Synthetic dimensions with multi-mode ring resonators

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@ Synthetic dimensions in quantum engineered systems, ETH, Switzerland
22 November 2017

Outline

- Modes of a ring-resonator as a synthetic dimension
  - TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)
- Angular coordinate of a ring-resonator as a synthetic dimension
  - TO, & Carusotto, PRL **118**, 013601 (2017)
- Harmonic oscillator states as synthetic dimensions
  - Price, TO, & Goldman, PRA **95**, 023607 (2017)
- Four-dimensional quantum Hall effect and synthetic dimension
  - Price, Zilberberg, TO, Carusotto, & Goldman, PRL **115**, 195303 (2015)

Synthetic dimensions with photonics

- Optomechanics — photon & phonon degrees of freedom

  Schmidt, Kessler, Peano, Painter, & Marquardt, Optica **2**, 635 (2015)
- Optical cavities — orbital angular momentum degree of freedom

  Luo, Zhou, Li, Xu, Guo, & Zhou, Nature Comm. **6**, 7704 (2015)
- Photonic lattice — increasing connectivity

  Tsomokos, Ashhab, & Nori, PRA **82**, 052311 (2010)

  Jukic & Buljan, PRA **87**, 013814 (2013)

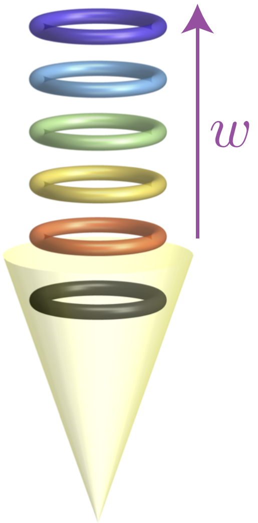
- Ring resonator arrays — different frequency modes or angular coodinates of a resonator

  TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)

  TO, & Carusotto, PRL **118**, 013601 (2017)

  Yuan, Shi, & Fan, Opt. Lett. **41**, 741 (2016), and series of works

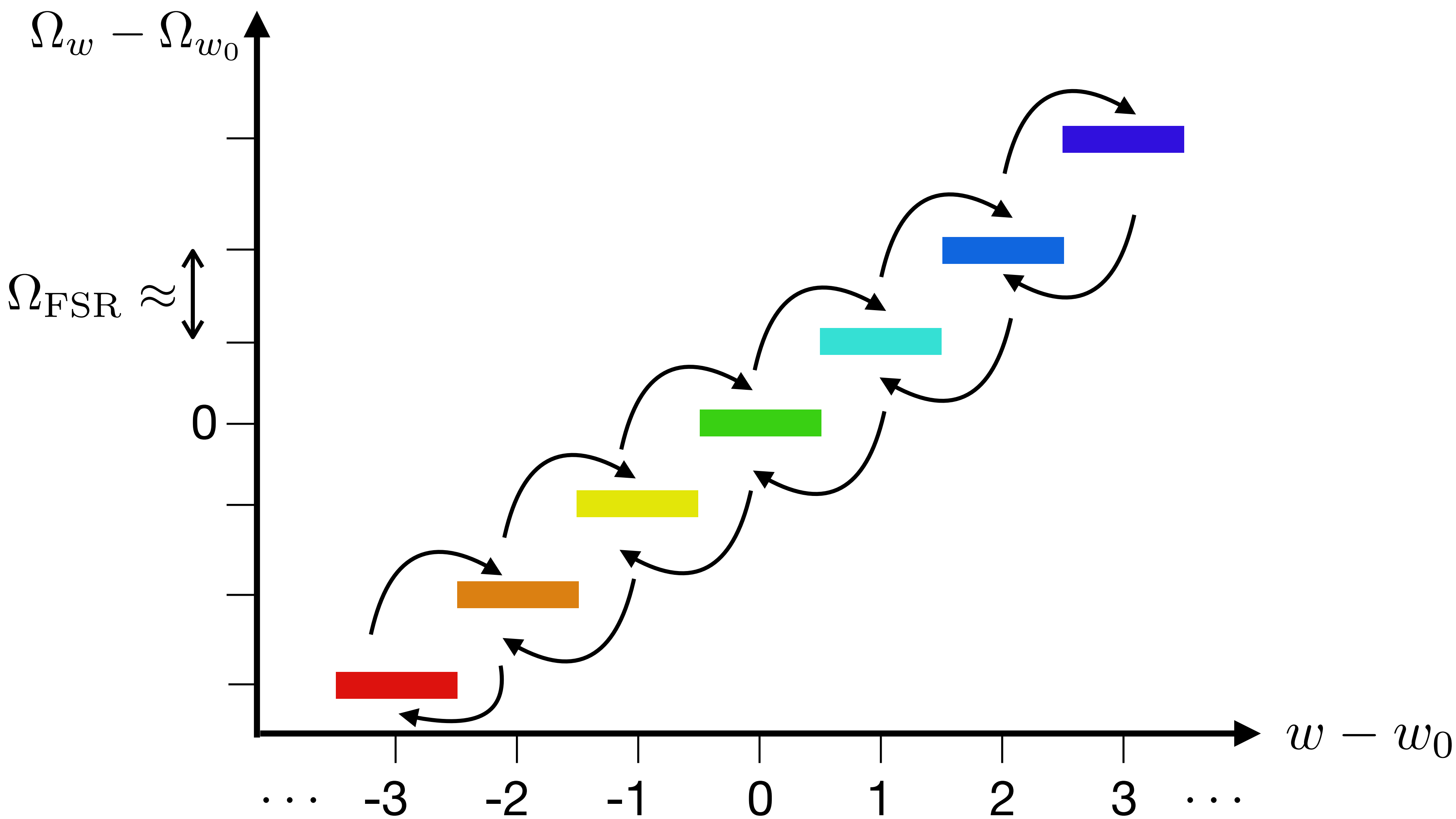
Hafezi et al, Nat. Photon. **7**, 1001, (2013)

