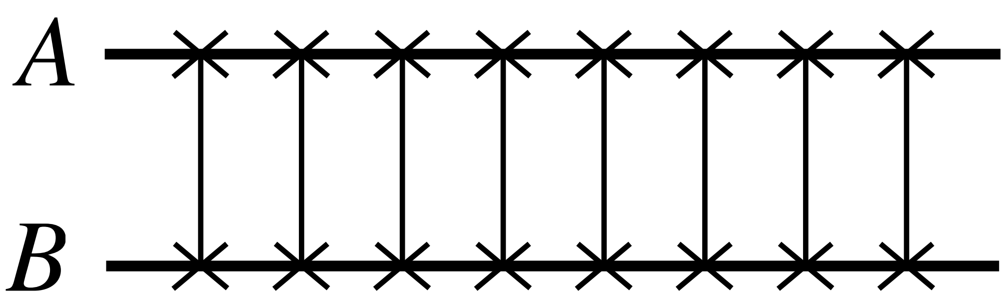


Local SWAP:

$$\mathrm{SWAP}_j |\psi_j\rangle_A \otimes |\phi_j\rangle_B = |\phi_j\rangle_A \otimes |\psi_j\rangle_B$$

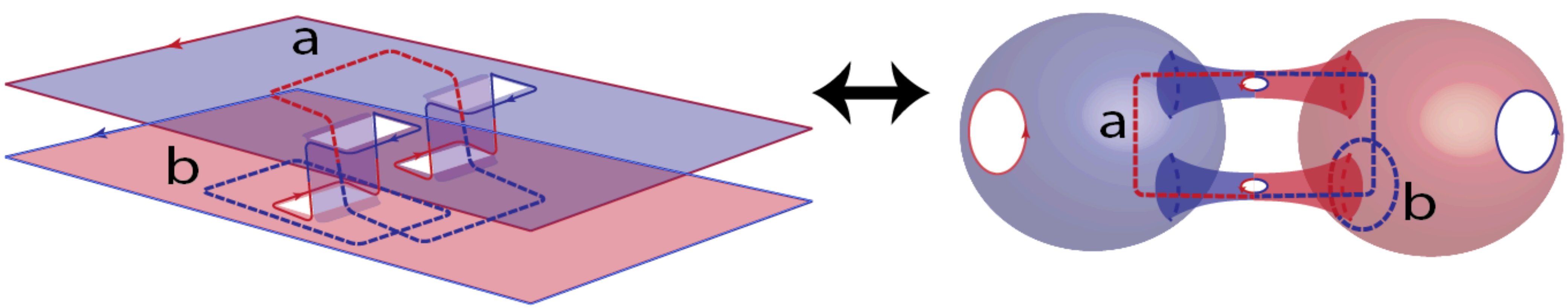
Transversal SWAP:

