Synthetic dimensions and topology:
Towards Laughlin-like physics

Leonardo Mazza

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Département de Physique
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Monday, November 20, 2017

Synthetic dimensions in quantum engineered systems

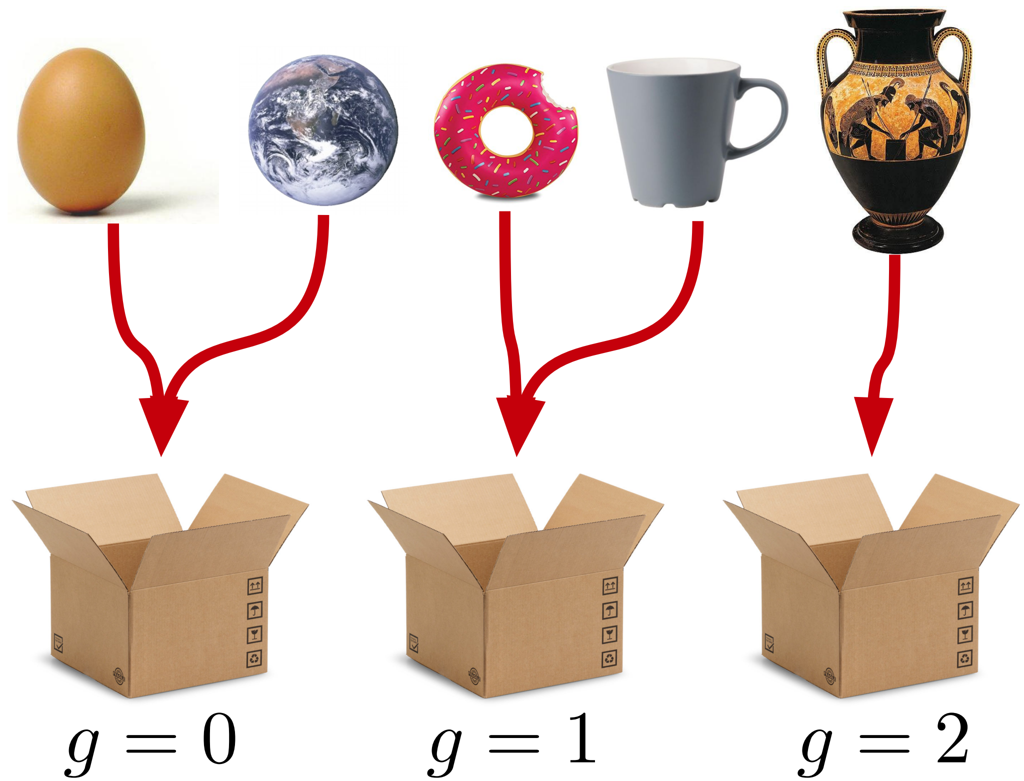
ETH

Topology?

Gauss and Bonnet theorem

$$\frac{1}{4\pi}\int_{\mathcal{S}}\Omega(\mathbf{r})\mathrm{d}^2r = 1-g$$

Classification of surfaces according to their **genus**



Topological protection: deformations of an object do not modify the integral of the curvature

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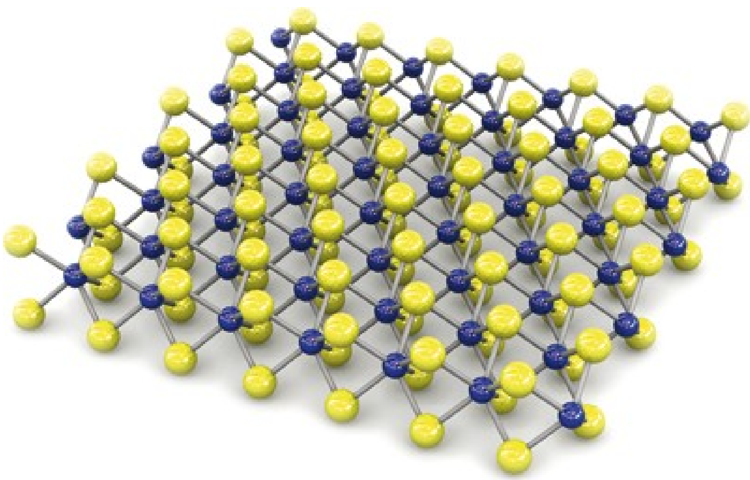
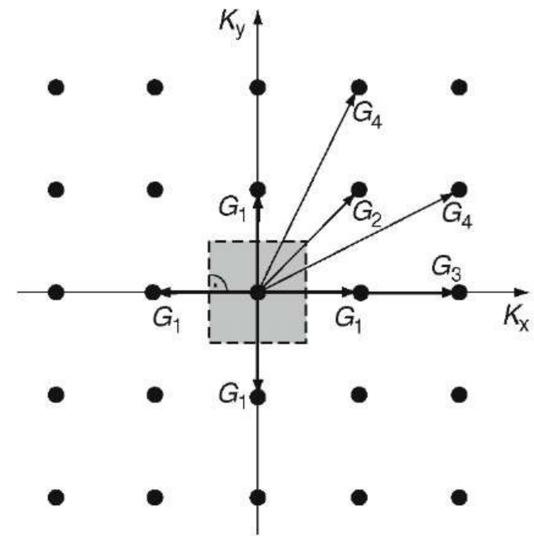
How can physics benefit from topology?



Two-dimensional material



Two-dimensional Brillouin Zone

Let us assume energy bands well separated in energies.
Focus on the lowest one, with Bloch states $|\Psi_{\mathbf{k}}\rangle$

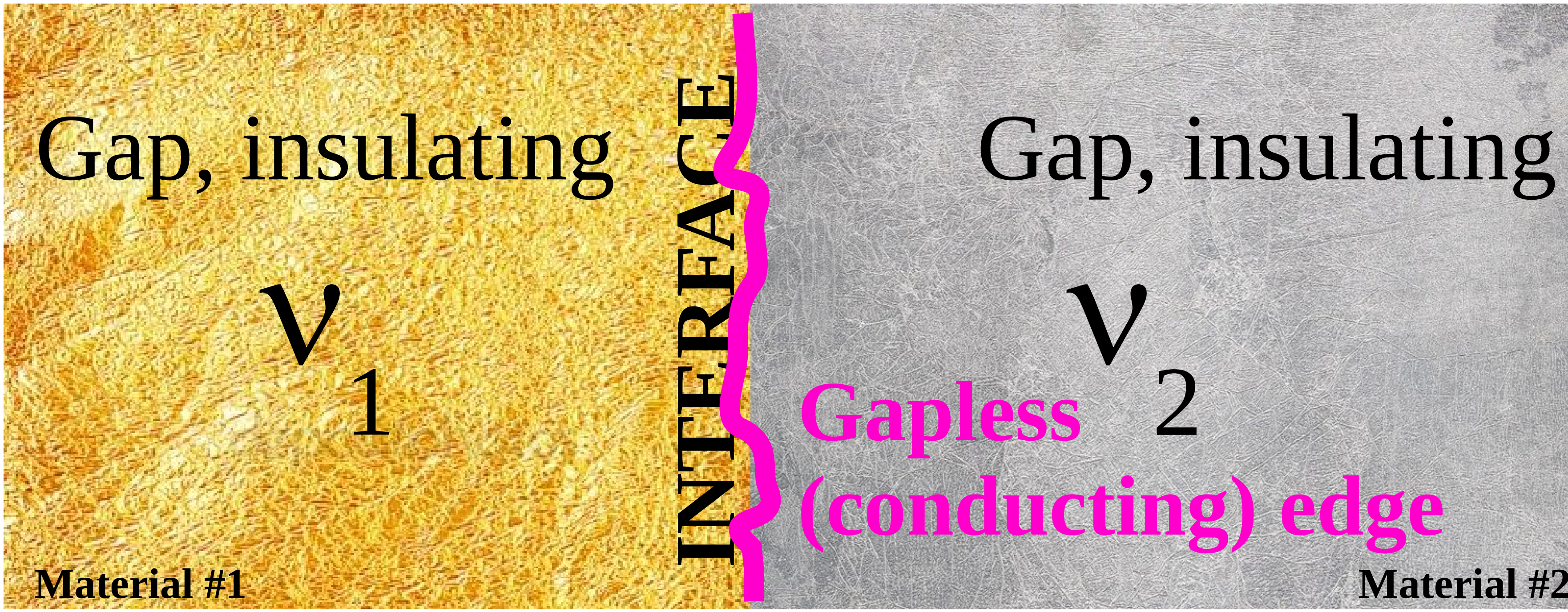
$$i\int_{BZ}\left[\partial_{k_x}\langle\Psi_{\mathbf{k}}|\partial_{k_y}|\Psi_{\mathbf{k}}\rangle-\partial_{k_y}\langle\Psi_{\mathbf{k}}|\partial_{k_x}|\Psi_{\mathbf{k}}\rangle\right]\mathrm{d}\mathbf{k}=\nu\in\mathbb{Z}$$

A physical quantity related to this mathematical object will be **topologically** robust to small perturbations and imperfections

- Example: the conductance of the quantum Hall effect

A striking consequence

A striking consequence



Interface between two gapped topologically distinct materials:
there must be a “phase transition” in between.

Topologically protected gapless edge modes

This talk: topological edge modes of the fractional quantum Hall effect in synthetic dimensions

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Outline

Introduction #1:
The edge modes of the fractional quantum Hall effect

Introduction #2:
Synthetic dimensions

Results:
Edge modes of the fractional quantum Hall effect in synthetic dimensions

Conclusions and Perspectives

The fractional quantum Hall effect and its gapless edge modes

Introduction

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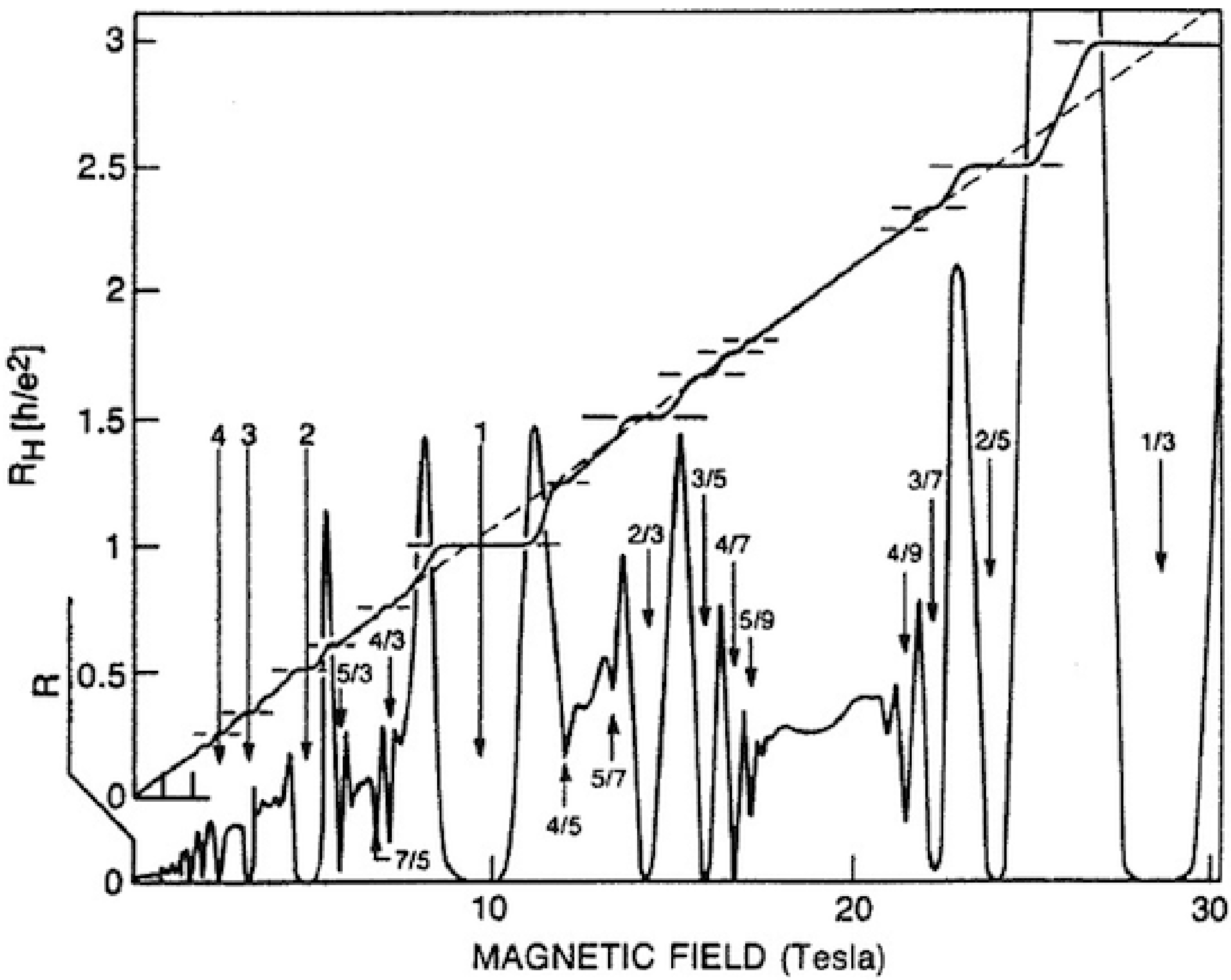
The fractional quantum Hall effect



$$\nu = \frac{N}{N_\Phi}$$

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

- Two-dimensional electron gas with perpendicular magnetic field
- Gapped phases at some fractional fillings ν = p/q
- Strongly correlated wavefunctions
- Crucial role of interactions

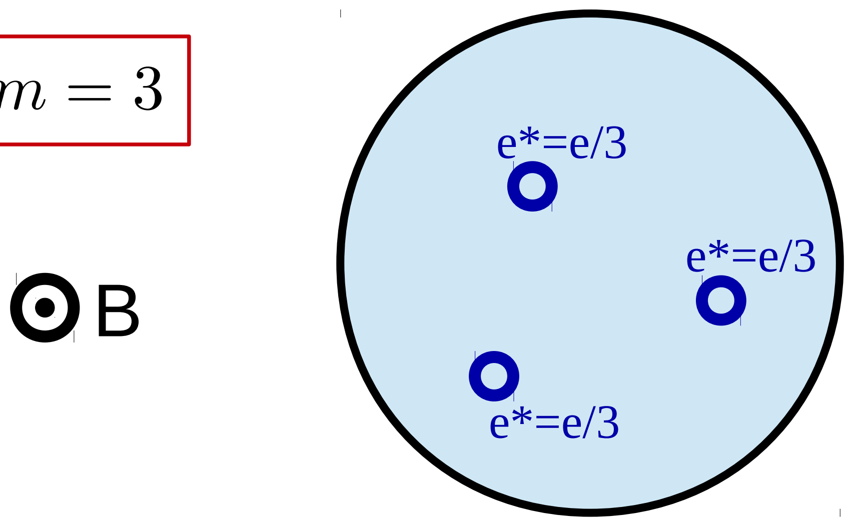
The Laughlin wavefunction

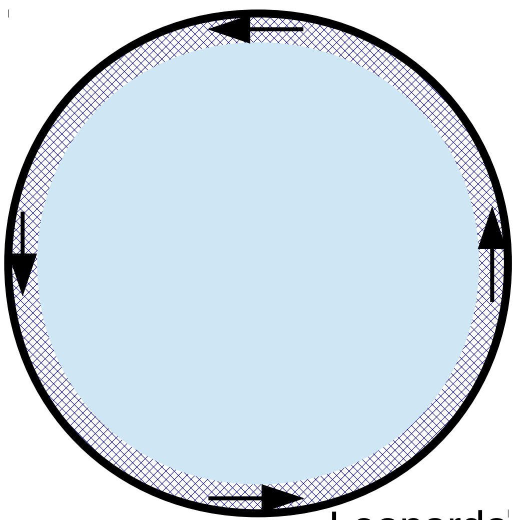
$$\boxed{\nu = \frac{1}{m} \quad m \text{ odd}}$$

$$\Psi_{\text{Laughlin}}(z_1, \dots, z_N) = \prod_{i<j} (z_i - z_j)^m \prod e^{-|z_i|^2/4\ell_B^2}$$