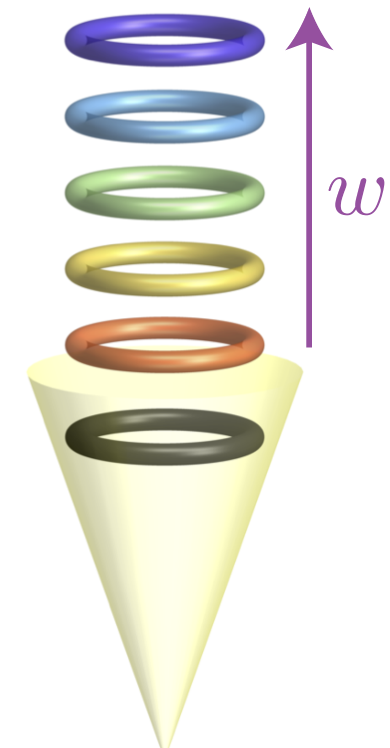


Multi-mode ring-resonator

Consider a ring resonator, and use different angular momentum modes as a synthetic dimension

$$\Omega_w \approx \Omega_{w_0} + \underbrace{\Omega_{\mathrm{FSR}}}_{\text{Free spectral range}}(|w| - w_0) + \cdots$$





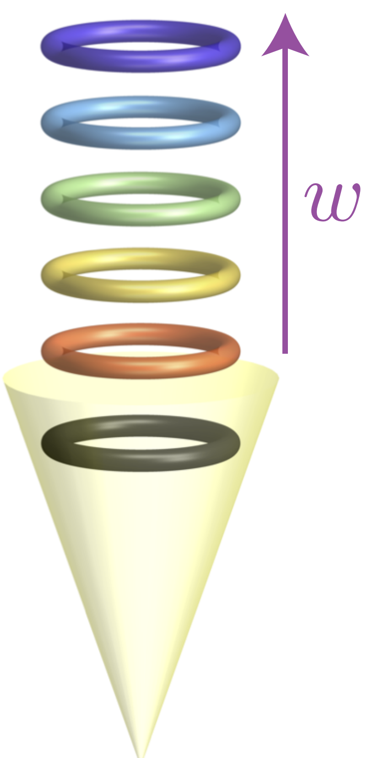
Hafezi et al, Nat. Photon. **7**, 1001, (2013)

Coupling different modes

Different modes can be coupled via some external modulation:

Nonlinearity with an external laser TO, *et al.*, PRA **93**, 043827 (2016)

Electro-optic phase modulators Yuan, *et al.*, Opt. Lett. **41**, 741 (2016)



The effective Hamiltonian of a cavity is

$$\boxed{H = \sum_w \Omega_w a_w^\dagger a_w - \mathcal{J} e^{-i(\Omega_{\mathrm{FSR}} t - \theta)} a_{w+1}^\dagger a_w + h.c.}$$

Move to a rotating frame $b_w \equiv a_w e^{i(\Omega_{w_0} + \Omega_{\mathrm{FSR}}(w - w_0))t}$

The effective Hamiltonian is

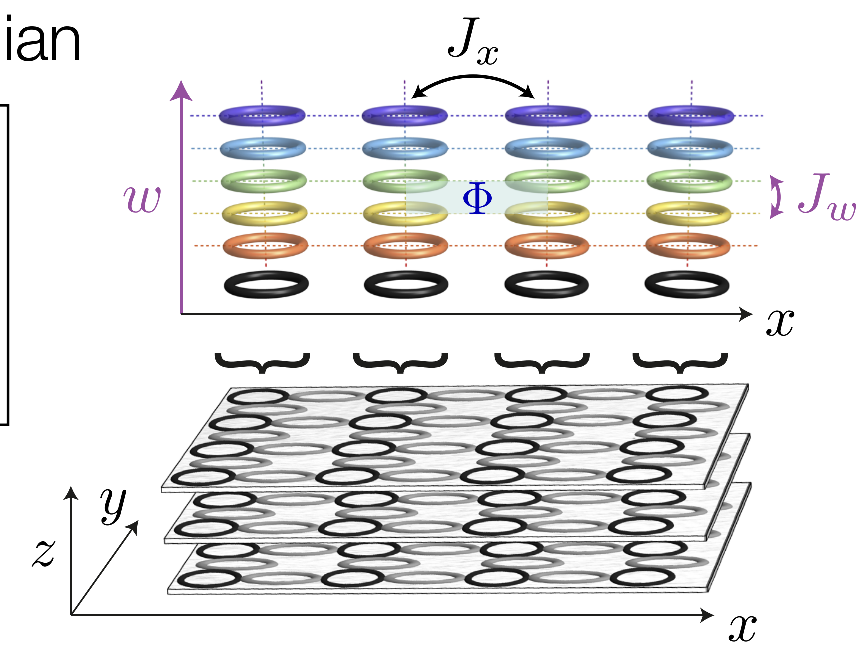
$$\boxed{H = -\sum_w \mathcal{J} e^{i\theta} b_{w+1}^\dagger b_w + h.c.}$$