$$\overline{\mathrm{SWAP}}(A,B) = \prod_{j\in AB} \mathrm{SWAP}_j$$



- Non-local boundary conditions
- T-transformation
- Higher genus

➡ <u>Genon</u>

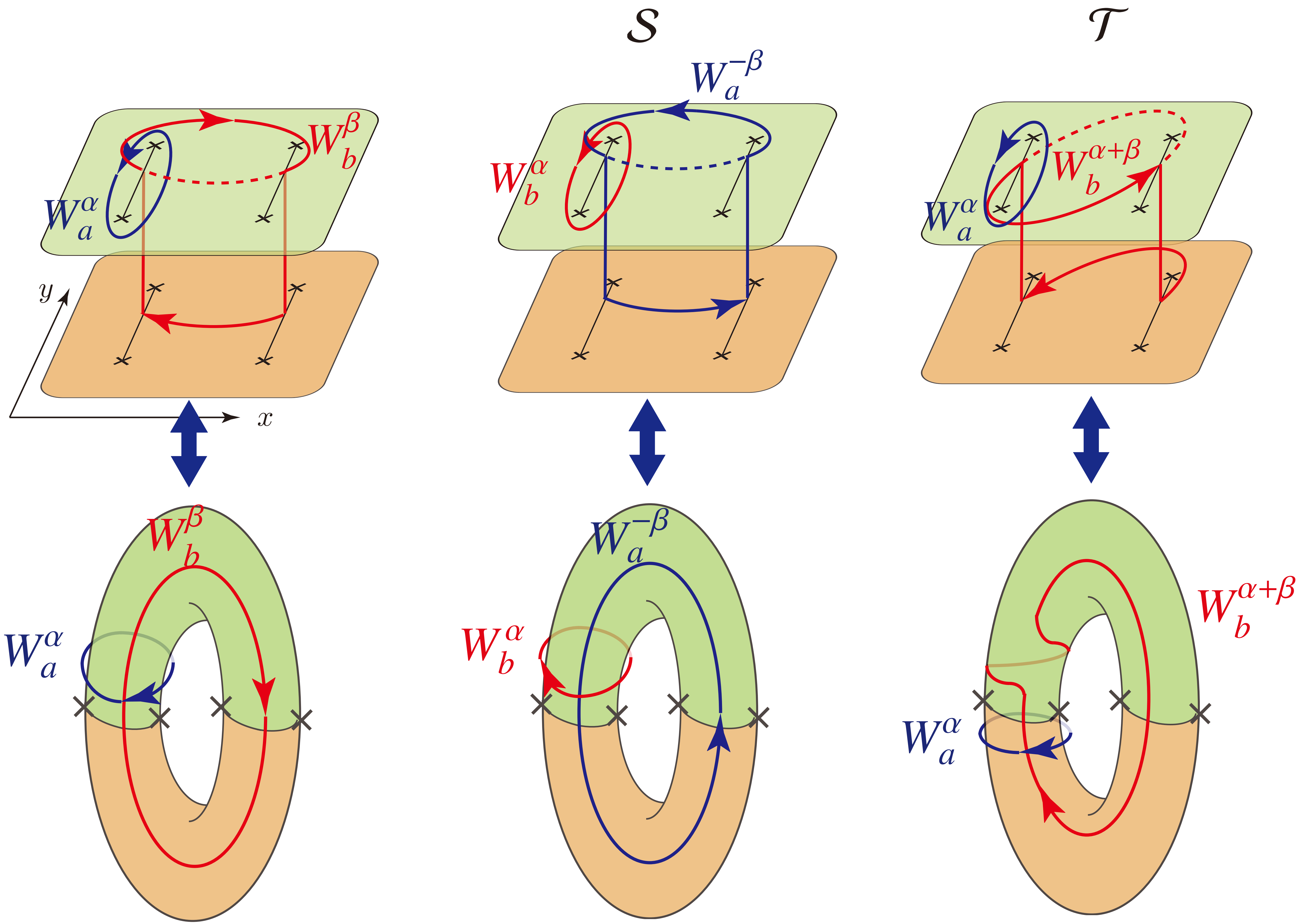


Adding a staircase (one pair of gates) adds genus by 1.