1) Quasi-hole excitations in the bulk with fractional charge e* = e/m
   - Motivate the introduction of the concept of anyons
2) Gapless edge modes (chiral) due to a boundary confining potential

$$\boxed{m = 3}$$





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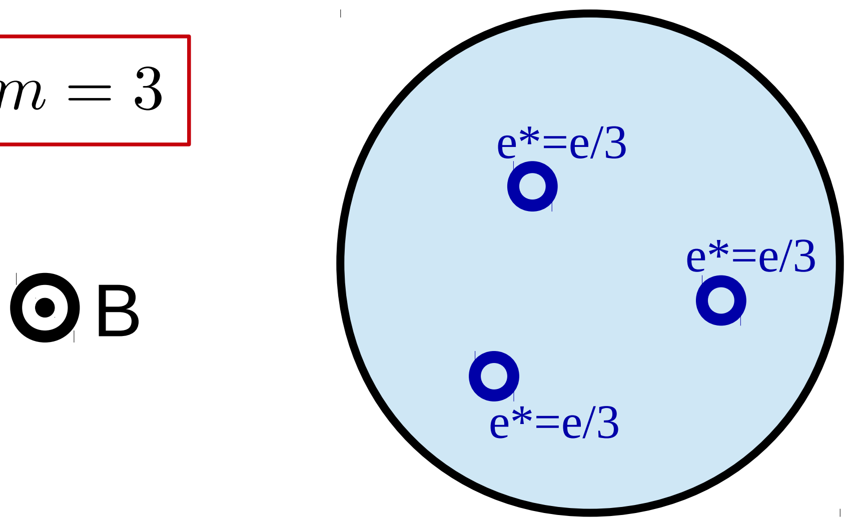

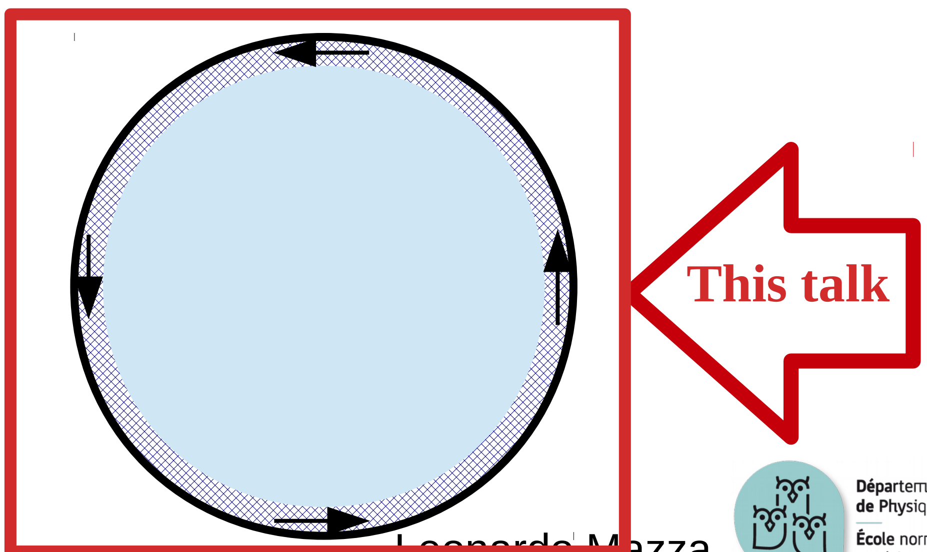

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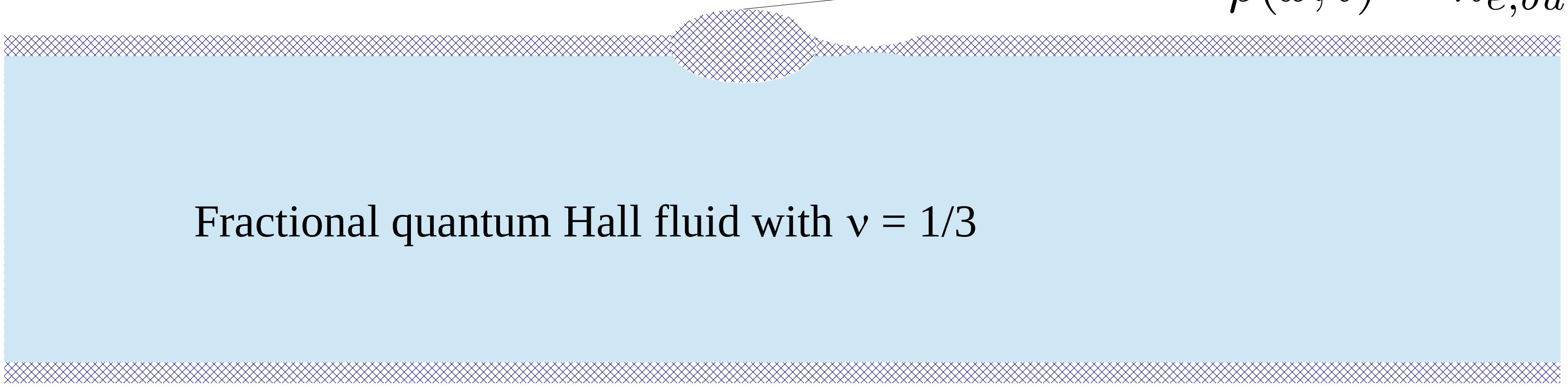
Hydrodynamics of edges

Boundary deformation

$\rho(x,t) = n_{e,bulk} h(x.t)$



$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x} = 0$$ and the velocity is $v = \frac{|E|}{|B|}$

$$H \sim \int \rho^2 dx$$

Bosonic excitations with linear spectrum

$$\hat{H} \simeq E_0 + \sum_{k>0} vk \hat{b}_k^\dagger \hat{b}_k$$

Elementary excitations with unconventional properties

$$\psi_{qp}(x) \sim e^{i\phi[\hat{b}_k, \hat{b}_k^\dagger](x)}$$

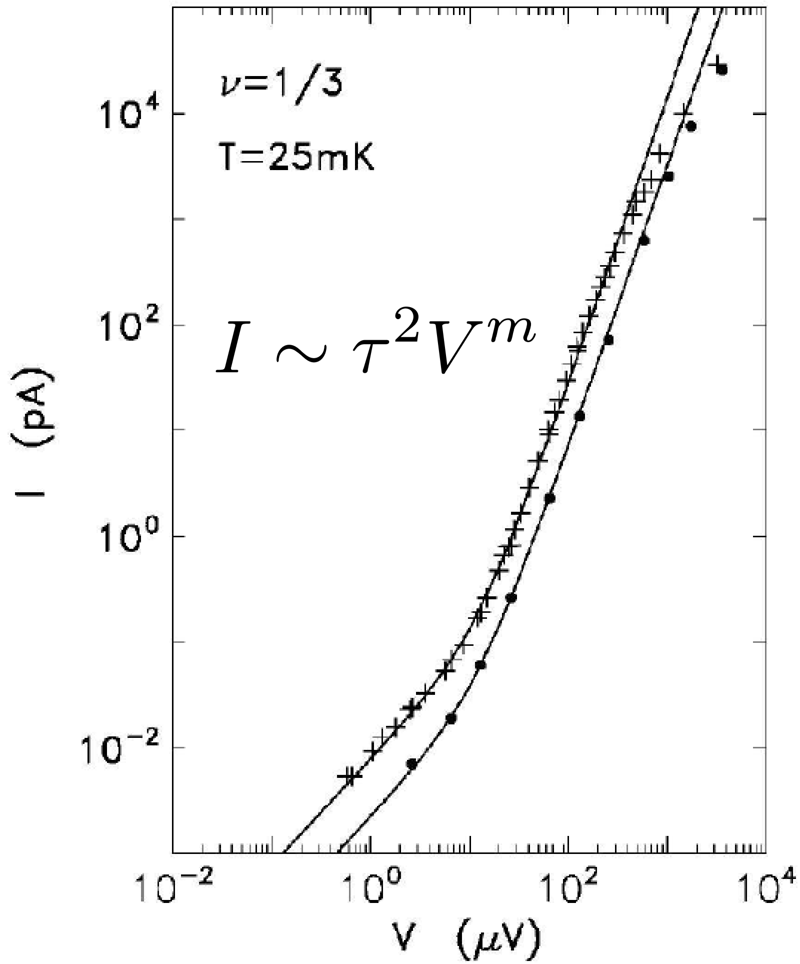
$$\psi_{qp}(x)\psi_{qp}(x') = e^{\frac{i\pi}{3}} \psi_{qp}(x')\psi_{qp}(x)$$

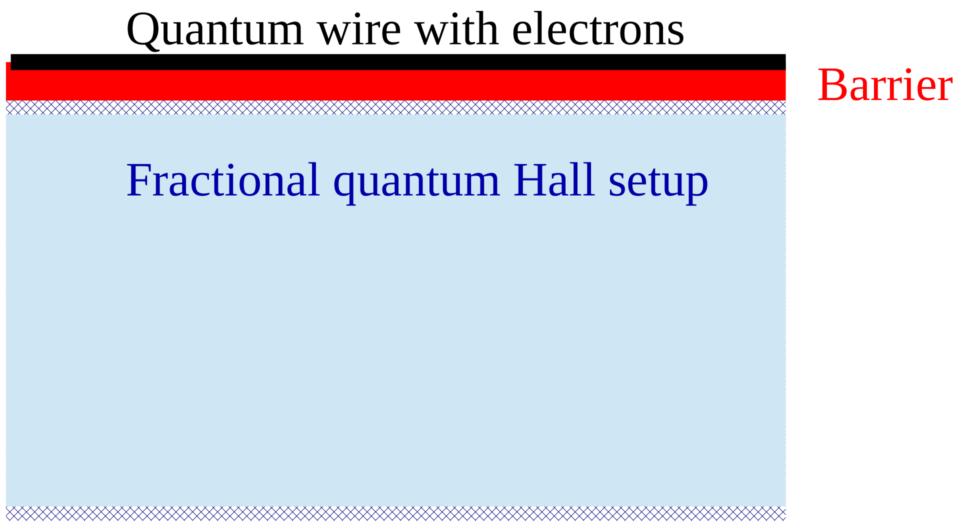
$$\Psi_{\mathrm{F}}(x) \sim \psi_{qp}^3(x)$$

Wen PRB (1990)

Violation of Ohm’s law

Measurement the tunneling current-voltage (*I*-*V*) characteristics for electron tunneling from a bulk doped-GaAs normal metal into the abrupt edge of a fractional quantum Hall effect.





Electrons can tunnel through the barrier

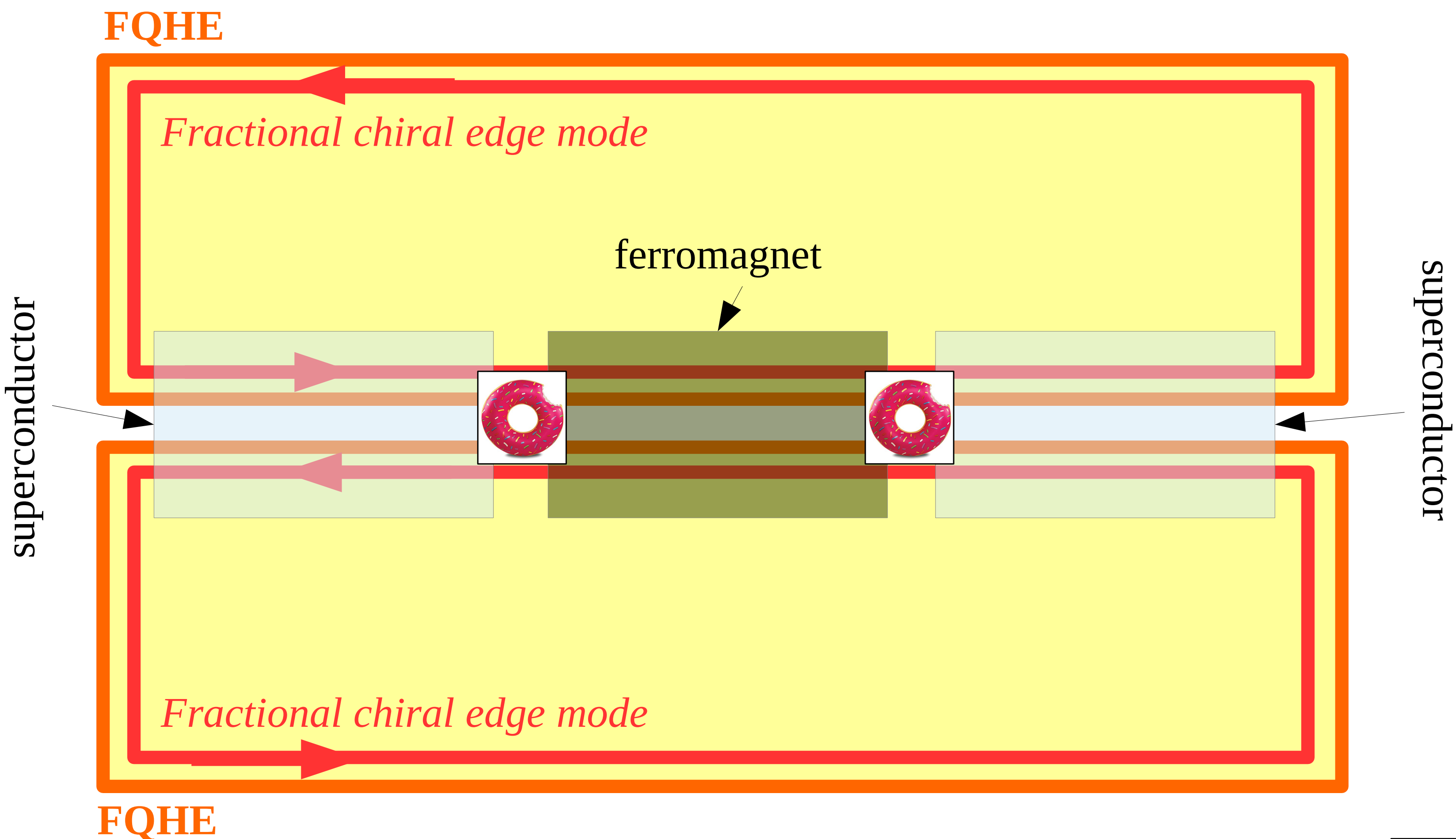
- from the wire to the Hall bar
- and viceversa

Striking violation of Ohm law!

Fitted exponent is around 2.7...

Chang, Pfeiffer, West, PRL (1996)

And much more!



Fractionalized Majorana fermions, namely **Parafermions**

Clarke, Alicea, Shtengel, Nature Communications **4**, 1348 (2012)

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Goal of this talk:

To identify a setup with synthetic dimension with the same low-energy theory

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Synthetic dimensions in ultra-cold gases

Introduction

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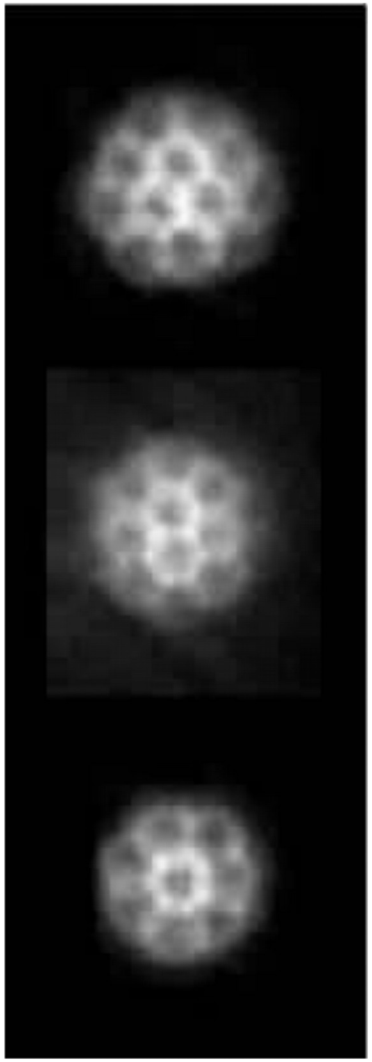
Artificial gauge fields in cold atoms

Ultra-cold quantum gases for the study of quantum many-body physics.

- **Quantum simulation (Feynman)**
  - Genuine many-body quantum systems
  - Unprecedented control
  - High-fidelity measurements

- **Problem: atoms are neutral**
  - No coupling to a magnetic field of motional degrees of freedom
    - Quantum Hall physics precluded?