- 1D tight-binding Hamiltonian with hopping phases -

Coupling different modes

Spatially aligning resonators, one can build up to 4D Hamiltonian

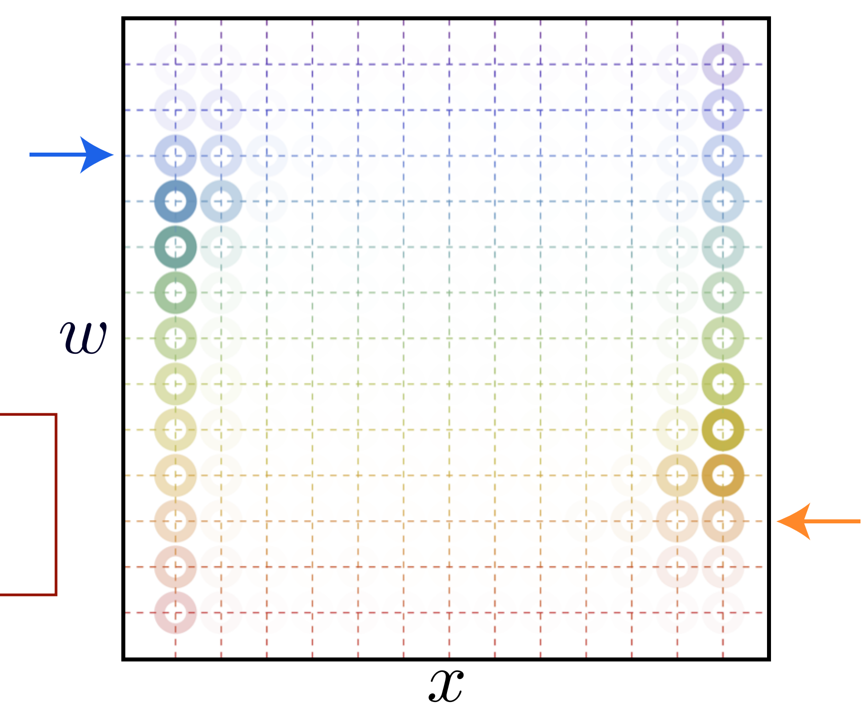
$$\boxed{\begin{aligned} H = \sum_{\mathbf{r},w} & -\mathcal{J}_x b^\dagger_{\mathbf{r}+\hat{e}_x,w} b_{\mathbf{r},w} - \mathcal{J}_y b^\dagger_{\mathbf{r}+\hat{e}_y,w} b_{\mathbf{r},w} \\ & - \mathcal{J}_z b^\dagger_{\mathbf{r}+\hat{e}_z,w} b_{\mathbf{r},w} - \mathcal{J}_w e^{i\theta(\mathbf{r})} b^\dagger_{\mathbf{r},w+1} b_{\mathbf{r},w} + h.c. \end{aligned}}$$



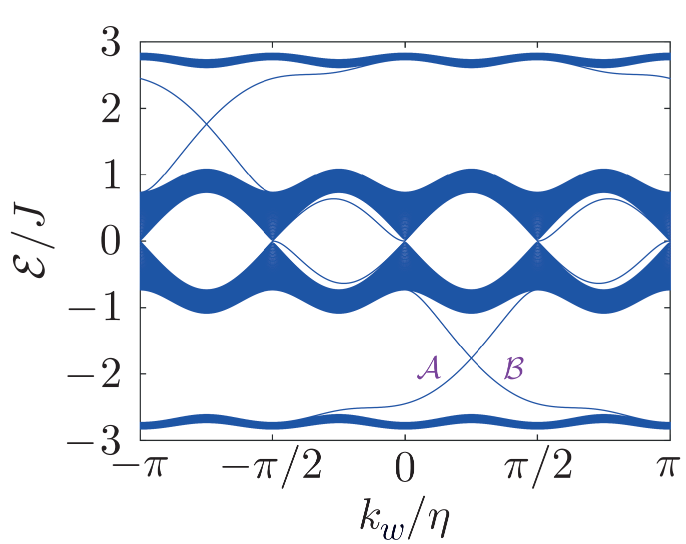
Hopping phases in x, y, z directions can also be added by other methods

e.g. Hafezi et al., Nature Photon. **7**, 1001, (2013)

1D chain of resonators + 1 synthetic dimension = 2D Harper-Hofstadter model



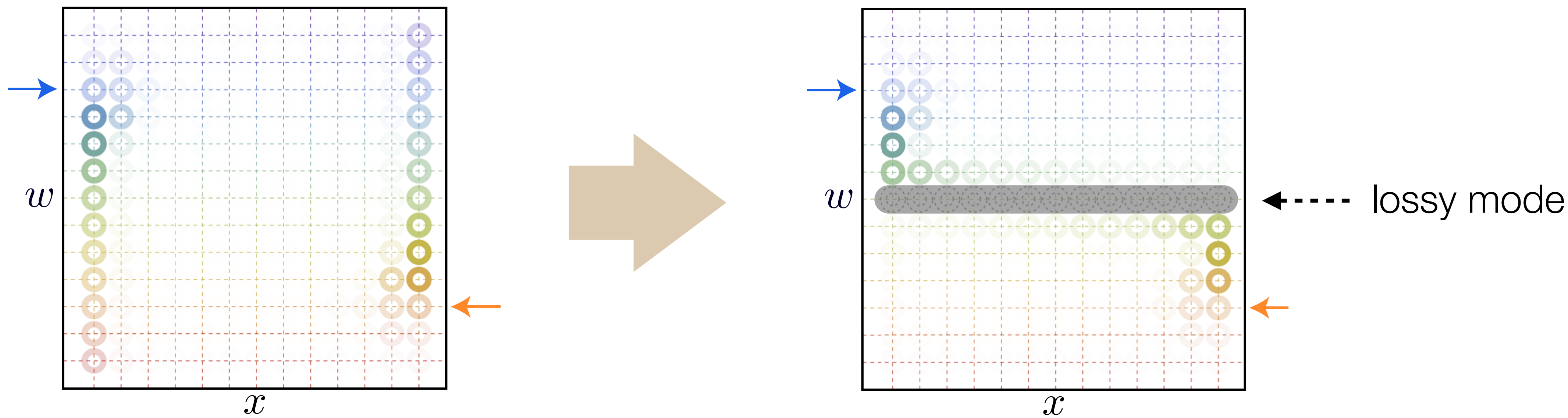
photon intensity with drive & dissipation