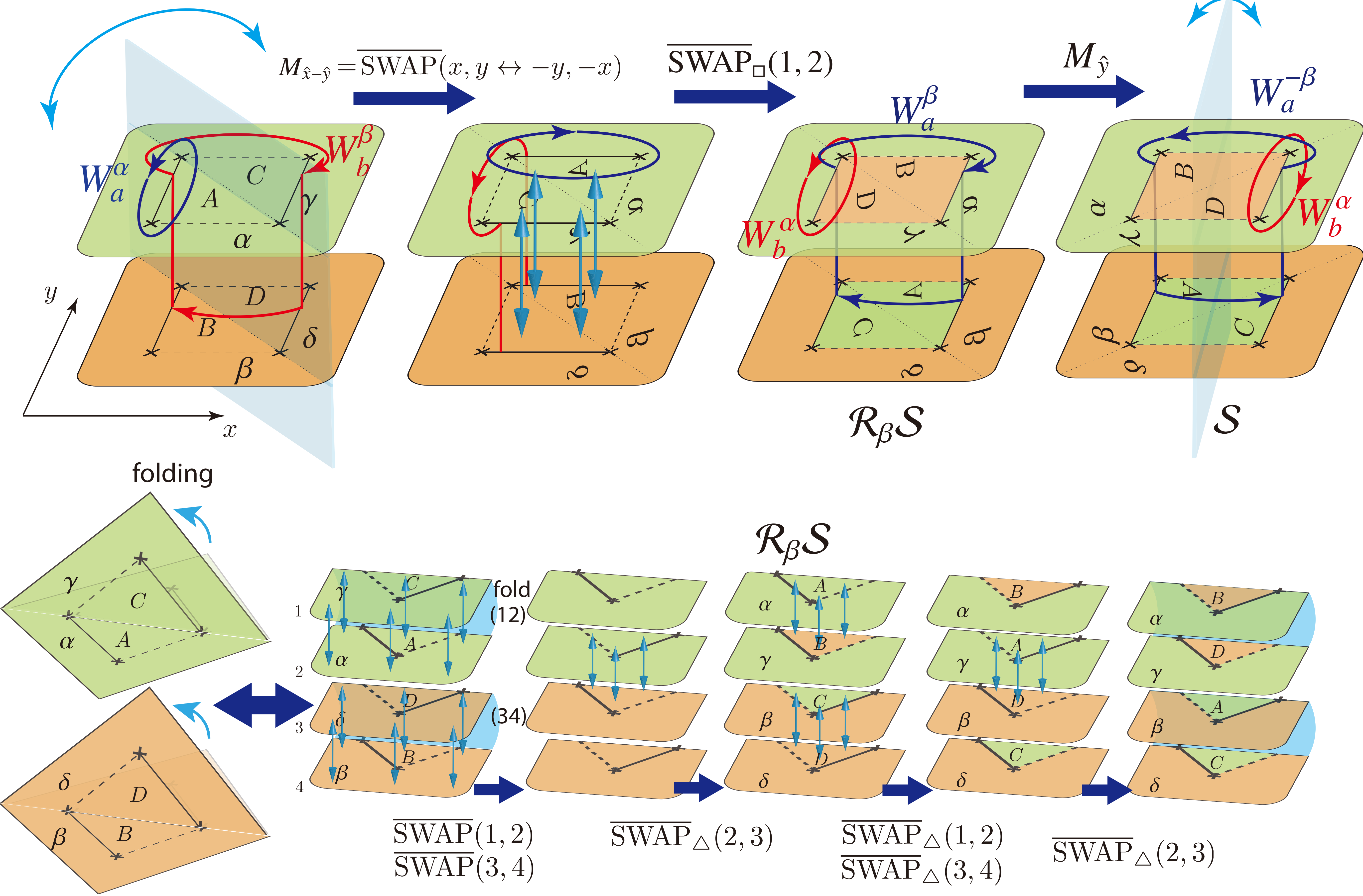
One staircase = 2 genon

Bombin, Kitaev, Barkeshli, Qi, …

Create effective torus with genons



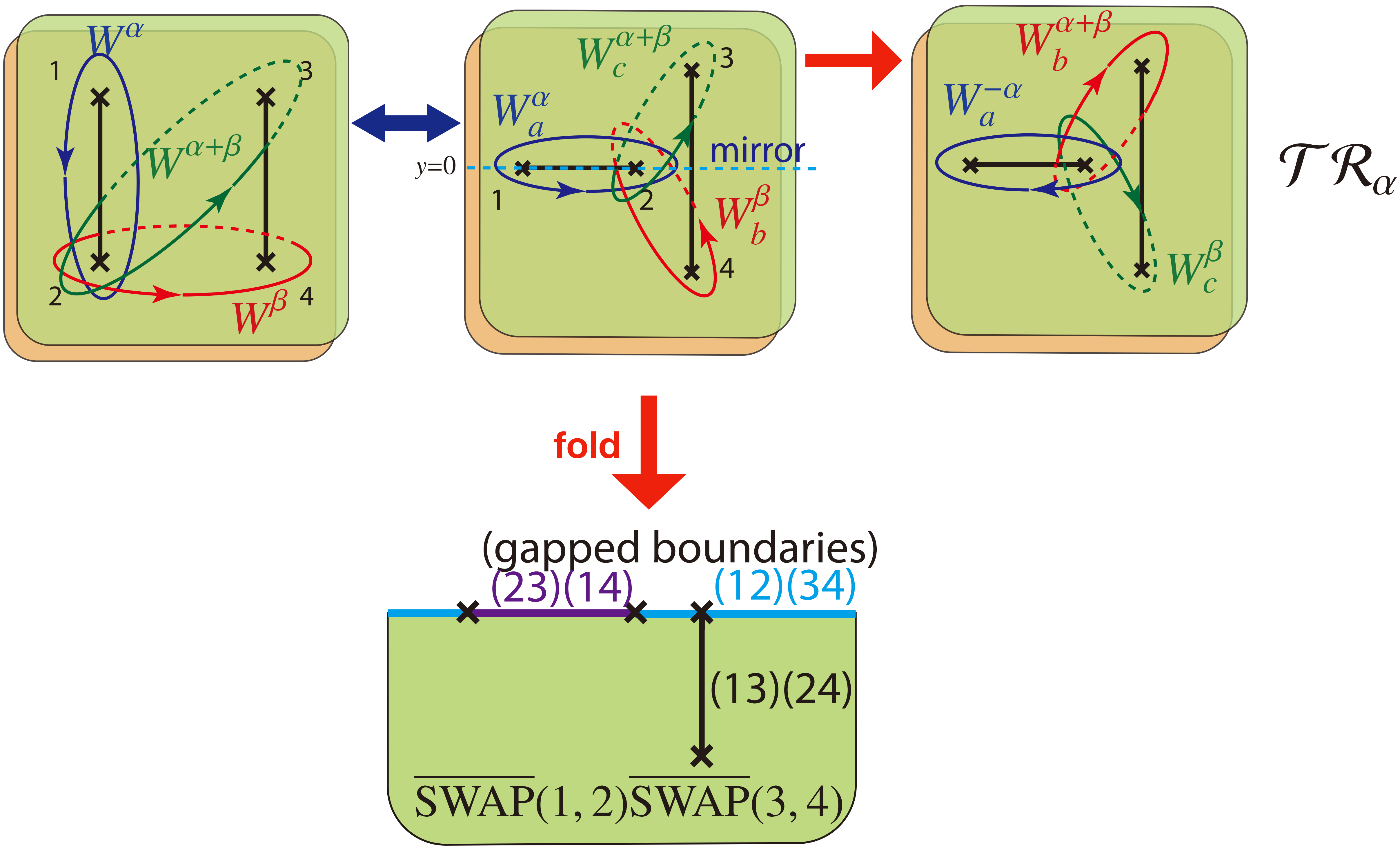
Modular S by folding and SWAP



Protected logical operation with constant-depth circuit

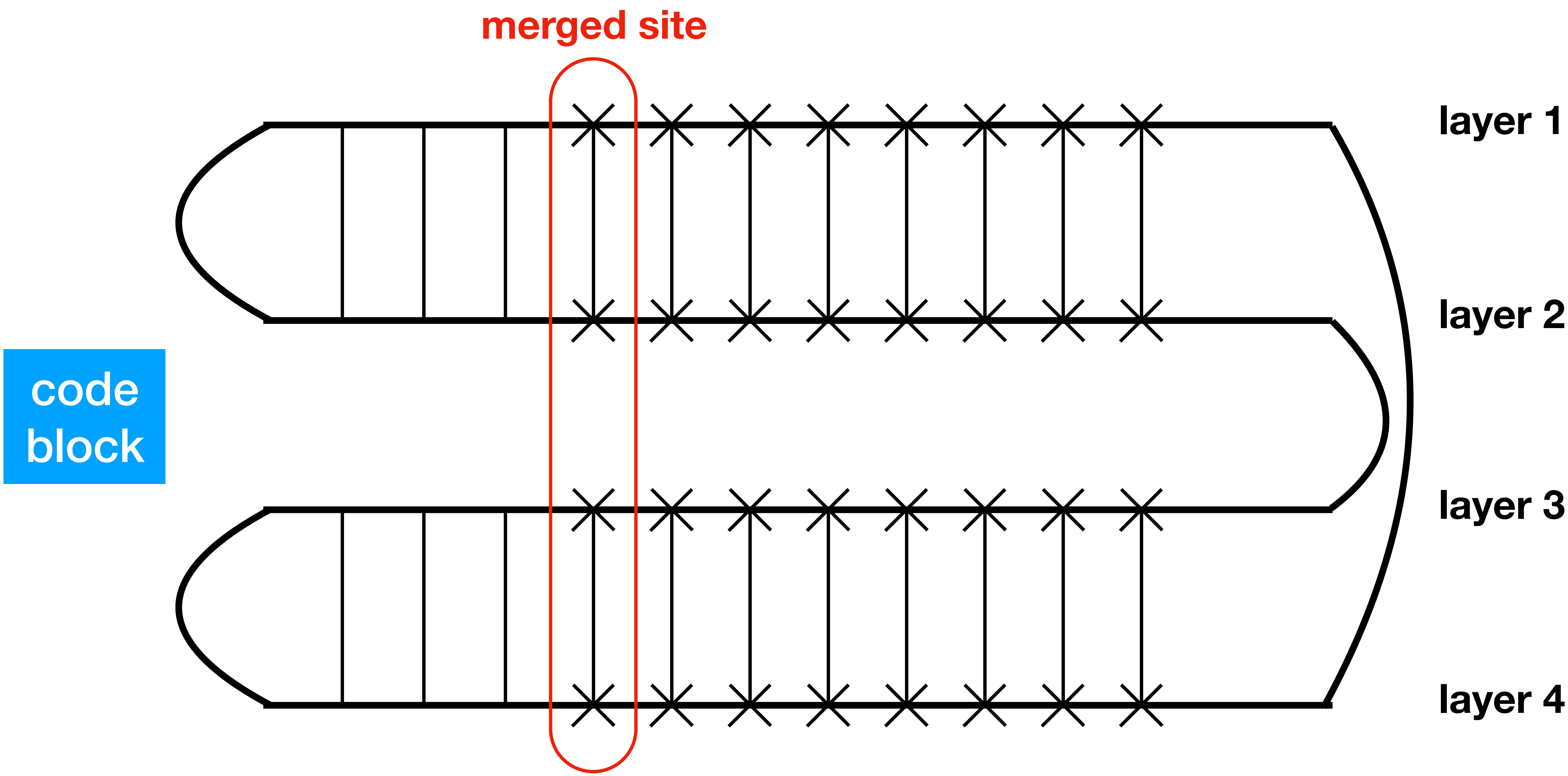
RT by SWAP and folding

$M_{\hat{x}} = \overline{\mathrm{SWAP}}(y \leftrightarrow -y)$



one can generate T with additional moving back to square geometry to apply R

Modular transformations as transversal logical gates