Solution: Engineering of ARTIFICIAL magnetic fields

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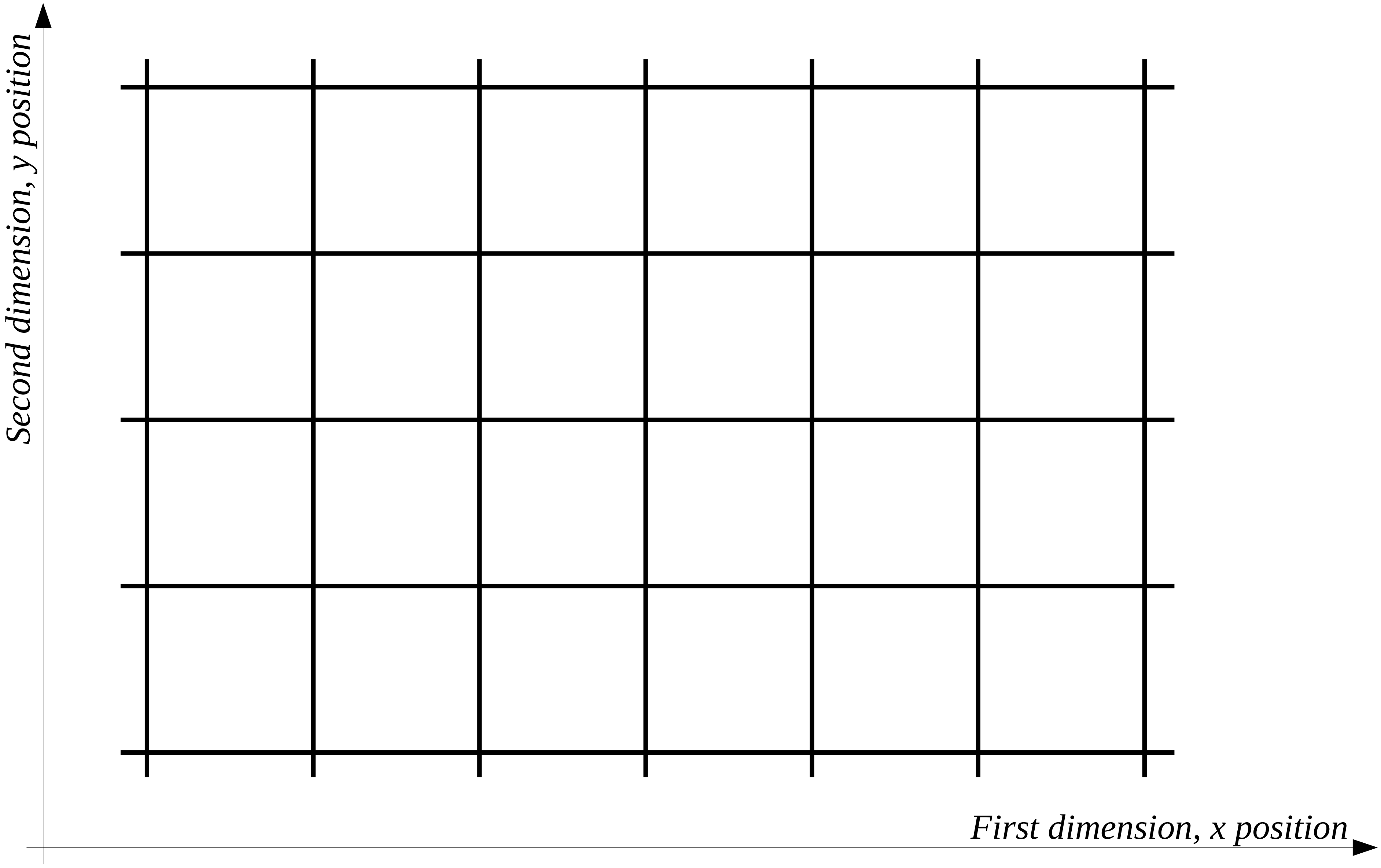
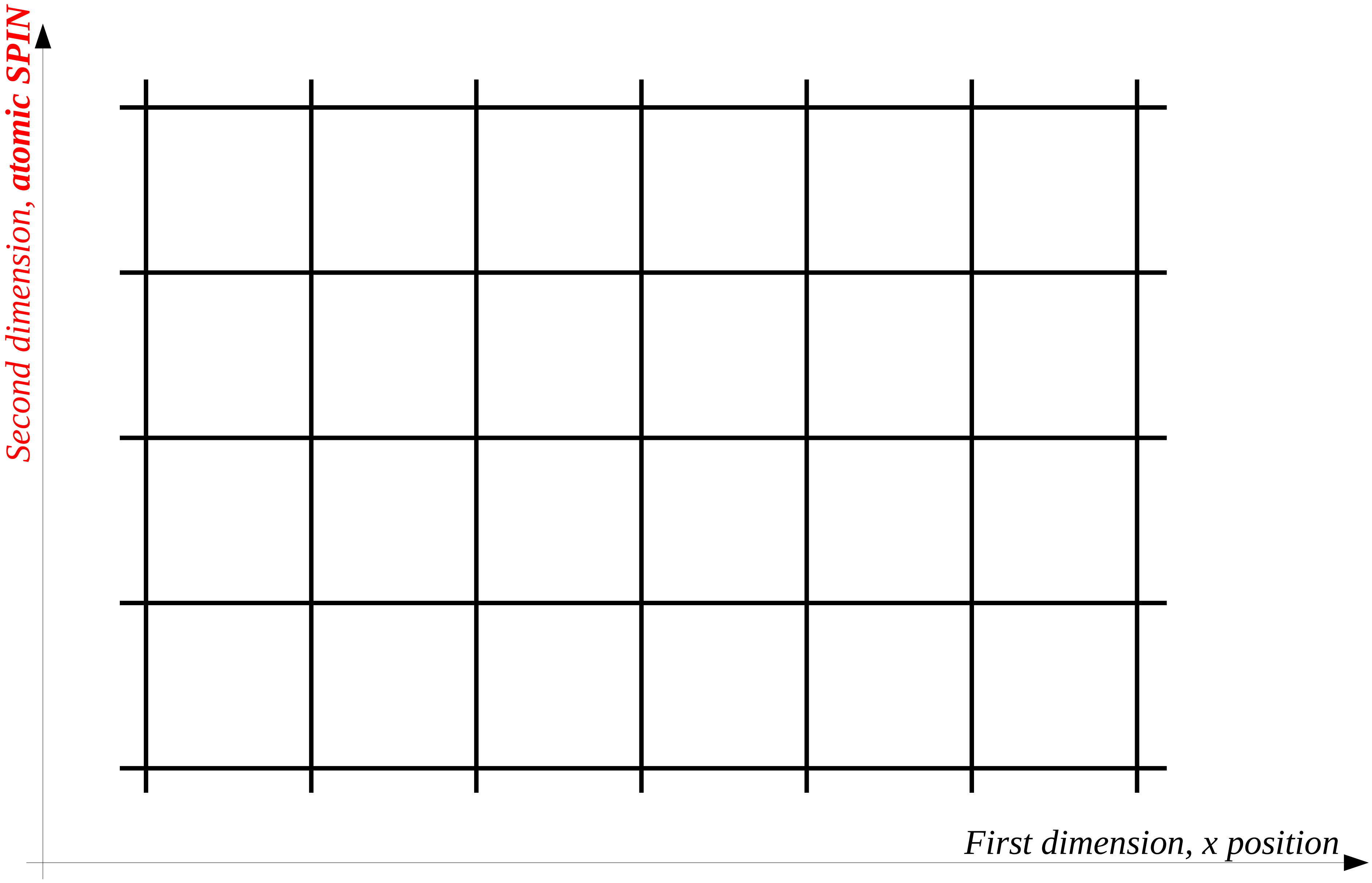
Synthetic dimensions



Synthetic dimensions



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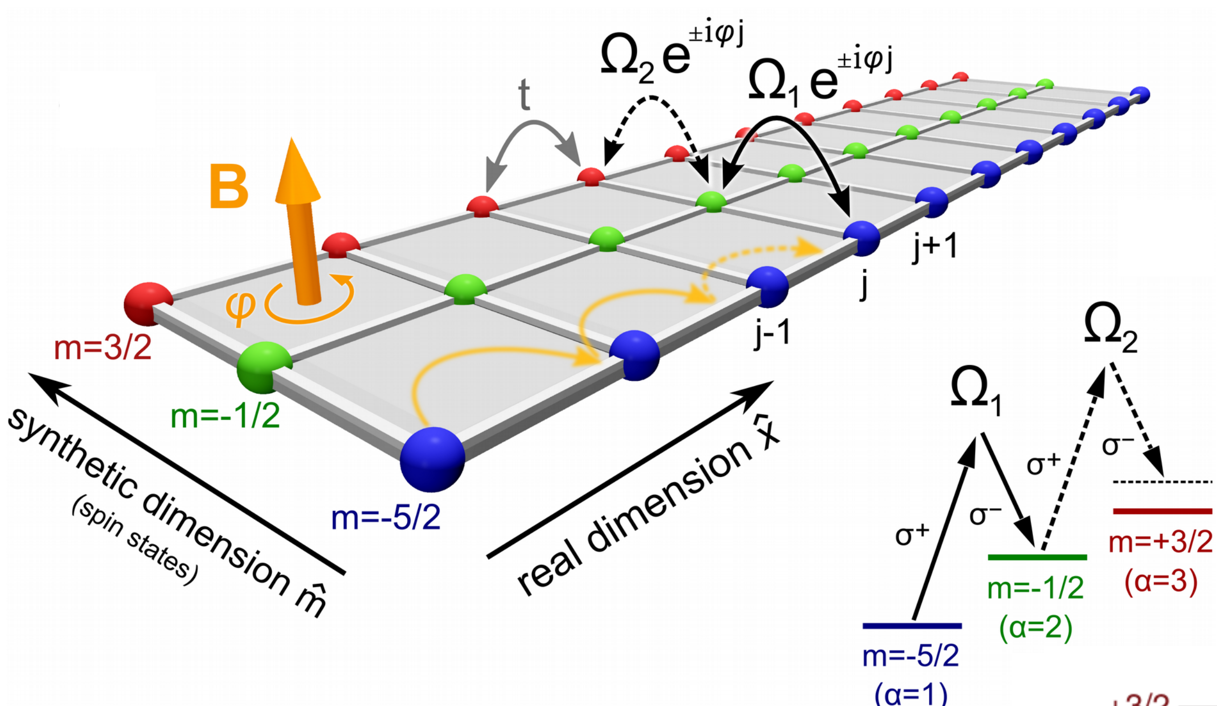
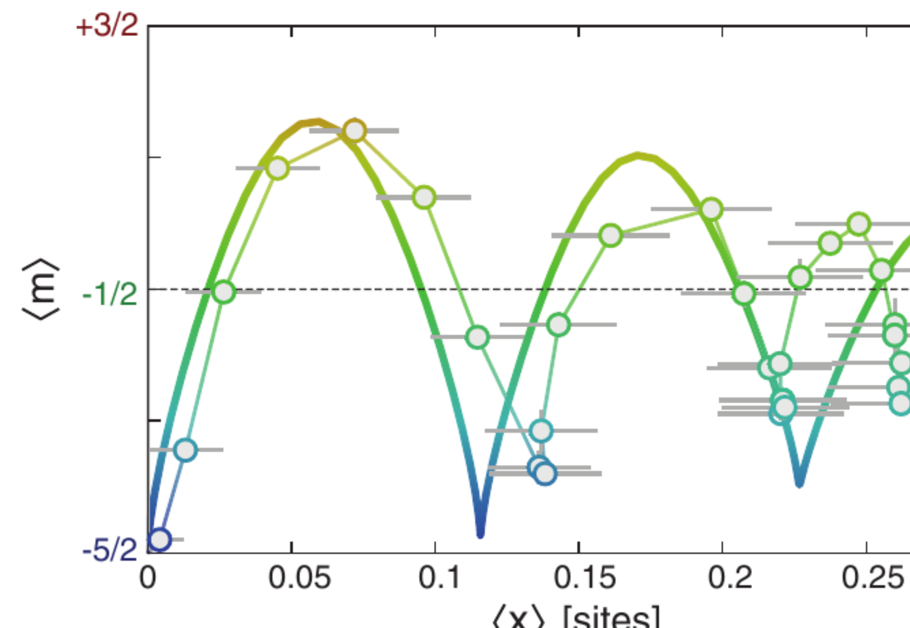
Experiments



- Three hyperfine states of fermionic ytterbium ^{173}Yb
- **Hopping in real space:** natural motion
- **Hopping in synthetic space:** laser induced, it can be a complex number and represent an artificial gauge field

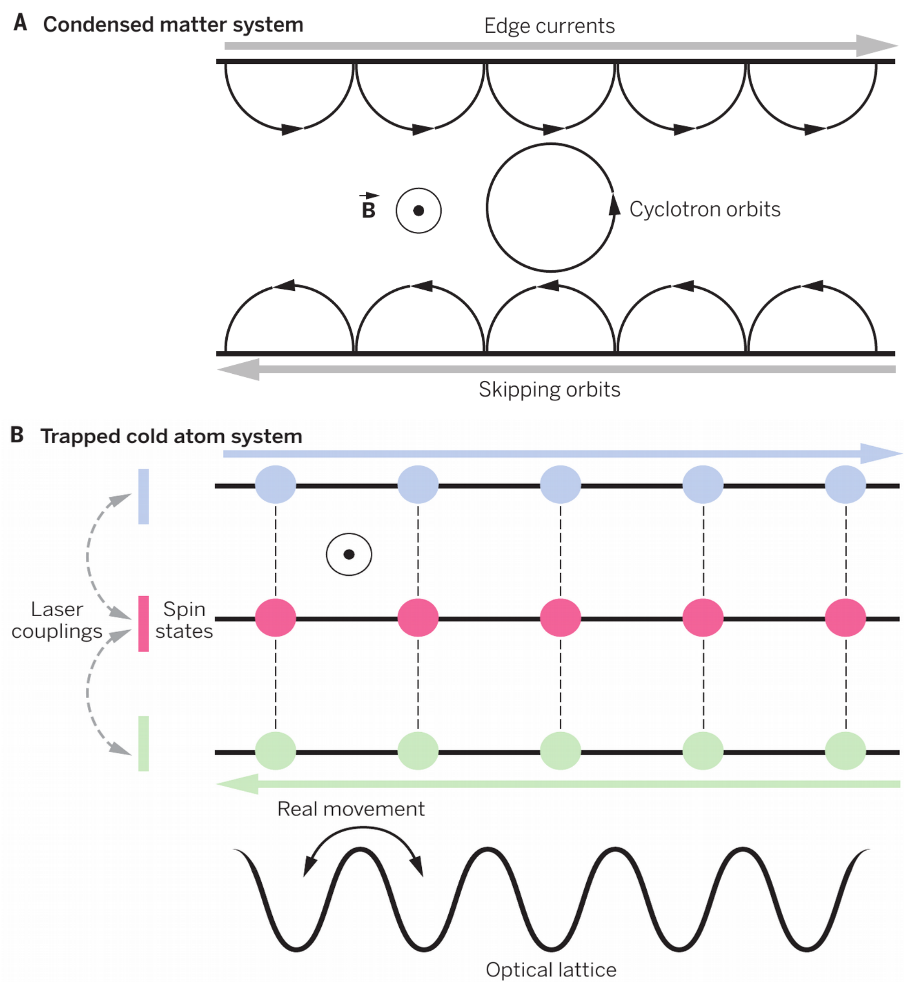


- Position in synthetic space given by the magnetic properties of the gas
- Theory using free-fermions

Mancini, Pagano, Cappellini, et al. Science **349** 1510 (2015)

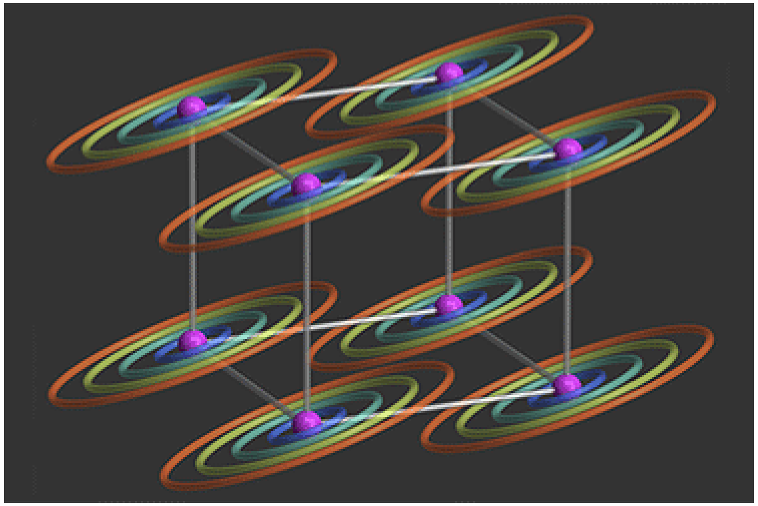
Why synthetic dimensions

- **Artificial gauge field**
- **Hard-wall boundaries in cold atoms**



Boada, Celi, Latorre, Lewenstein PRL 108, 133001 (2012)
Celi, Massignan, Ruseckas, et al. PRL 112, 043001 (2014)

- **Four-dimensional (or higher) physics**



Price, Zilberberg, Ozawa, et al. Phys. Rev. Lett. **115**, 195303 (2015)
Price, Ozawa, Goldman, PRA 95 023607 (2017)

- **Extensions to photonic systems**

Carusotto and Zilberberg

Limitations:

- Intrinsically discrete
- Typically short
- Strange form of interactions

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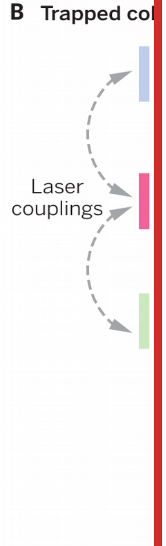
Why synthetic dimensions