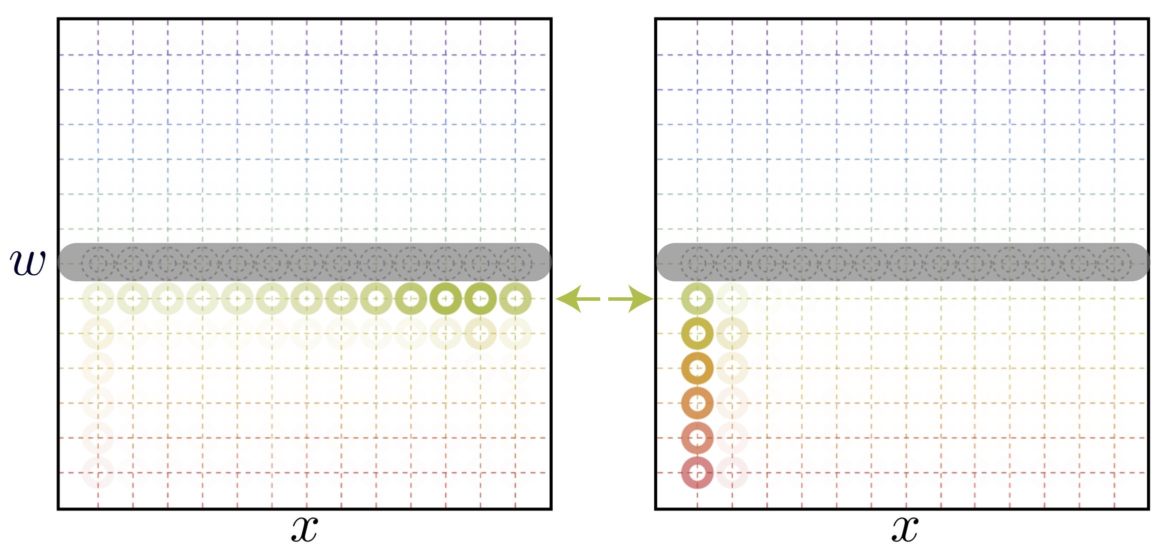
Band structure of ϕ = 1/4

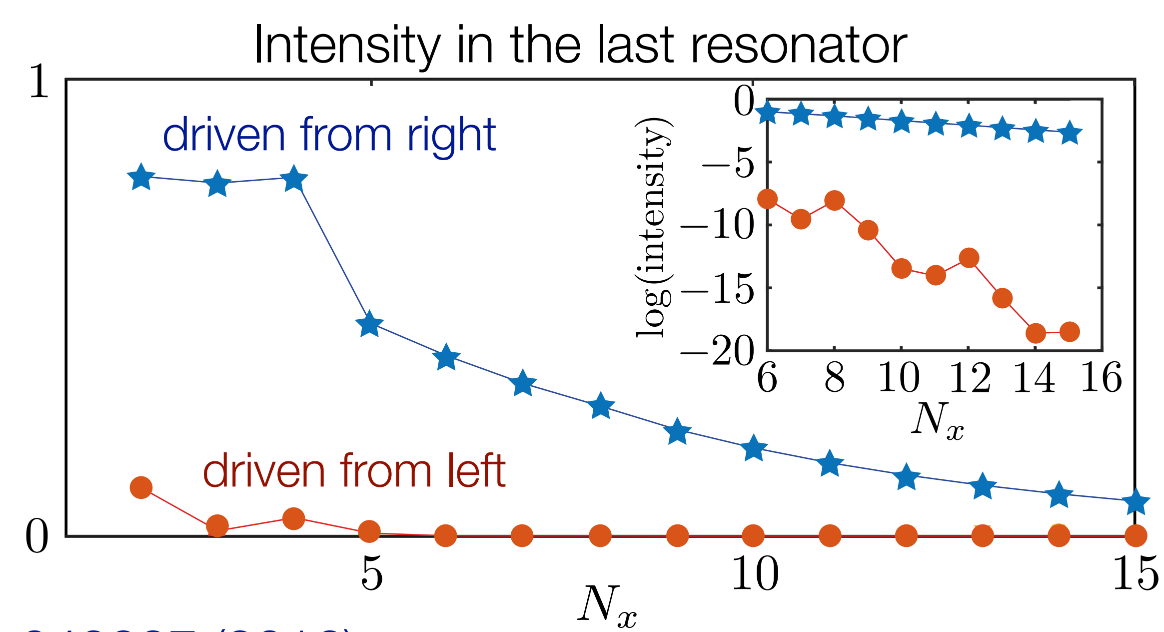
1+1D lattice with an edge

One can introduce an artificial “edge” in w-direction by making one mode very lossy



The system can be used an optical isolator:

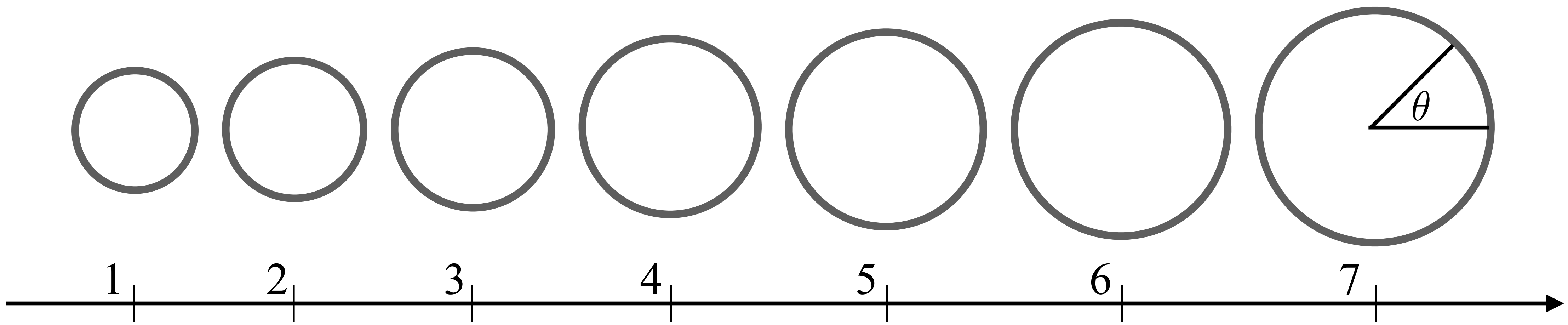




TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)