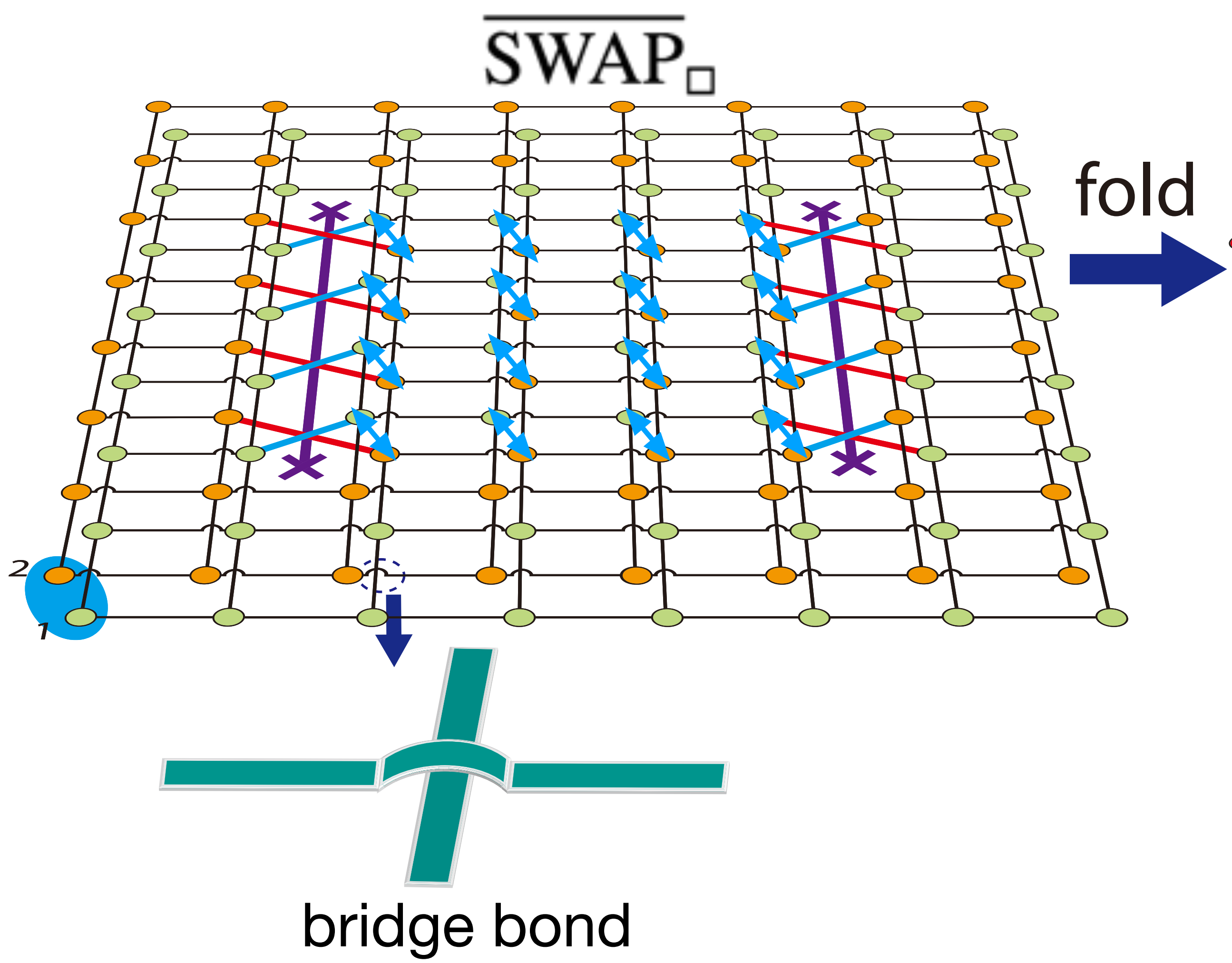


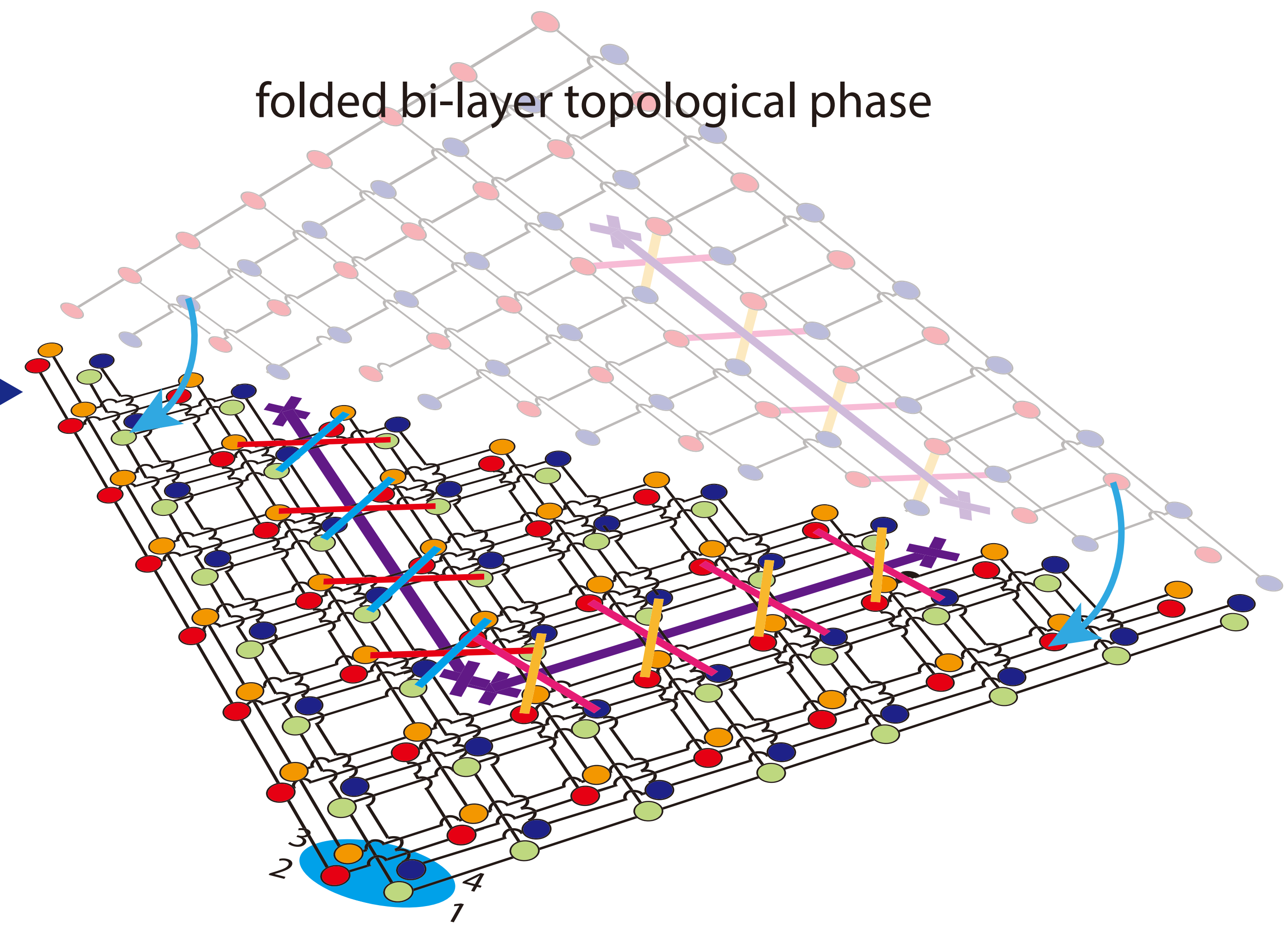
Error propagation bounded by the Lieb-Robinson light cone

Experimental realization

qubit array of superconducting qubits

embed bi-layer topological phase
on a single-layer chip





Realize topological states with superconducting qubits:

Theory:
E. Kapit, M. Hafezi, and S. H. Simon, PRX 4,031039 (2014)

Experiments:
Owens and Schuster et al. arXiv:1708.01651 (U Chicago)
Roushan and Martinis et al. *Nature Physics* **13**, 146–151 (2017)

Developed in Martinis's lab:

Chen et al. APL 104,052602 (2014)

Foxen et al. arXiv:1708.04270 (2017)

Transversal SWAP operation

Tunneling Hamiltonian: $$H_{\text{tunnel}} = -J \sum_{j \in AB} (a^\dagger_{j,A} a_{j,B} + \text{H.c.})$$

Tunneling: $$\overline{U}^{(1)}(t) = e^{-iH_{\text{tunnel}}t} \qquad t = \pi/(2J)$$

phase shift: $$\overline{U}^{(2)}(t) = \prod_{j \in AB} e^{it(a^\dagger_{j,A} a_{j,A} + a^\dagger_{j,B} a_{j,B})}$$

$$\overline{\text{SWAP}} = \overline{U}\left(\frac{\pi}{2J}\right) = \overline{U}^{(1)}\left(\frac{\pi}{2J}\right)\overline{U}^{(2)}\left(\frac{\pi}{2J}\right)$$
$$= \prod_{j \in AB} e^{-i\frac{\pi}{2}(a^\dagger_{j,A} a_{j,B} + \text{H.c.})} e^{i\frac{\pi}{2}(a^\dagger_{j,A} a_{j,A} + a^\dagger_{j,B} a_{j,B})} = \prod_{j \in AB} e^{i\pi a^\dagger_{j,-} a_{j,-}}$$

$$a_{j,-} = \frac{1}{\sqrt{2}}(a_{j,A} - a_{j,B})$$

↓

parity of the anti-symmetric mode

(useful for measurements)

Also controlled version: H. Pichler, G. Zhu, A. Seif, P. Zoller, MH, Phys. Rev. X 6, 041033 (2016)

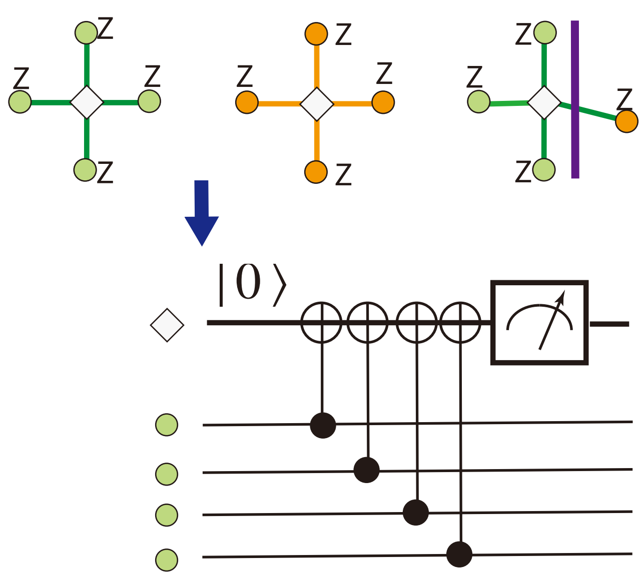
Experimental realization: topological codes