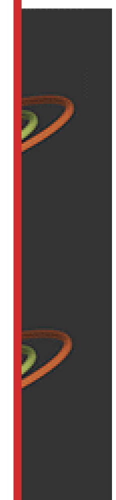
- **Artificial gauge field**
- **Har**

- **Four dimensional (or higher) physics**

A Condensed

B Trapped co

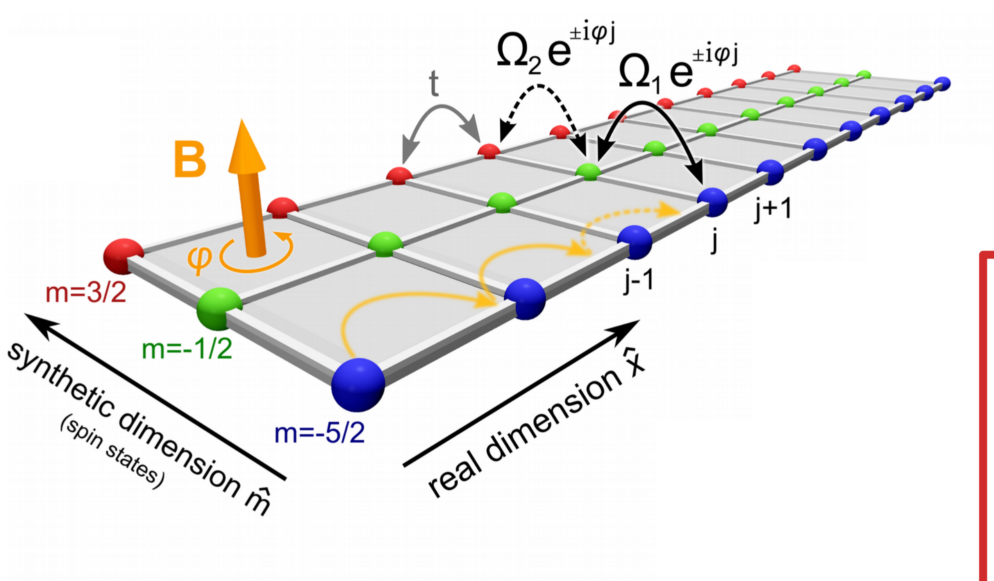




195303 (2015)

Relevant for this talk:

- Synthtic dimensions can realize synthetic flux ladders
- There is interaction between the legs of the ladder



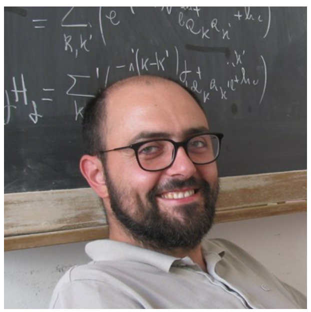
Advertisement:
See the talk of Saro Fazio for other theory ideas when putting periodic boundary conditions in the synthetic dimension

Boada, Celi, Latorre, Lewenstein PRL 108, 133001 (2012)
Celi, Massignan, Ruseckas, et al. PRL 112, 043001 (2014)

Fractional edge states in synthetic dimension

M. Calvanese Strinati, E. Cornfeld, D. Rossini, S. Barbarino, M. Dalmonte, R. Fazio,
E. Sela and LM, Phys. Rev. X **7** 021033 (2017)




Marcello Calvanese Strinati, Davide Rossini,
Simone Barbarino, Marcello Dalmonte, Rosario Fazio (Pisa and Trieste)


Eyal Cornfeld, Eran Sela (Tel Aviv University)

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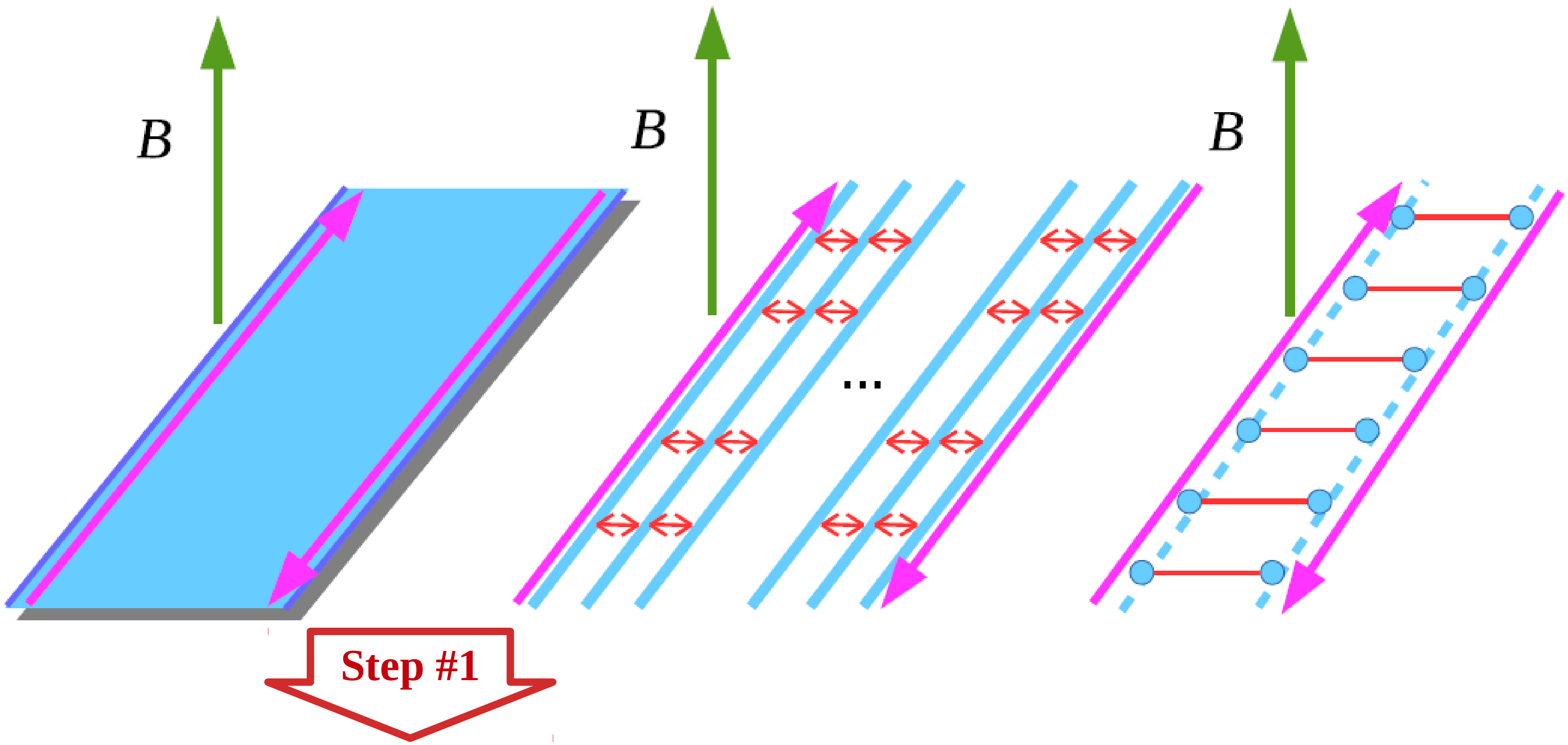
From 2D physics to ladders



Coupled-wire construction of the quantum Hall effect

- Bosonization of each wire (two counterpropagating modes with linear dispersion relation)
- Perturbative insertion of inter-wire coupling and detection of quantum-Hall instabilities

VOLUME 88, NUMBER 3 PHYSICAL REVIEW LETTERS 21 JANUARY 2002

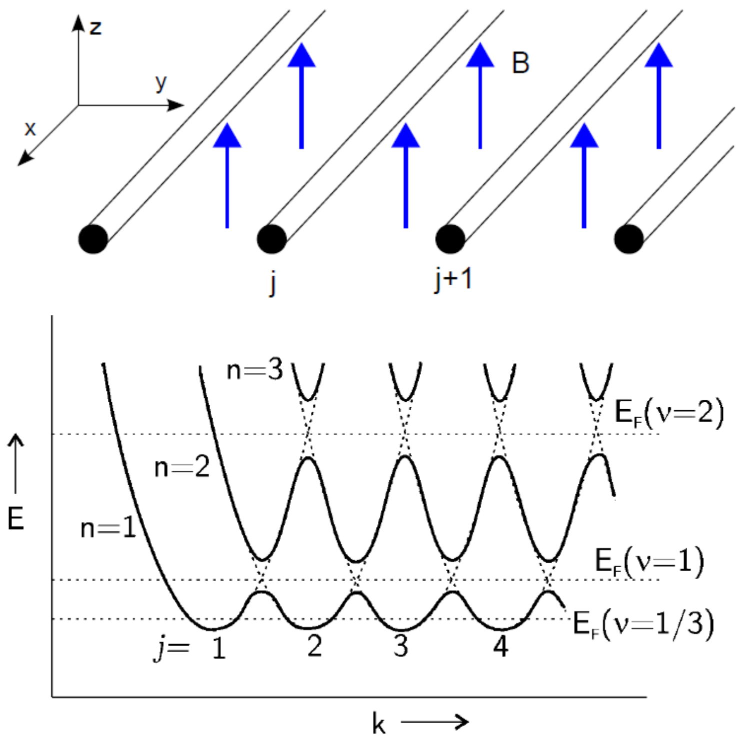
Fractional Quantum Hall Effect in an Array of Quantum Wires

C. L. Kane, Ranjan Mukhopadhyay, and T. C. Lubensky
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 27 August 2001; published 4 January 2002)

We demonstrate the emergence of the quantum Hall (QH) hierarchy in a 2D model of coupled quantum wires in a perpendicular magnetic field. At commensurate values of the magnetic field, the system can develop instabilities to appropriate interwire electron hopping processes that drive the system into a variety of QH states. Some of the QH states are not included in the Haldane-Halperin hierarchy. In addition, we find operators allowed at any field that lead to novel crystals of Laughlin quasiparticles. We demonstrate that any QH state is the ground state of a Hamiltonian that we explicitly construct.

DOI: 10.1103/PhysRevLett.88.036401 PACS numbers: 71.10.Pm, 71.27.+a, 73.43.Cd, 73.43.–f

From 2D physics to ladders



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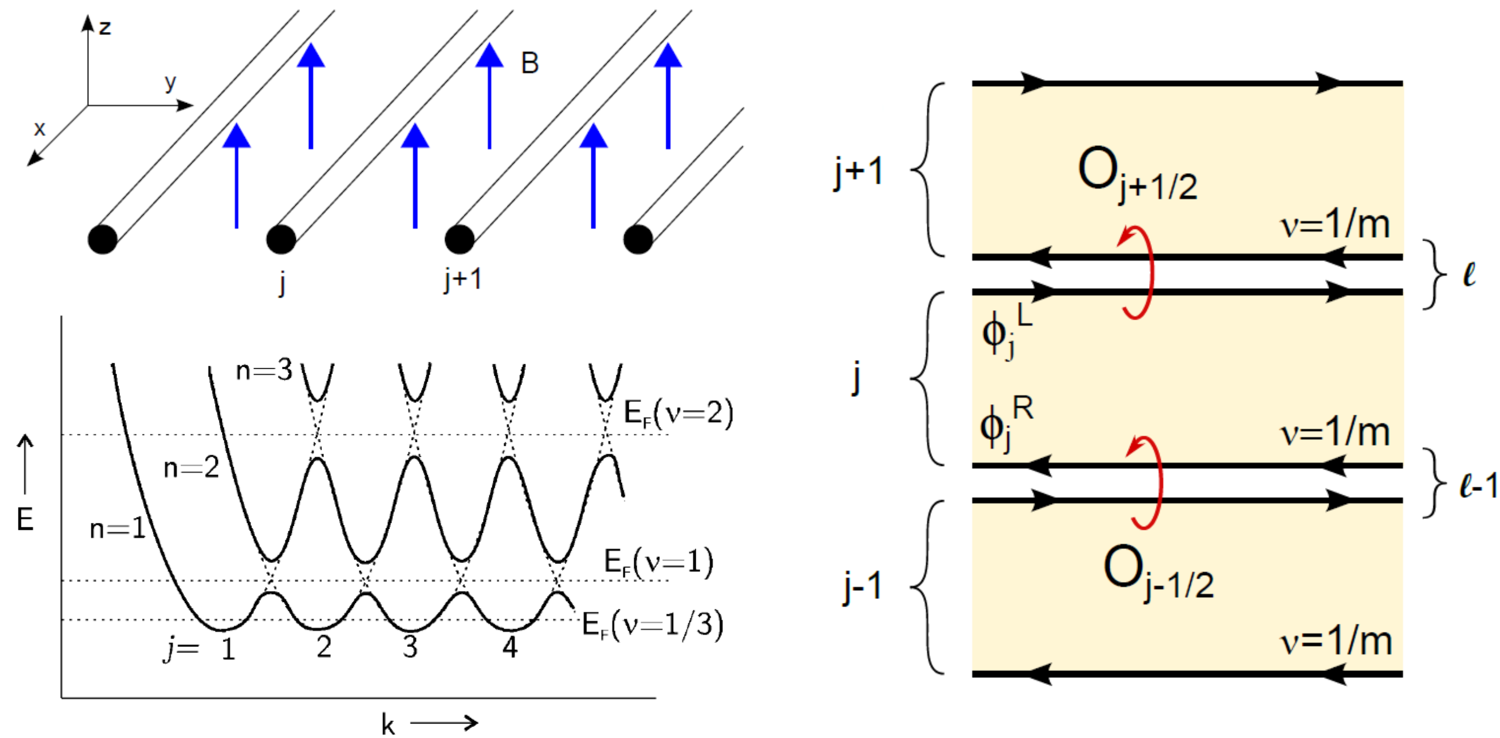

$$\psi_e \equiv e^{im\tilde{\Phi}_{L,1}}$$

Coupled-wire construction of the quantum Hall effect

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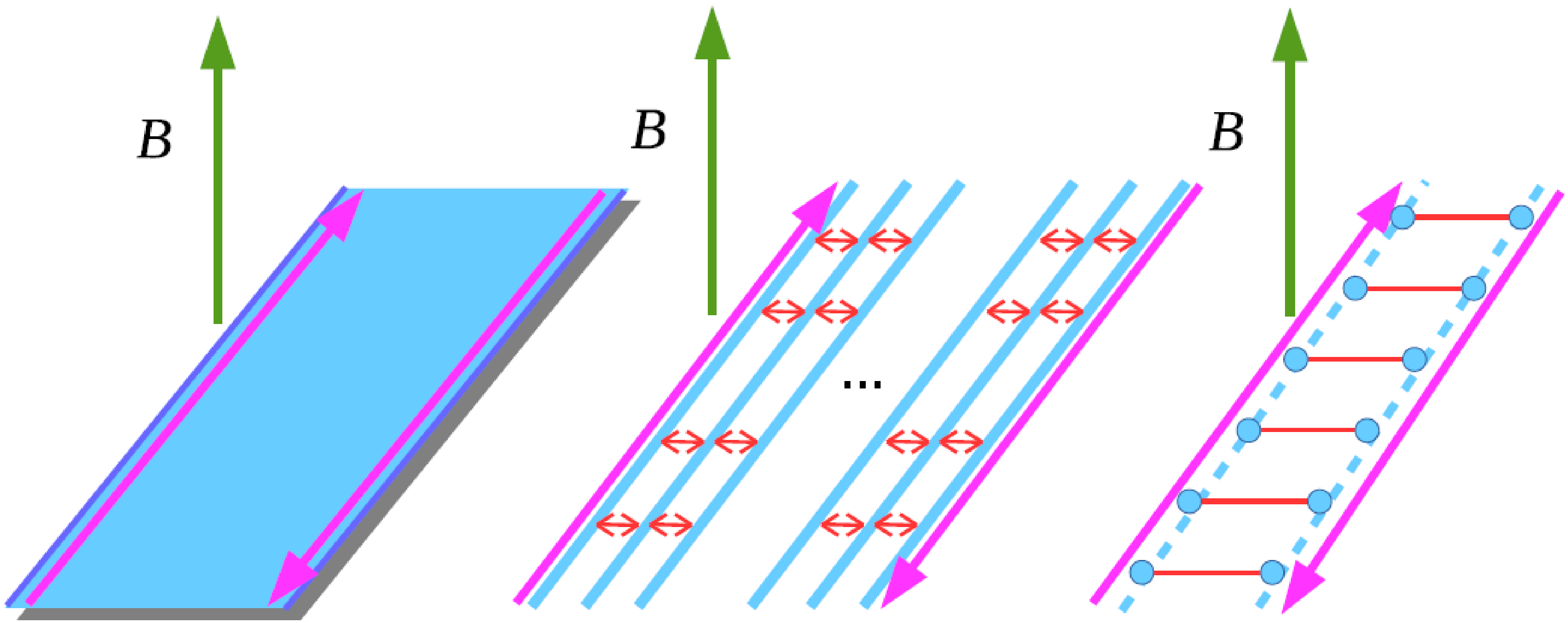


Step #2

An infinite array of wires is not necessary in order to have fractional edge modes

- Two are enough: a flux ladder! clearly to be engineered with synthetic dimensions ☺
- Topological protection is lost: no spatial distance of counterpropagating edge modes, back-scattering possible