Angular coordinate as synthetic dimensions I

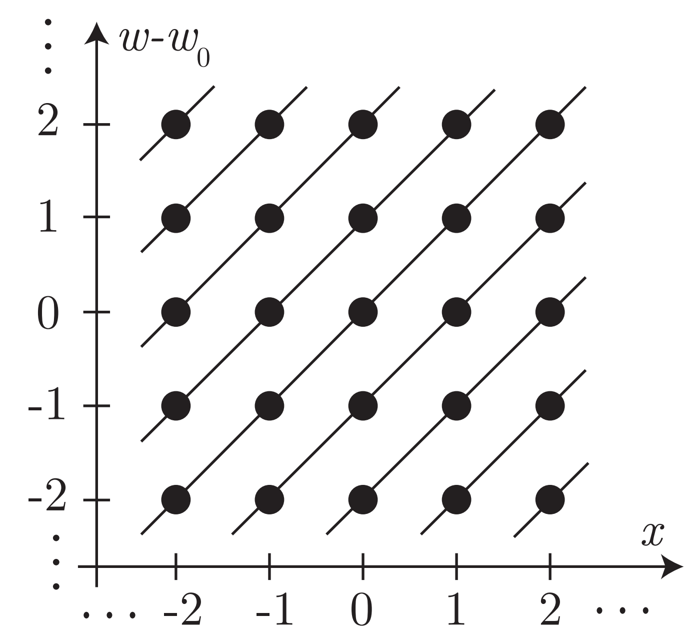
Align ring resonators with different sizes and shapes



$$\Omega_{1,w} \approx \Omega_0 + \Omega_{\mathrm{FSR}}(w - w_0) + D(w - w_0)^2/2 + \cdots$$

$$\Omega_{2,w} = \Omega_{1,w} - \Omega_{\mathrm{FSR}}$$

$$\Omega_{x+1,w} = \Omega_{x,w} - \Omega_{\mathrm{FSR}}$$



Then, neighboring resonators follow $\Omega_{x,w} \approx \Omega_{x+1,w+1}$

A photon with mode w at site x hops to mode w+1 at site x+1

TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

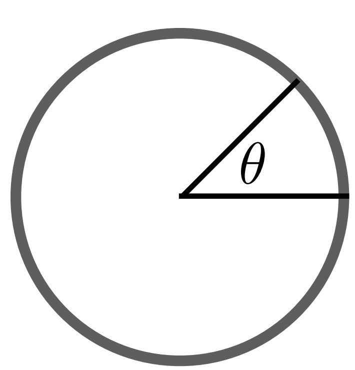
Angular coordinate as synthetic dimensions II

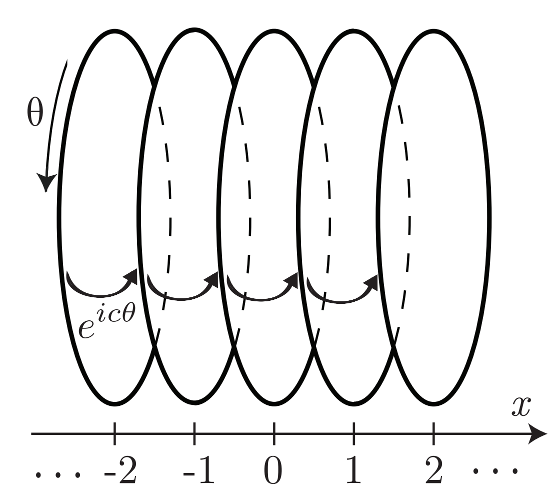
Effective tight-binding Hamiltonian is

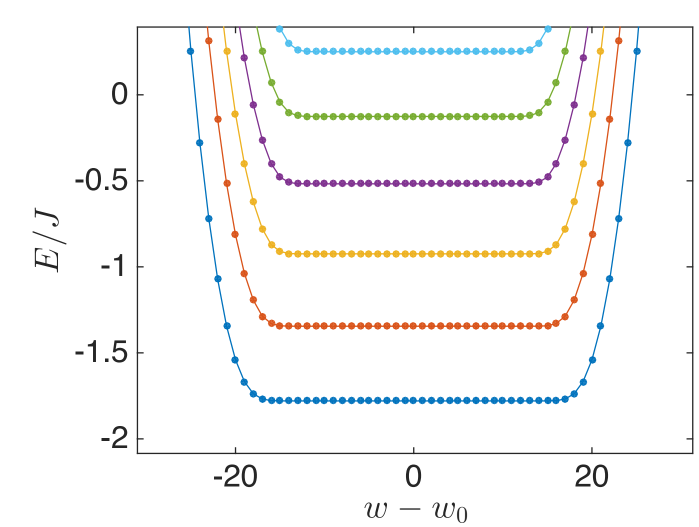
$$\mathcal{H} = \sum_{x,w} \frac{D}{2}(w-w_0)^2 b^\dagger_{x,w} b_{x,w} \boxed{-J\sum_{x,w}\left(b^\dagger_{x+1,w+1}b_{x,w} + h.c.\right)} + \underbrace{\frac{U}{4\pi}\sum_{x}\sum_{w_1+w_2=w_3+w_4} b^\dagger_{x,w_1}b^\dagger_{x,w_2}b_{x,w_3}b_{x,w_4}}_{\text{angular momentum conserving interaction term}}$$

In the space of θ (angular coordinate),

$$\mathcal{H} = \sum_{x}\int_0^{2\pi} d\theta \left[\underbrace{\frac{D}{2}\left\{i\nabla_\theta b^\dagger_x(\theta)\right\}\left\{-i\nabla_\theta b_x(\theta)\right\}}_{\text{kinetic energy in synthetic dimension}} \boxed{\underbrace{-J\left\{e^{i\theta}b^\dagger_{x+1}(\theta)b_x(\theta) + h.c.\right\}}_{\text{hopping with phase}}} + \underbrace{\frac{U}{2}b^\dagger_x(\theta)b^\dagger_x(\theta)b_x(\theta)b_x(\theta)}_{\text{zero-range interaction term}}\right]$$