embedded bi-layers of topological error-correction code on a single-layer

qubit layer 1 — qubit layer 2 — ancilla

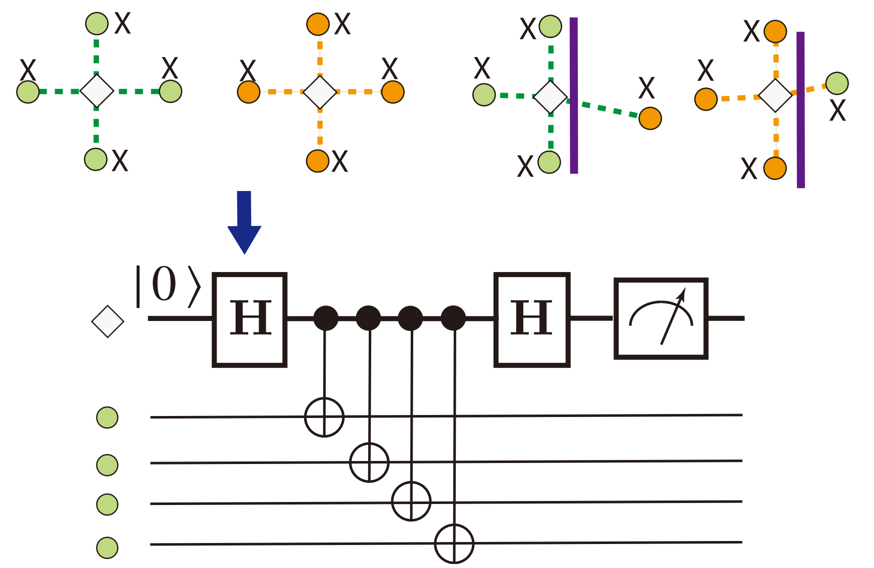
Z-stabilizer measurement:

layer 1 layer 2 (twist 1→2) (twist 2→1)



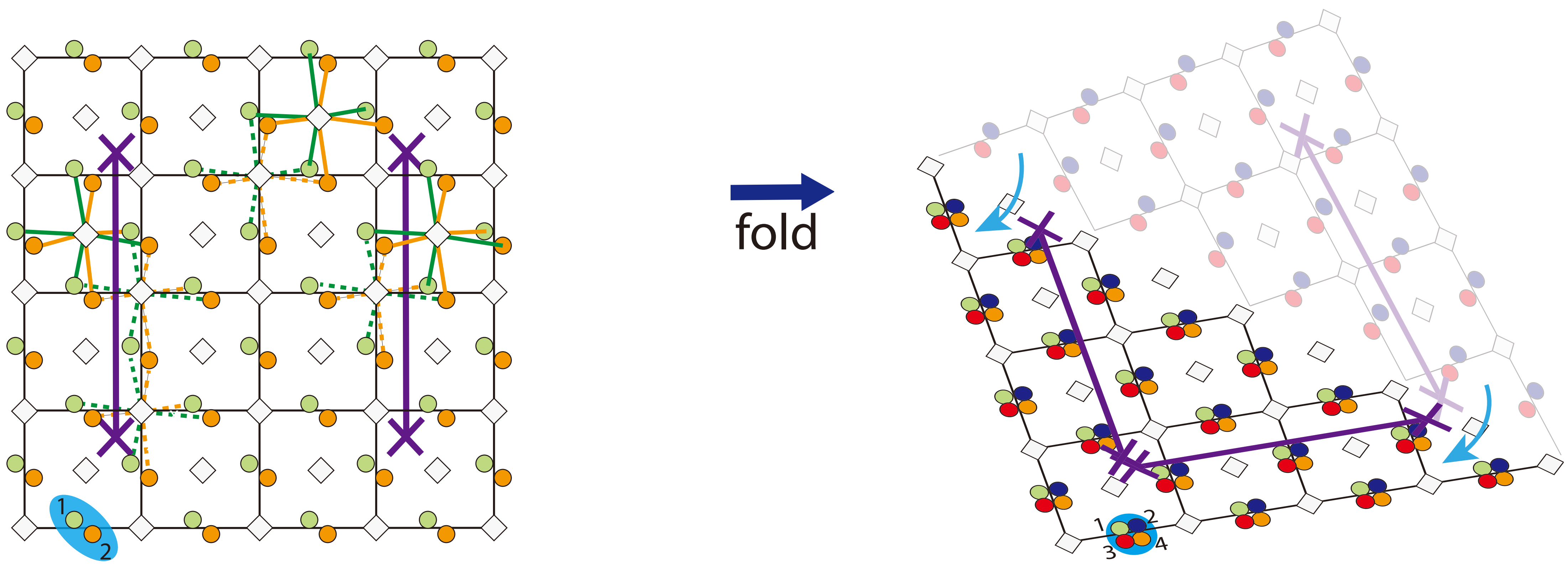
X-stabilizer measurement:

layer 1 layer 2 (twist 1→2) (twist 2→1)