Laughlin-like physics

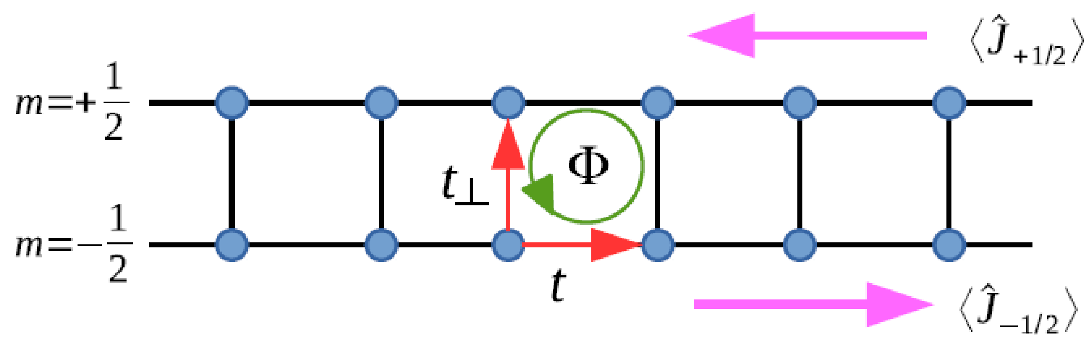
PHYSICAL REVIEW B 89, 115402 (2014)

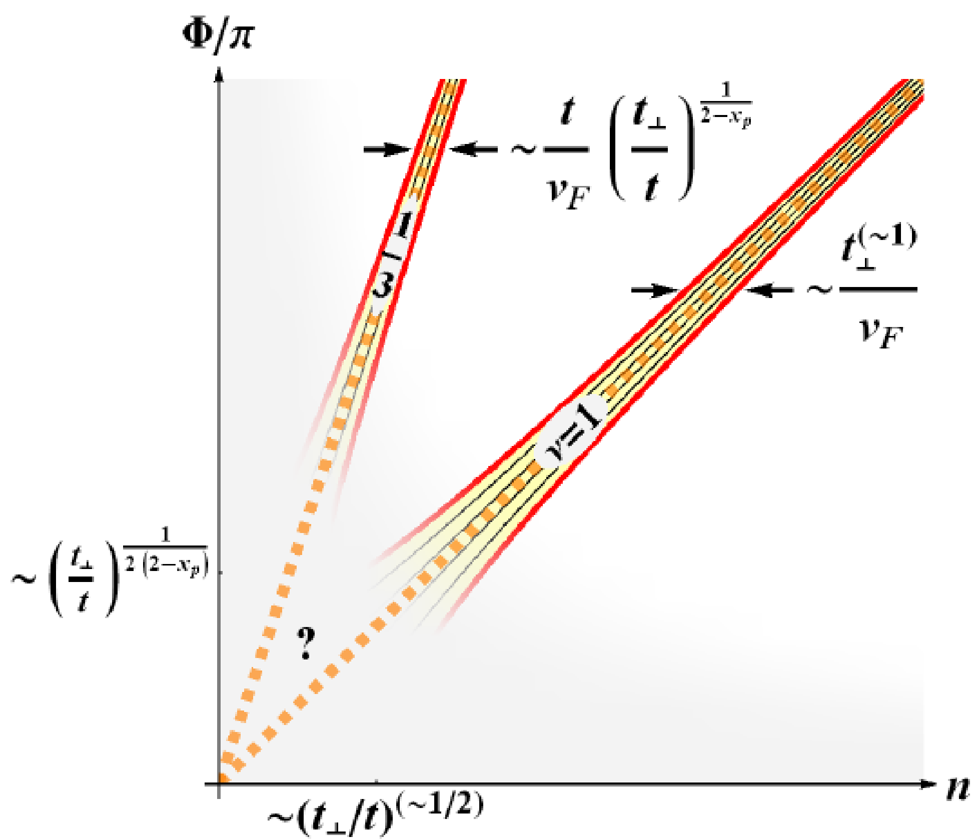
Fractional helical liquids in quantum wires

Yuval Oreg,[1] Eran Sela,[2] and Ady Stern[1]
[1] *Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[2] *Raymond and Beverly Sackler School of Physics and Astronomy, Tel-Aviv University, Tel Aviv, 69978, Israel*
(Received 13 January 2014; published 4 March 2014)

...but see also works by Le Hur and Petrescu

Laughlin-like states in ladders





- Fractional edge modes appearing at **ν = 1/p**
  - For bosons: p is even
  - For fermions: p is odd
- p = 1 is analogous to the integer quantum Hall effect and no interactions are necessary
- p > 1 requires interactions and can be efficiently addressed with matrix-product states
- The calculation is perturbative in $t_\perp$ and is based on bosonization

Cornfeld and Sela PRB 2015

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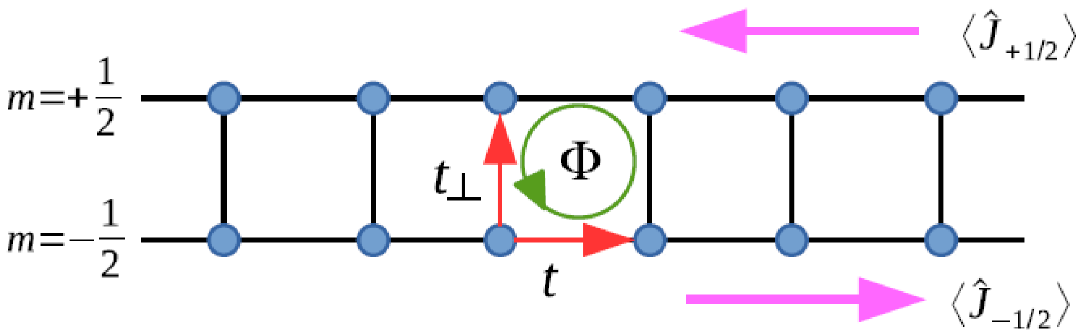
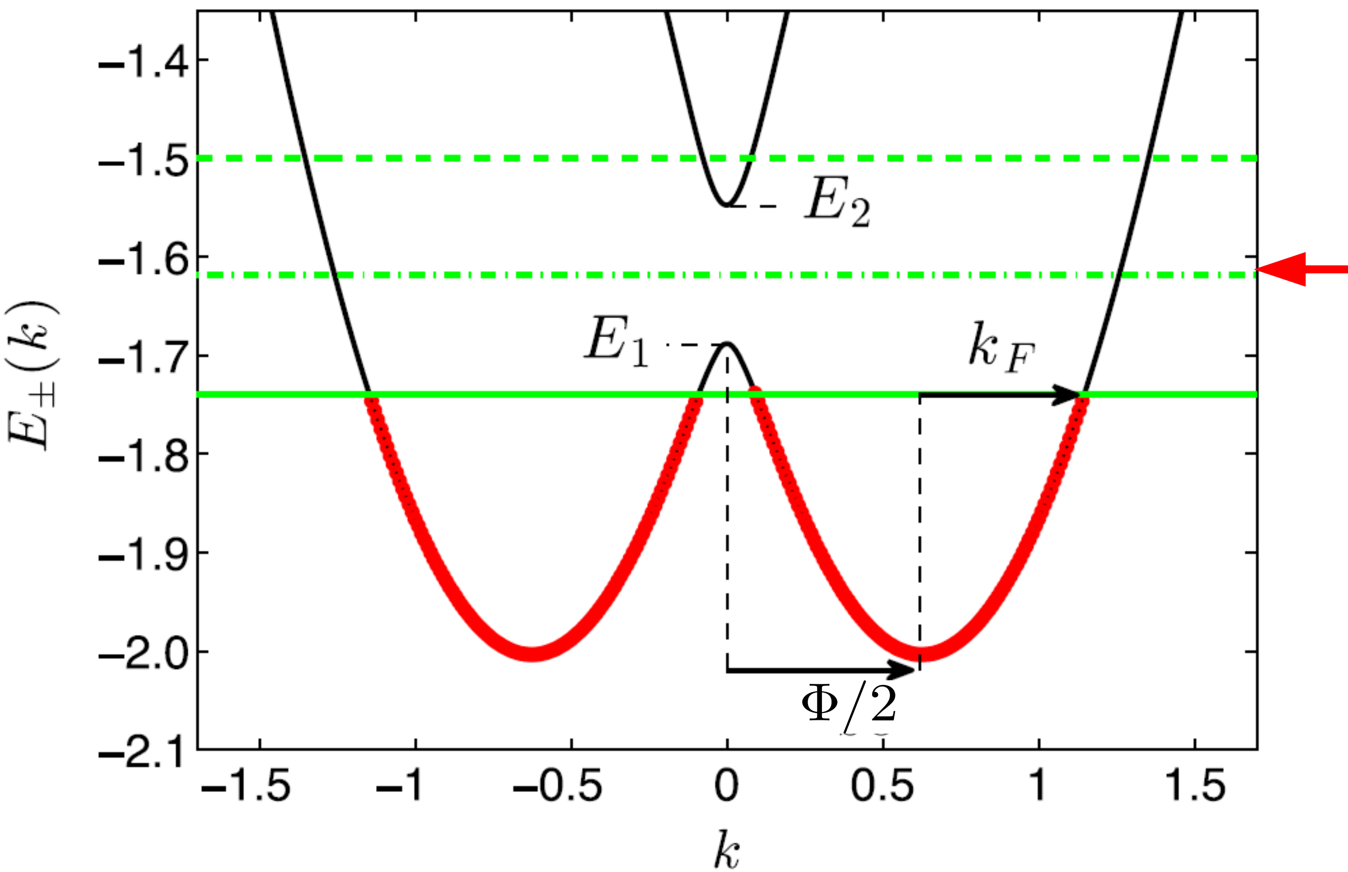
“Laughlin”-like physics @ ν =1

Free fermions for ν=1



$$\hat{H} = -t \sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + H.c. \right]$$
$$+ t_\perp \sum_j \left[c^\dagger_{j,\frac{1}{2}} c_{j,-\frac{1}{2}} + H.c. \right]$$



IQHE-like region	Two counterpropagating modes with almost opposite polarization
$k_F = \frac{\Phi}{2}$ $n = \frac{\Phi}{\pi}$ $\nu = 1$	In spin language, a **helical region**

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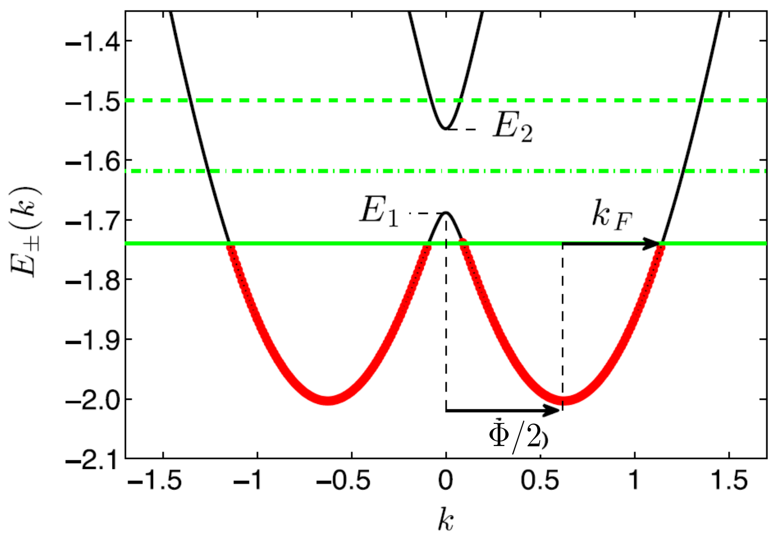
Observable quantities (beyond band structure)

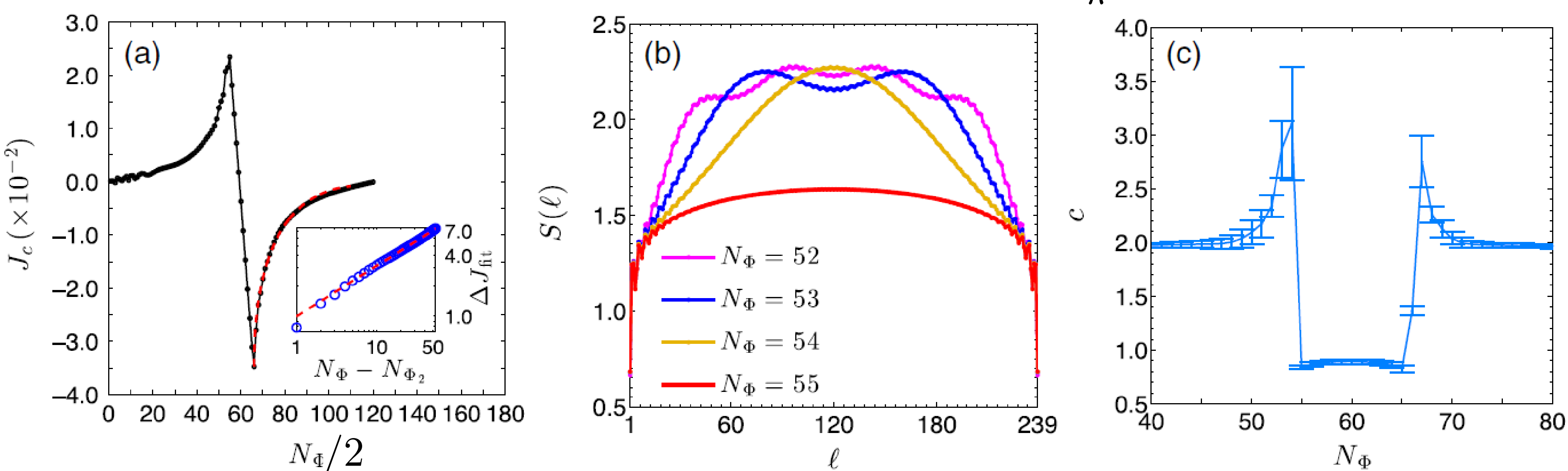


Observable signatures of the presence of the IQHE-like region

- Chiral current
- Entanglement entropy
- Central charge

by varying the chemical potential (or the flux) across the commensurate value

$$n = \frac{\Phi}{\pi}$$



Benchmark for the simulations in the interacting regime!

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Laughlin-like physics @ $\nu = 1/2$

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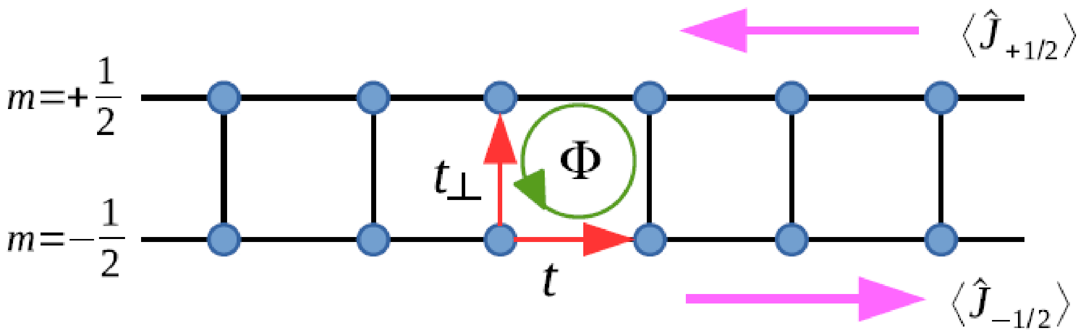
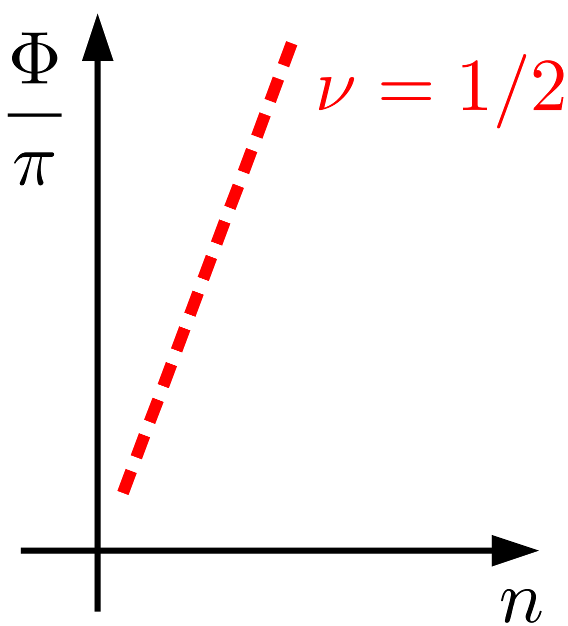
The bosonic flux ladder



$$\hat{H} = -t \sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{b}^\dagger_{j,m} \hat{b}_{j+1,m} + H.c. \right] + t_\perp \sum_j \left[b^\dagger_{j,\frac{1}{2}} b_{j,-\frac{1}{2}} + H.c. \right] + V_\perp \sum_j \hat{n}_{j,+\frac{1}{2}} \hat{n}_{j,-\frac{1}{2}}$$