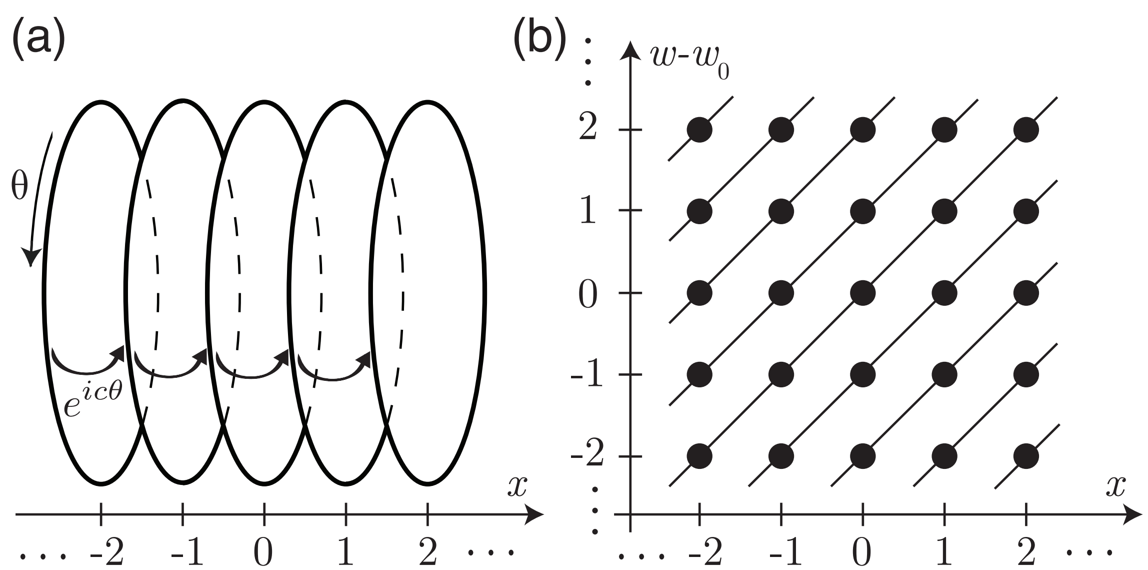


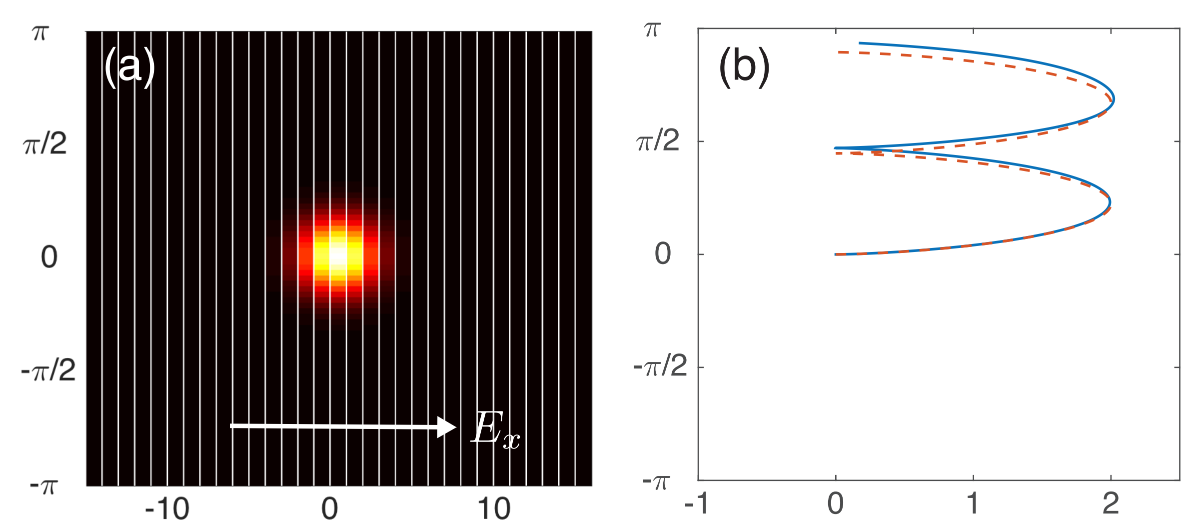


TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

Angular coordinate as synthetic dimensions II

- Synthetic dimension is continuous and periodic
- The interaction is zero-range in both dimensions
- Coupled wire setup

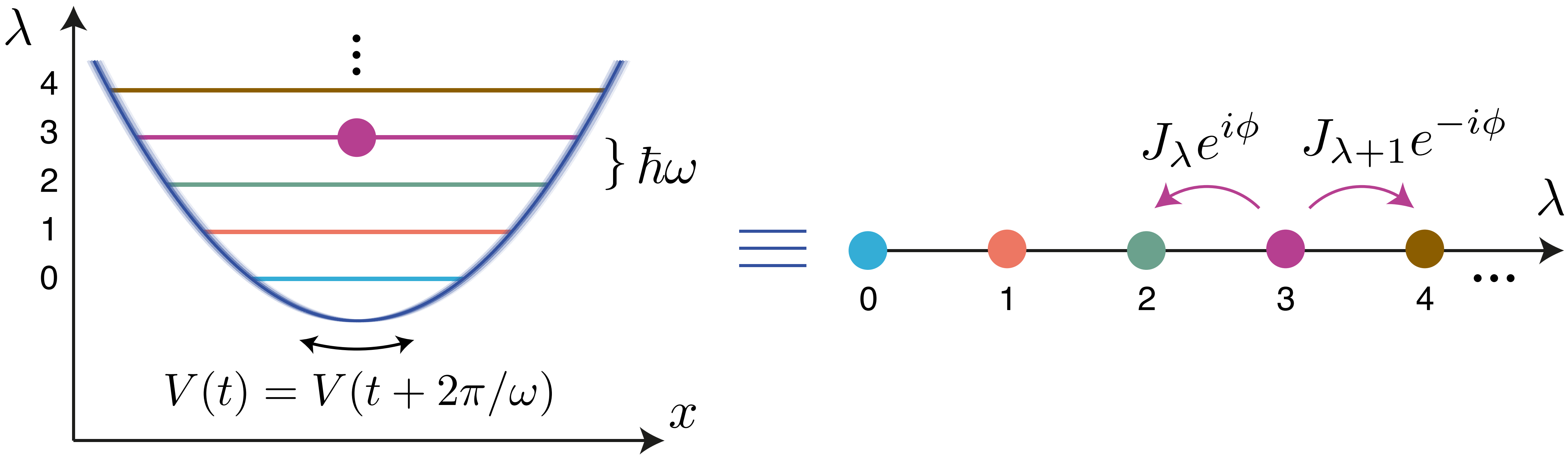




Q. Can one make both the real and synthetic dimensions continuous?

TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

Harmonic oscillator states as synthetic dimensions



- Consider harmonic oscillator states as lattice sites in the synthetic direction
- Couple different states by shaking the potential resonantly with the level spacing