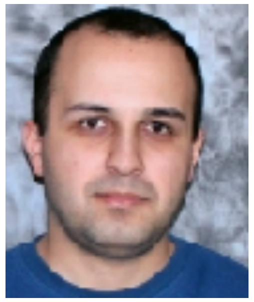
Multi-mode resonators as ancilla

Naik and Schuster et al. (U Chicago)
arXiv:1705.00579



- Photons and electronic quantum Hall states

  - Optical probe of IQHE states
    M. Gullans, J. Taylor, A Imamoglu, P. Ghaemi, MH arXiv:1701.03464 (PRB)
  - Driven FQH states and bilayer physics
    A. Ghazaryan, T. Grass, M. J. Gullans, P. Ghaemi, MH arXiv:1612.08748 (PRL)



- *Quantum Origami*: Applying Transversal Gates and Measuring Topological Order (Modular transformation)
  G. Zhu, MH, M. Barkeshli arXiv:1701.03464

- Hwanmun Kim
- Alireza Seif
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- Guanyu Zhu

- Julia Sell
- Jon Vannucci
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- Vikram Orre
- Bin Cao
- Chris Flower

- J. Fan, J. Taylor, A. Migdall, M. Gullans, E. Waks, M. Barkeshli (NIST/JQI)
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- A. Vaezi (Stanford)
- A. Imamoglu (ETH)
- A. Ghazaryan, P. Ghaemi (CUNY)

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A zoo of transversal logical gates

Models	logical basis	logical string	transversal logical gate by modular $\mathcal{S}$	logical gate by modular $\mathcal{T}$	universality of MCG_Σ
$\nu = 1/2$ FQH ($U(1)_2$ CS)	$\|a\rangle_\alpha$ (qubit) $a = 0, 1$ (semion)	$W^\alpha = \overline{X}$ $W^\beta = \overline{Z}$	Hadamard $\overline{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	phase $\overline{P} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	Clifford
$\nu = 1/k$ FQH ($U(1)_k$ CS)	$\|a\rangle_\alpha$ (qudit) $a = 0, 1, ..k-1$	$W^\alpha = \overline{X}$ $W^\beta = \overline{Z}$	Fourier transform $\mathcal{S}_{aa'} = \frac{1}{\sqrt{k}} e^{i2\pi aa'/k}$	phase $\mathcal{T}_{aa'} = \delta_{aa'} e^{i2\pi a(a+k)/2}$	(generalized) Clifford
Doubled semion ($U(1)_2 \times \overline{U(1)_2}$ CS)	$\|n_s\rangle_\alpha$ (qubit) (semion No.)	$W^\alpha_s = \overline{X}$ $W^\beta_s = \overline{Z}$	Hadamard $\overline{H}$	phase $\overline{P}$	Clifford
Toric code (Z_2 spin liquid)	$\|n_e n_m\rangle_\alpha$ (2-qubit) $(n_e, n_m = 0, 1)$	$W^\alpha_{e,m} = \overline{X}_{1,2}$ $W^\beta_{m,e} = \overline{Z}_{1,2}$	$(\overline{H}_1 \otimes \overline{H}_2)\overline{\text{SWAP}}_{12}$	control-Z $\overline{CZ}$	subset of Clifford
Ising	$\|\mathbb{I}\rangle_\alpha, \|\sigma\rangle_\alpha, \|\psi\rangle_\alpha$ (qutrit)		$\mathcal{S} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$	half a $\pi/8$-phase $\mathcal{T}^2 = \overline{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$ in Span$\{\|\mathbb{I}\rangle_\alpha, \|\sigma\rangle_\alpha\}$	universal (+ measurement or braiding anyons)
Fibonacci	$\|\mathbb{I}\rangle_\alpha, \|\tau\rangle_\alpha$ (qubit)		$\overline{U}_{xz} = \frac{1}{\sqrt{2+\phi}}\begin{pmatrix} 1 & \phi \\ \phi & -1 \end{pmatrix}$ $\phi = \frac{1+\sqrt{5}}{2}$	phase $\overline{U}_z = \begin{pmatrix} 1 & 0 \\ 0 & e^{i4\pi/5} \end{pmatrix}$	universal

QEC realization of non-abelian codes: Levin-Wen Model / Turaev-Viro code

Levin and Wen (2005) Koenig, G. Kuperberg, and B. W. Reichardt (2010)

realization: $\text{Ising} \times \overline{\text{Ising}}$ $\text{Fib.} \times \overline{\text{Fib.}}$

Circumvents the no go theorem:

M. E. Beverland, O. Buerschaper, R. Koenig, F. Pastawski, J. Preskill, and S. Sijher, Journal of Mathematical Physics 57, 022201 (2016)