+ hard-core constraint



First prediction to be tested:

- Universal signatures in the current profile along the ν = ½ line

Cornfeld and Sela PRB 2015

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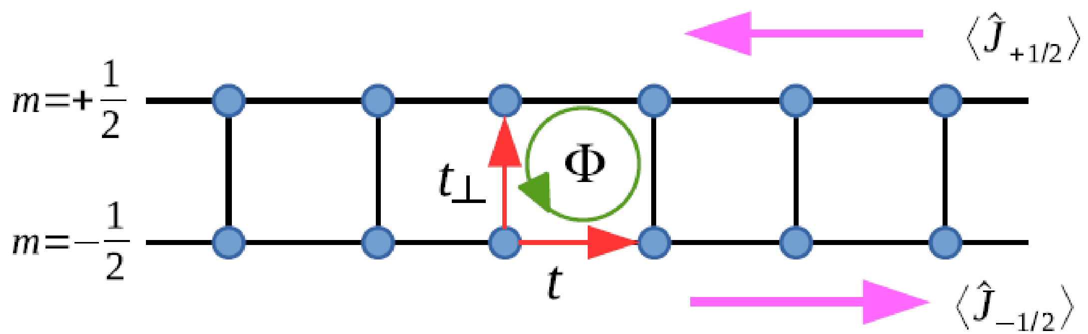
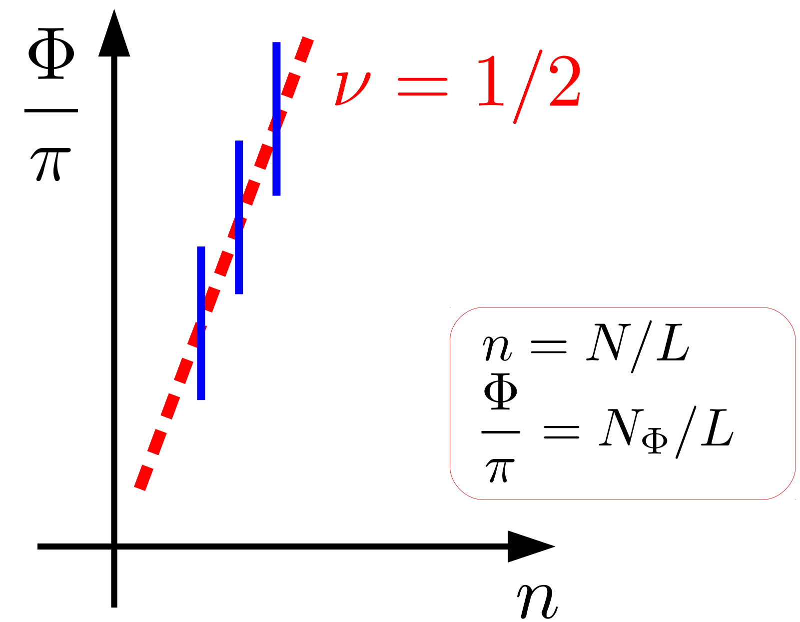
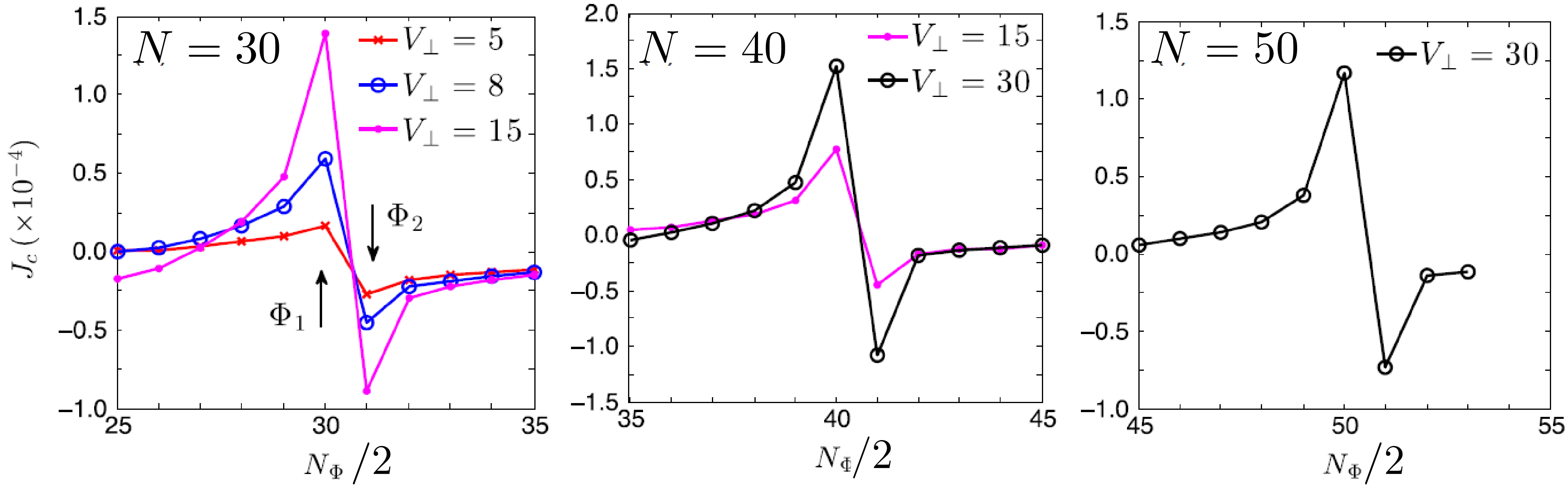
"Universal" signatures

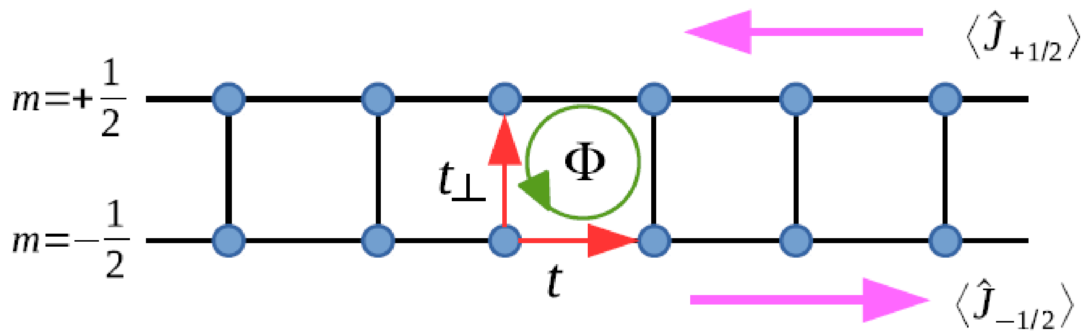


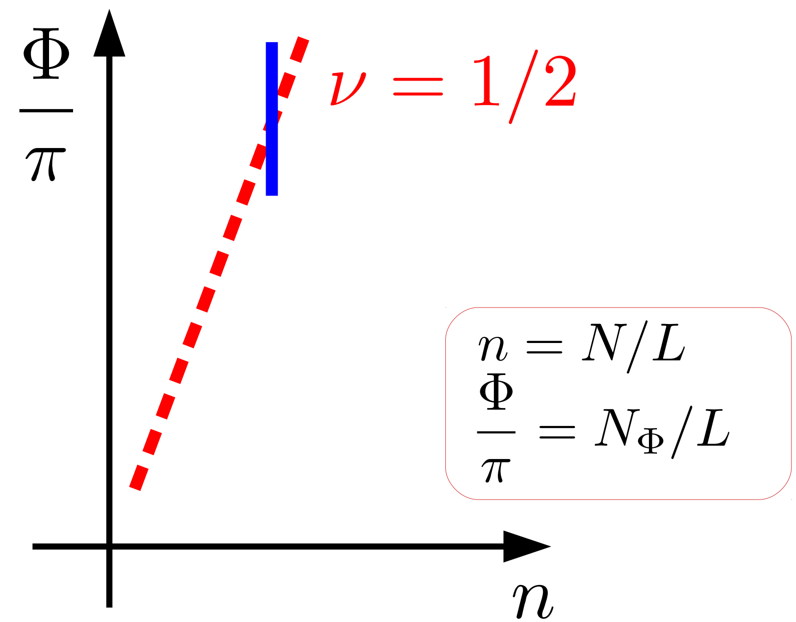


DMRG results for L = 120 and $t_\perp/t = 0.01$

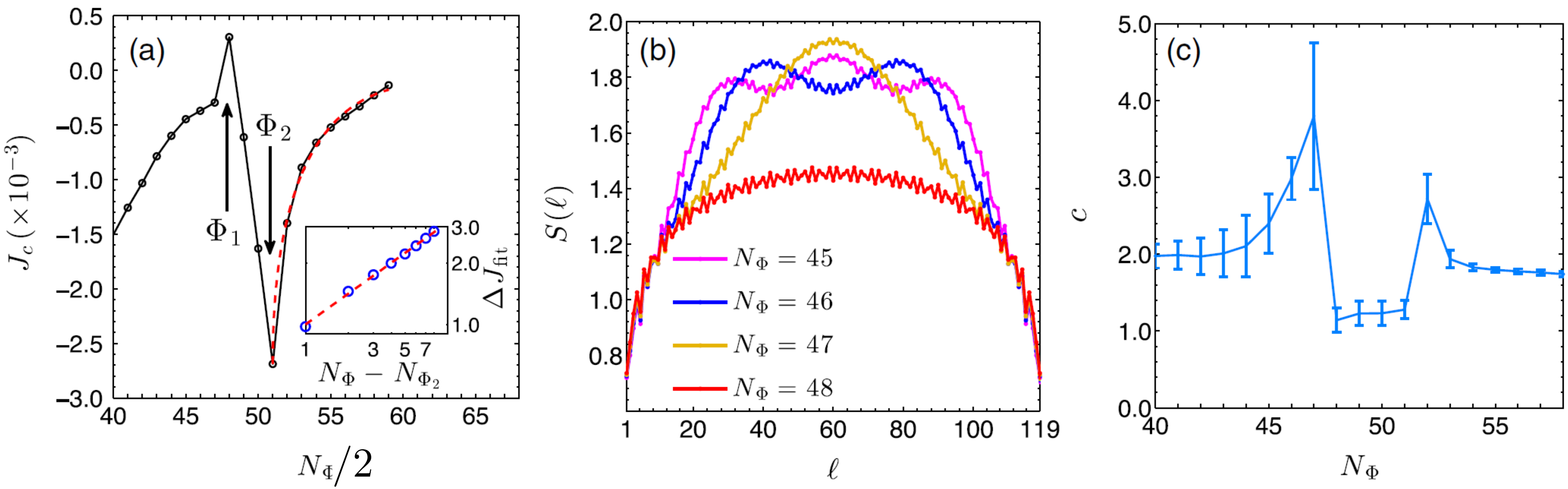


The bosonic Laughlin-like state





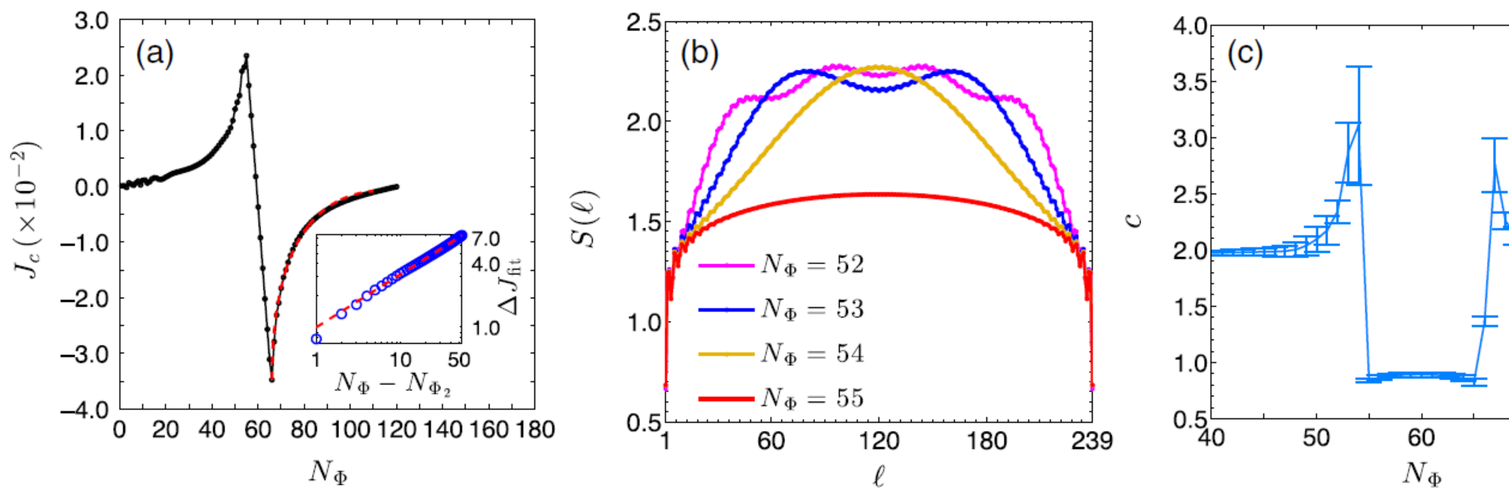
DMRG results for L = 120 and $t_\perp/t = 0.1$ $N = 50$



The bosonic Laughlin-like state

m=+

m=−



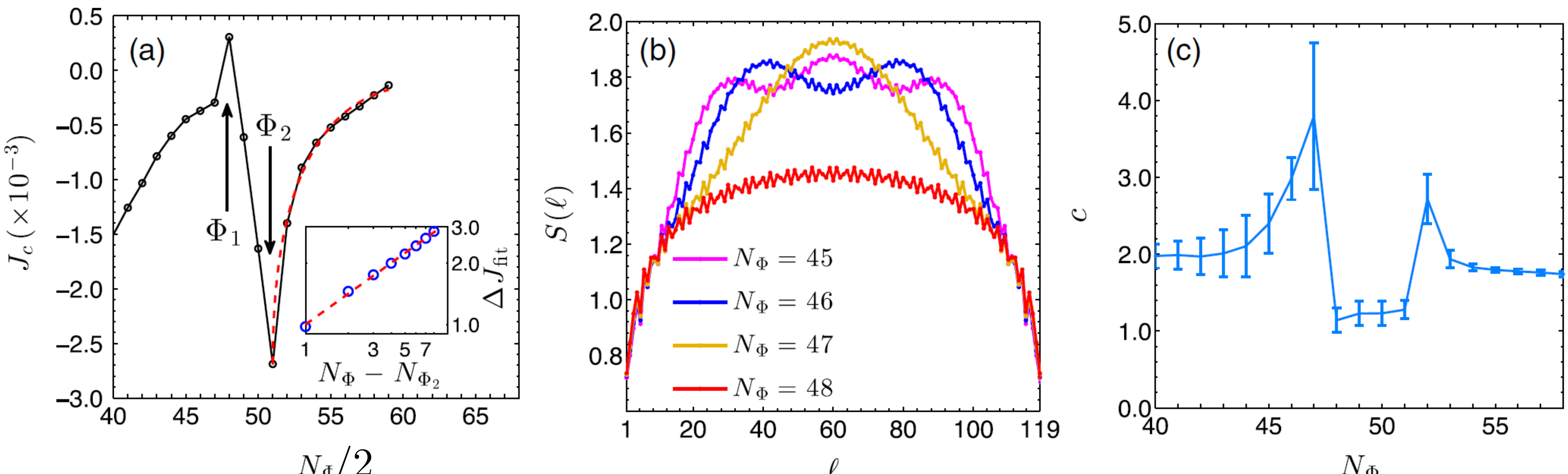
$/L$

N_Φ/L

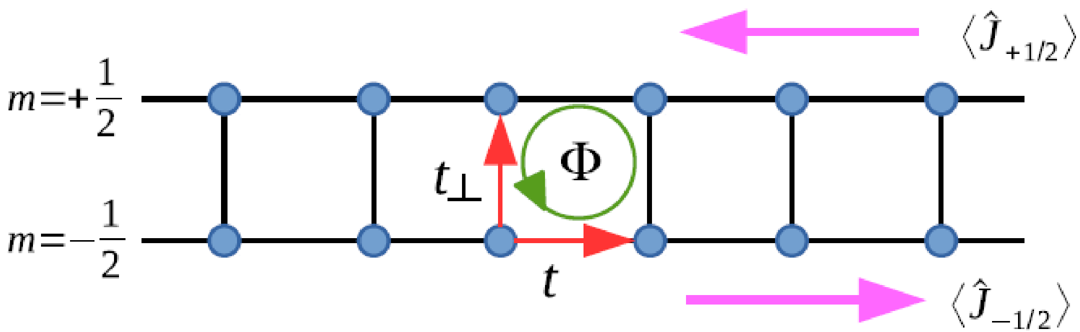
Free fermions: p = 1

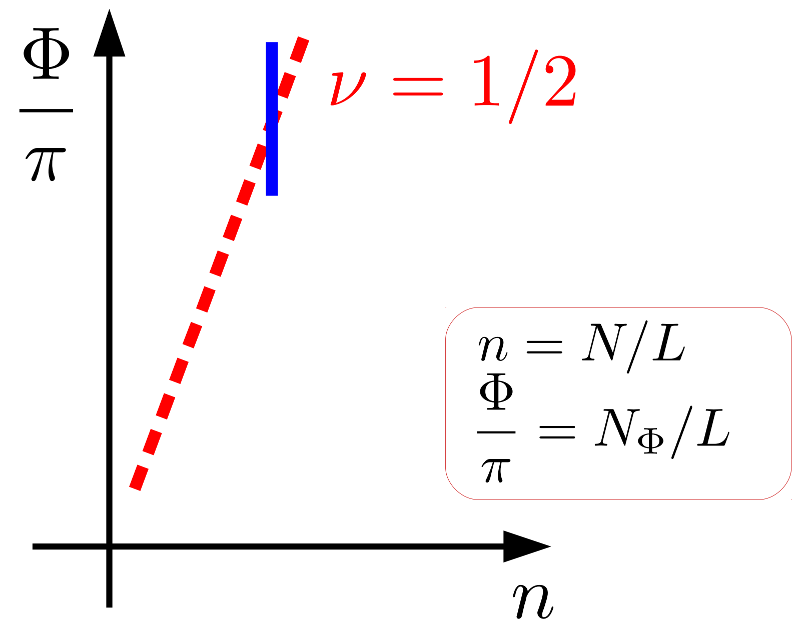
DMRG results for L = 120 and $t_\perp/t = 0.1$ $N = 50$