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \sum_{\lambda=0}^{\infty} \omega\lambda |\lambda\rangle\langle\lambda|$$

$$V(t) = \kappa x \cos(\omega t + \phi) = \frac{\kappa}{\sqrt{2m\omega}} \cos(\omega t + \phi) \sum_{\lambda=1}^{\infty} \sqrt{\lambda} \left(|\lambda\rangle\langle\lambda - 1| + h.c.\right)$$

Price, TO, & Goldman, PRA **95**, 023607 (2017)

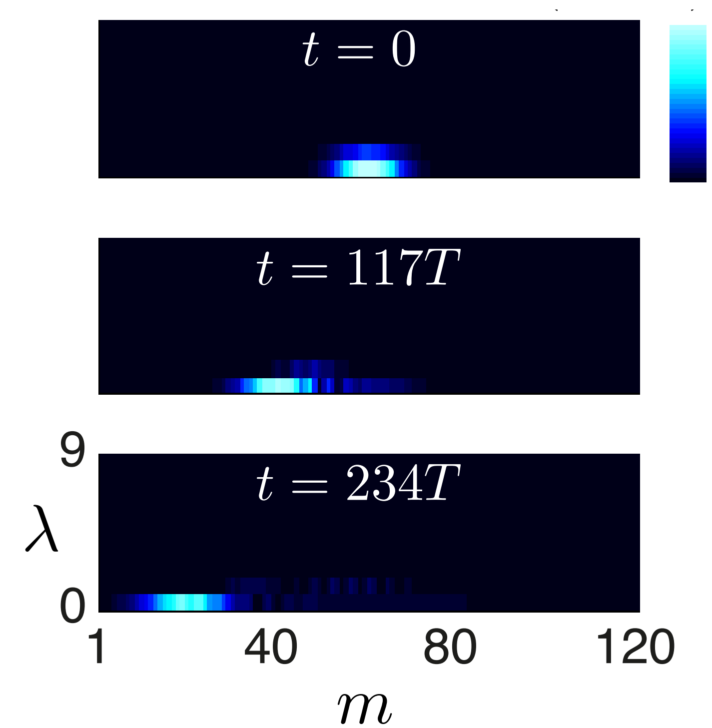
Harmonic oscillator states as synthetic dimensions

Change basis as $|\lambda\rangle \to e^{i\omega\lambda t}|\lambda\rangle$

Then, the Hamiltonian becomes (after the rotating wave approximation):

$$H = H_0 + V(t) \to \sum_\lambda \kappa\sqrt{\frac{\lambda}{8m\omega}}\left(|\lambda-1\rangle\langle\lambda|e^{i\phi} + h.c.\right)$$

1D tight-binding Hamiltonian with hopping phase



- Can in principle go up to 6D (3D optical lattice + 3 directions of harmonic potential)
- Interaction decays algebraically along the synthetic direction

Price, TO, & Goldman, PRA **95**, 023607 (2017)

Four dimensional quantum Hall effect

Price, *et al.*, PRL **115**, 195303 (2015)

The current responds nonlinearly to external perturbing fields

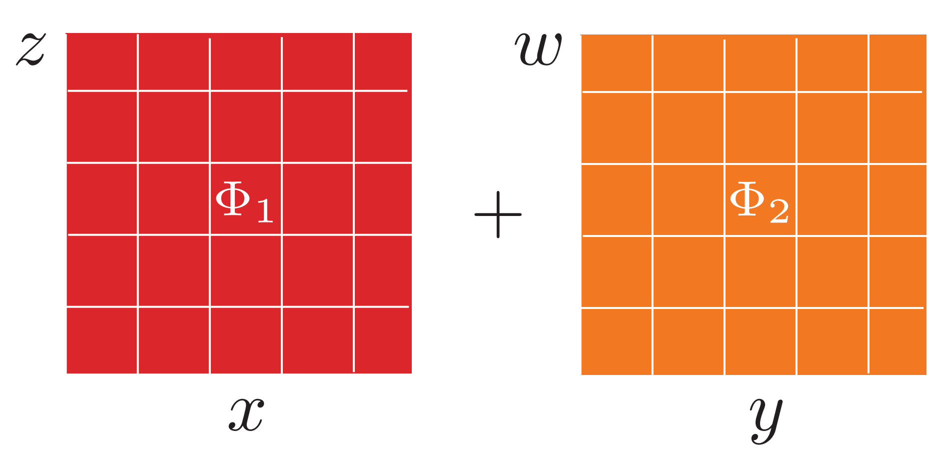
$$j^{\mu} = \boxed{E_{\nu} \frac{1}{(2\pi)^4} \int_{\mathrm{BZ}} \Omega_n^{\mu\nu} d^4k} + \boxed{\frac{\nu_2^n}{(2\pi)^2} \epsilon^{\mu\nu\rho\sigma} E_{\nu} B_{\rho\sigma}} \qquad B_{\rho\sigma} \equiv \partial_{\rho} A_{\sigma} - \partial_{\sigma} A_{\rho}$$