n

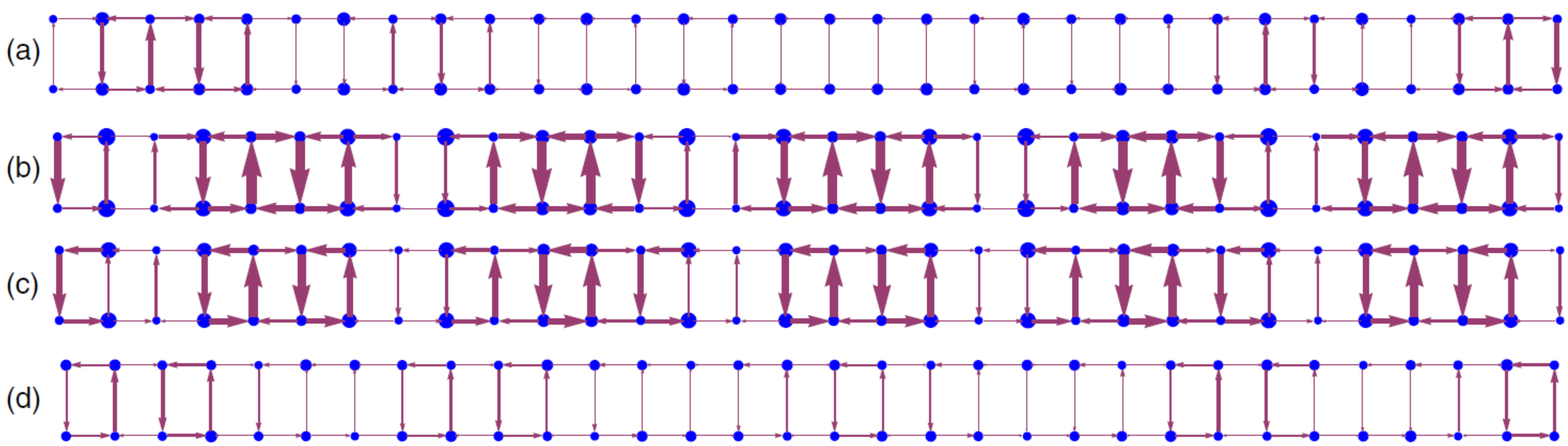


The bosonic Laughlin-like state

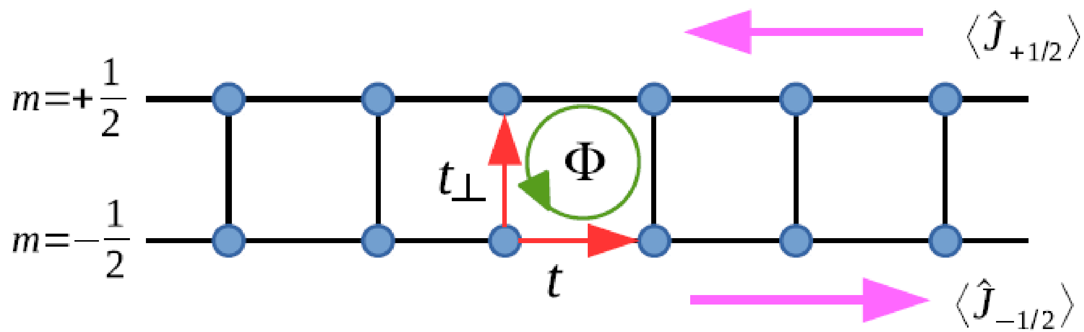


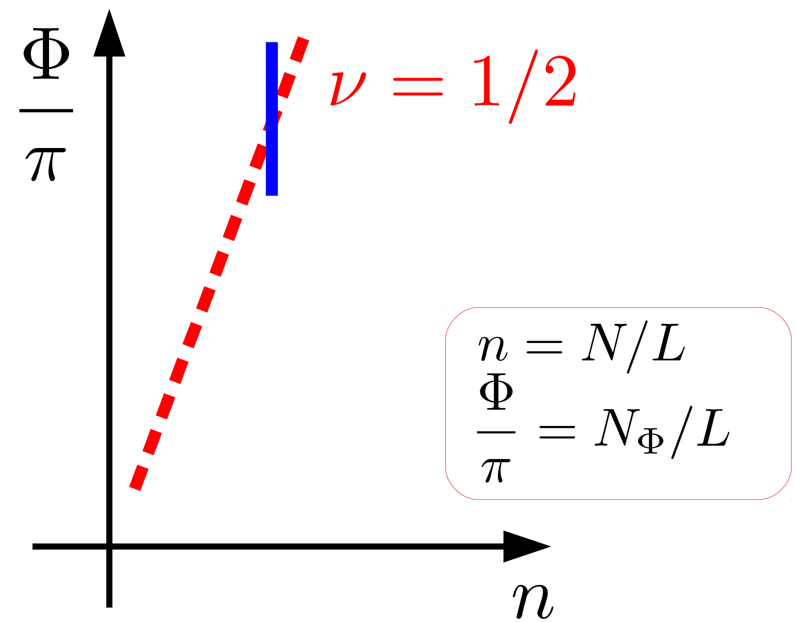


DMRG results for L = 120 and $t_\perp/t = 0.1$ $N = 50$

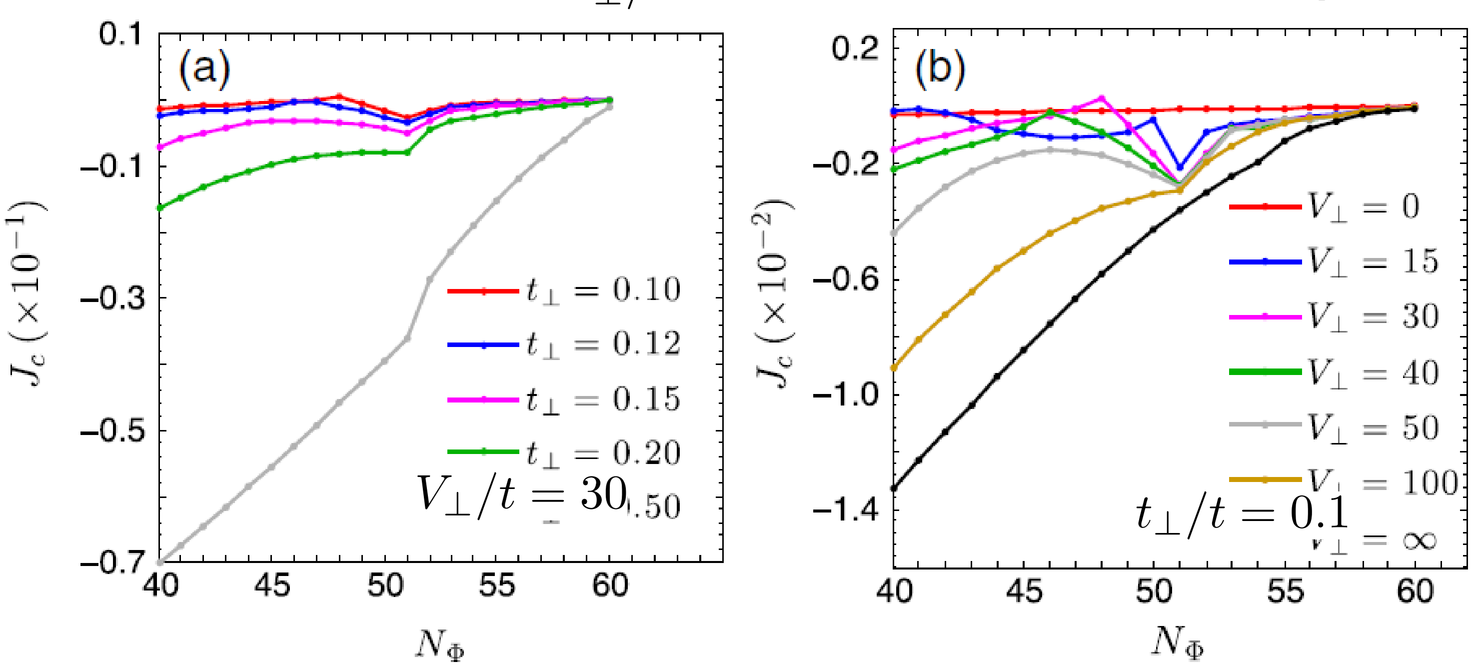


Beyond small inter-leg couplings





DMRG results for L = 120 and $t_\perp/t = 0.1$ $N = 50$



What happens to the Laughlin-like state when the double-cusp is lost? Alternative characterization?

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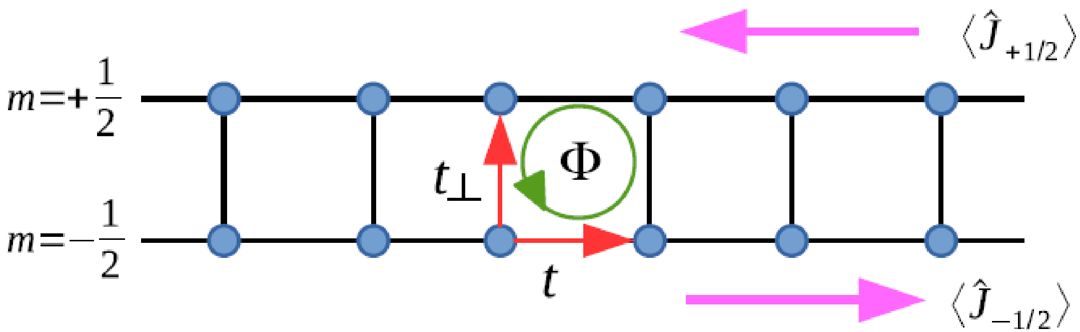
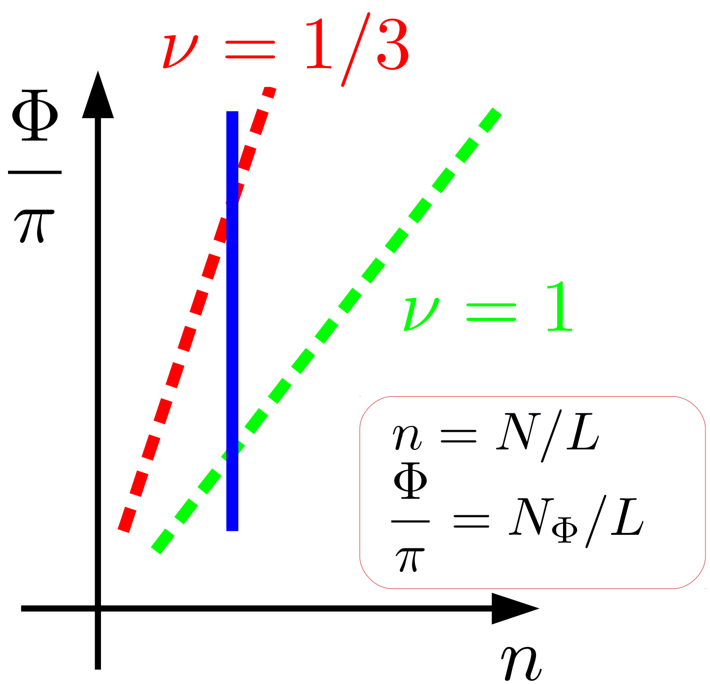
École normale supérieure

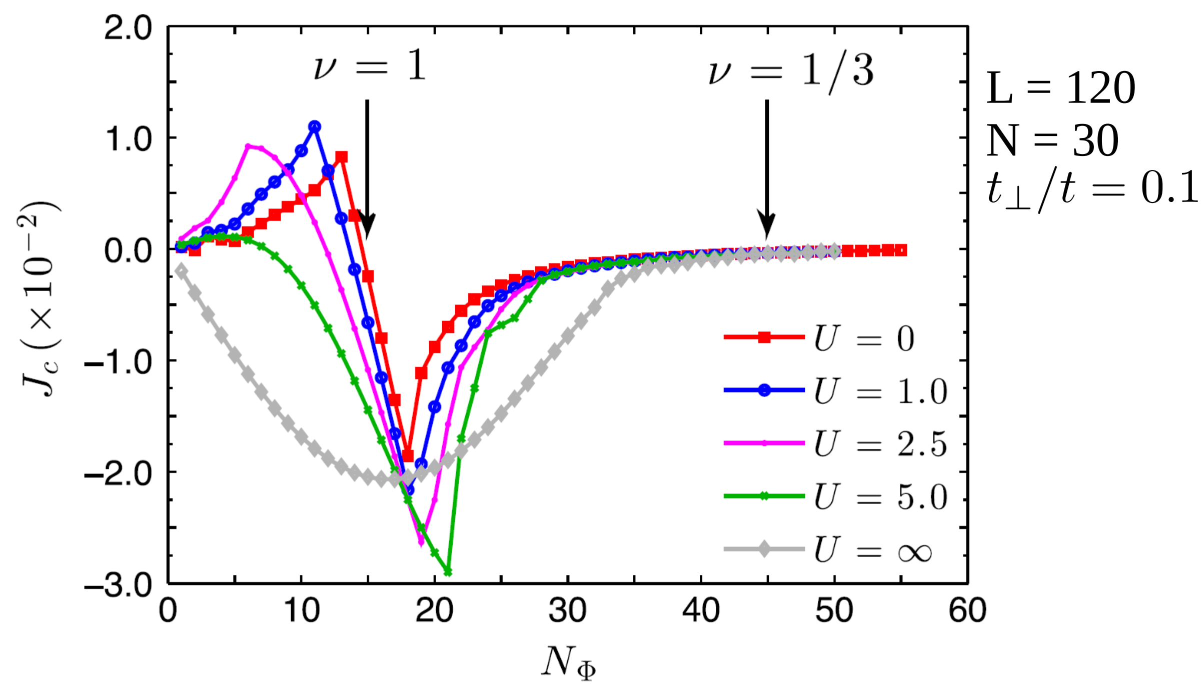
Laughlin-like physics @ ν =1/3

The fermionic Laughlin-like state?

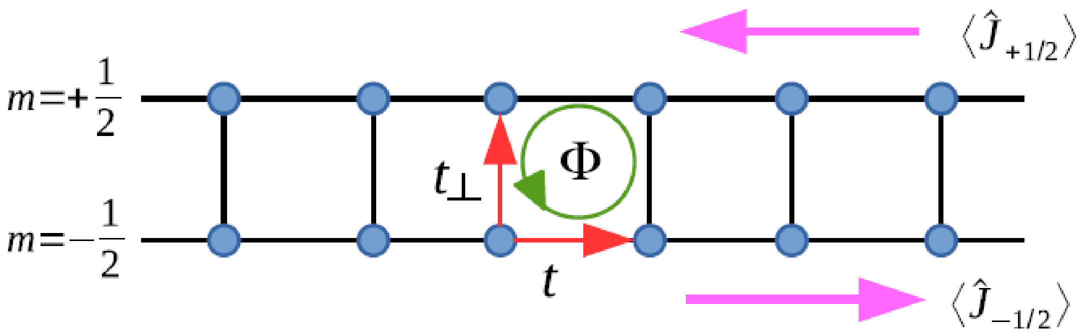


$$\hat{H} = -t \sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + H.c. \right] + t_\perp \sum_j \left[c^\dagger_{j,\frac{1}{2}} c_{j,-\frac{1}{2}} + H.c. \right] + U \sum_j \hat{n}_{j,+\frac{1}{2}} \hat{n}_{j,-\frac{1}{2}}$$





Long-range interactions



$$\hat{H} = -t\sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + H.c. \right] + t_\perp \sum_j \left[c^\dagger_{j,\frac{1}{2}} c_{j,-\frac{1}{2}} + H.c. \right] + U \sum_j \hat{n}_{j,+\frac{1}{2}} \hat{n}_{j,-\frac{1}{2}} + \sum_j \sum_{r\geq 0} V(r) \hat{n}_j \hat{n}_{j+r}$$

Exactly-solvable limit

- Shoulder potential

$$V(r) = \begin{cases} U & \text{for } r \leq \xi \\ 0 & \text{for } r > \xi. \end{cases}$$

- Nearest-neighbor Hard-core

$$U \gg t$$

Idea

the original model can be remapped into a model with:

- $\xi' = 0$
- $L' = L - (N-1)\xi$

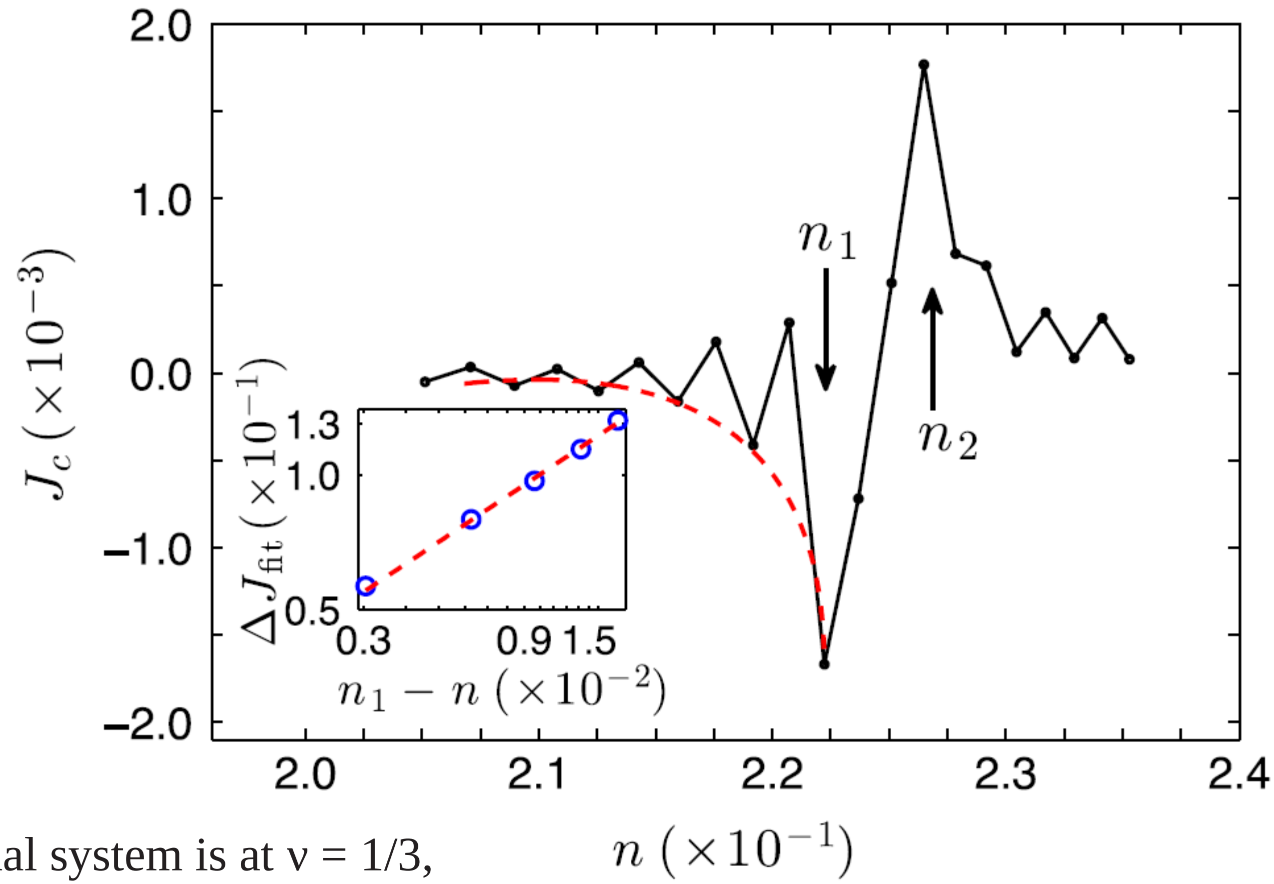
A model with ν = 1/p can be remapped to ν = 1

Long-range interactions

$m=+\frac{1}{2}$ $+ H.c.$

$m=-\frac{1}{2}$ $+ H.c.$

$)\hat{n}_j\hat{n}_{j+r}$



The actual system is at ν = 1/3,
with ξ = 3, L = 225, Φ ≃ 0.658π

Exa

- S

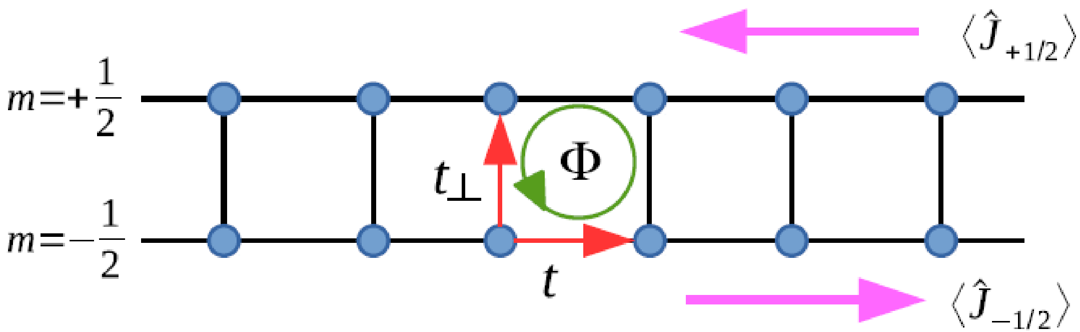
- Nearest-neighbor Hard-core

$$U \gg t$$

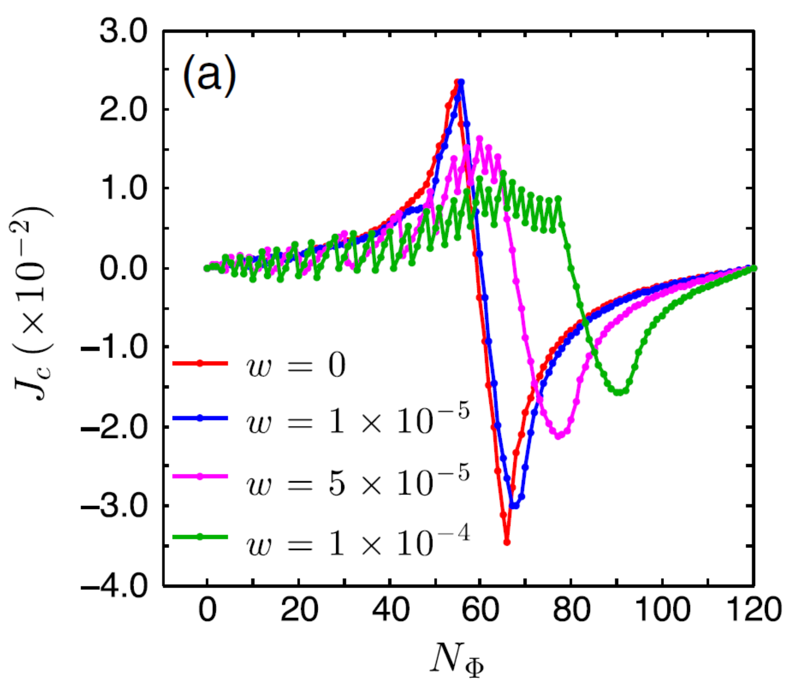
A model with ν = 1/p can be remapped to ν = 1

Experimental considerations

Harmonic confinement



$$\hat{H} = -t \sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + H.c. \right] + t_\perp \sum_j \left[c^\dagger_{j,\frac{1}{2}} c_{j,-\frac{1}{2}} + H.c. \right] + \sum_{m=\pm\frac{1}{2}} \sum_j w(j-j_0)^2 \hat{n}_{j,m}$$

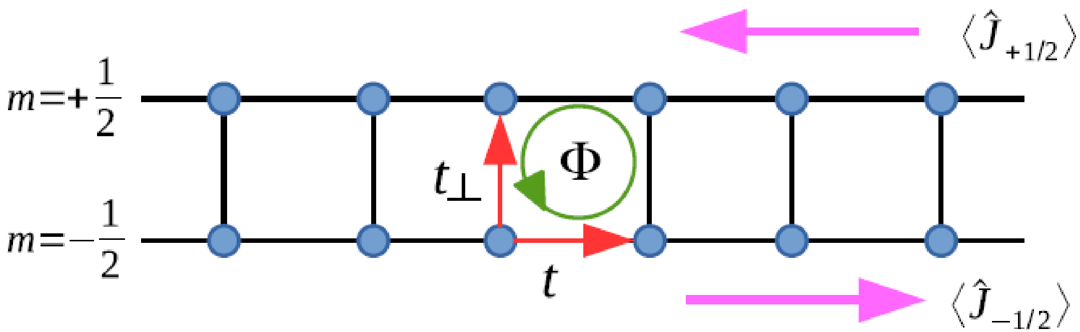


High sensibility even to very weak traps

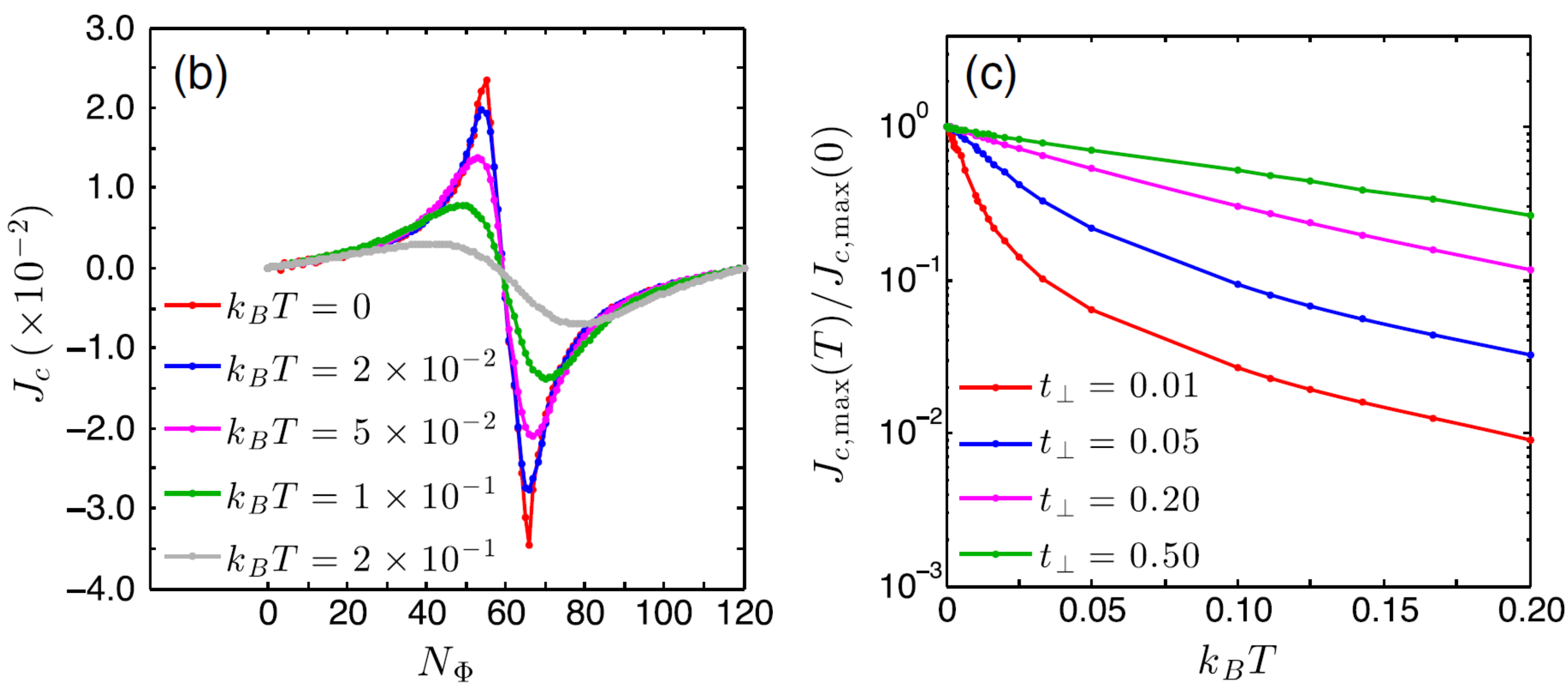
Box potential necessary

Free-fermion simulation with
L = 240,
N = 120,
and $t_\perp/t$ =0.2

Temperature



$$\hat{H} = -t \sum_j \sum_{m=\pm\frac{1}{2}} \left[e^{i\Phi m} \hat{c}^\dagger_{j,m} \hat{c}_{j+1,m} + H.c. \right] + t_\perp \sum_j \left[c^\dagger_{j,\frac{1}{2}} c_{j,-\frac{1}{2}} + H.c. \right]$$



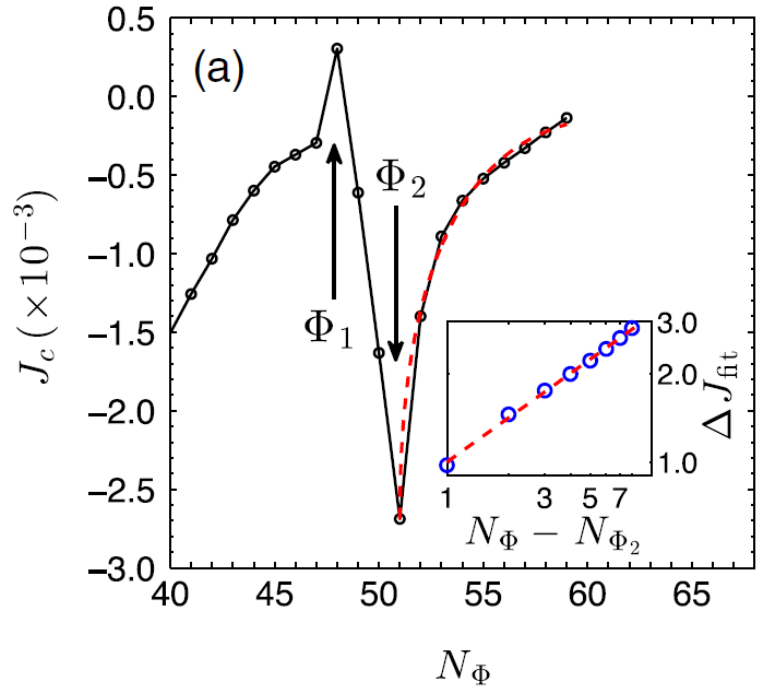
Conclusions

Our goal: fractional edge modes in synthetic ladders

Our result: synthetic dimensions are an interesting route to strongly-correlated physics

Key points:

- Signatures of Laughlin-like physics in bosonic and fermionic ladders
- Characterization in terms of a measurable observable (current)
- Experimental characterization: trap is bad, temperature is manageable



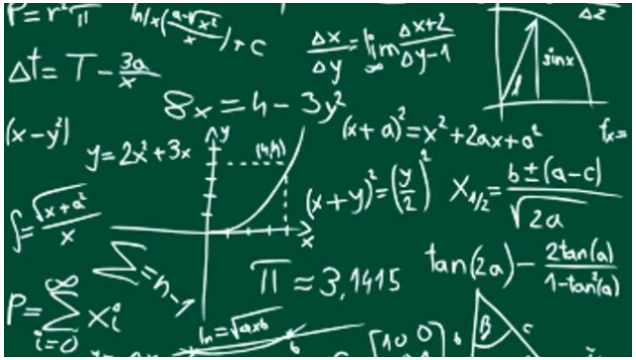
Calvanese Strinati, Cornfeld, Rossini, Barbarino, Dalmonte, Fazio, Sela and LM, Phys. Rev. X **7** 021033 (2017)

Perspectives

Is there an equilibrium observable that is quantized to the fractions described so far?

(my personal holy-grail)

Work in progress



Further **insights** in the physics of FQHE-like states in ladders:

- What happens increasing the inter-wire coupling?
- How many legs to move from Laughlin-like to Laughlin?
- Bulk fractional quasi-particles?
- Experiments?

Synthetic dimensions and topology:

an interesting collaboration

Thank you
for your attention

M. Calvanese Strinati, E. Cornfeld, D. Rossini, S. Barbarino, M. Dalmonte, R. Fazio, E. Sela and LM, Phys. Rev. X **7** 021033 (2017)