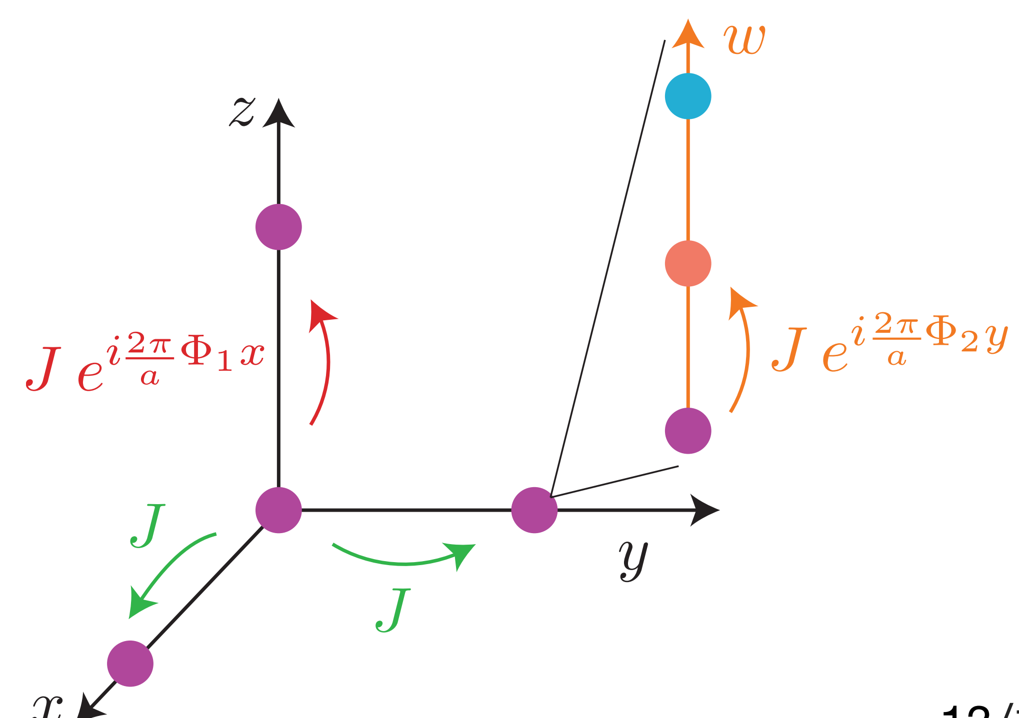
2D Quantum Hall Contribution   4D Quantum Hall Effect!

where, the 2nd Chern number is defined by

$$\nu_2^n \equiv \frac{1}{(2\pi)^2} \int_{\mathrm{BZ}} \{\Omega_n^{xy}\Omega_n^{zw} + \Omega_n^{wx}\Omega_n^{zy} + \Omega_n^{zx}\Omega_n^{yw}\} d^4k \in \mathbb{Z}$$

The minimum model requires magnetic fields through two disconnected planes





4D quantum Hall effect in ultracold atoms

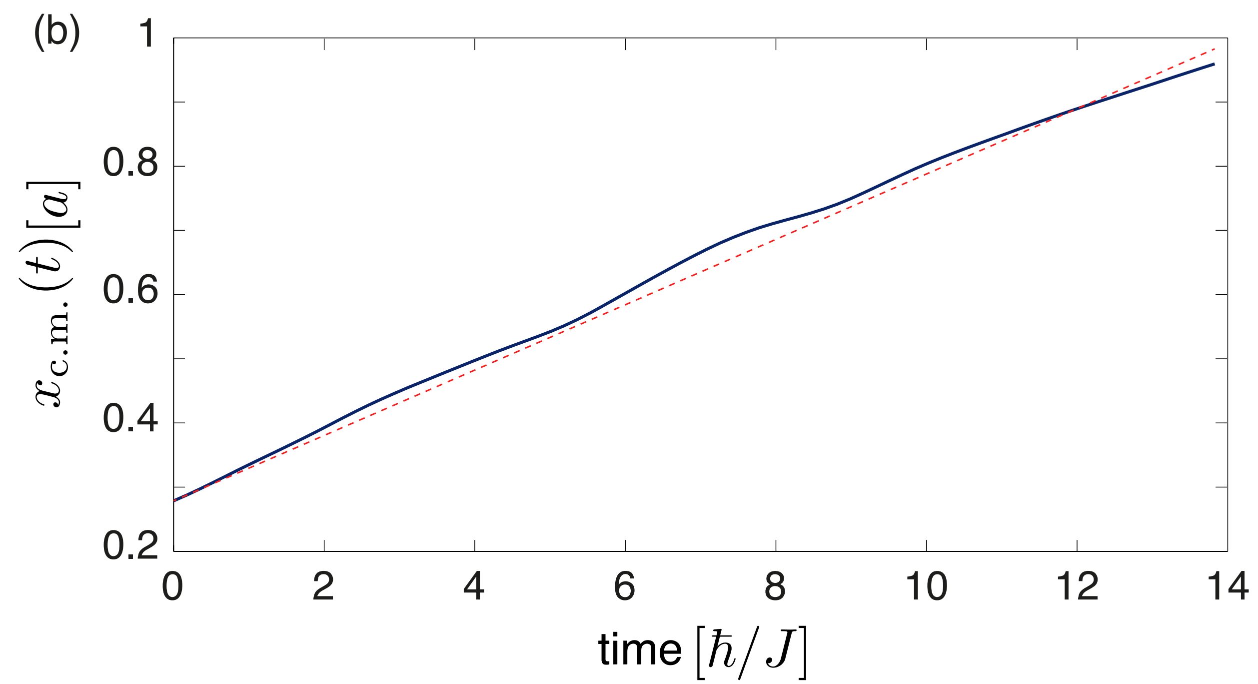
A minimum model for 4D quantum Hall effect requires magnetic fields in two disconnected planes: we add magnetic fields in x-z and y-w planes

$$\boxed{\mathcal{H} = -J \sum_{x,y,z,w} \left(c^\dagger_{\mathbf{r}+\hat{e}_x} c_{\mathbf{r}} + c^\dagger_{\mathbf{r}+\hat{e}_y} c_{\mathbf{r}} + e^{iB_{xz}x} c^\dagger_{\mathbf{r}+\hat{e}_z} c_{\mathbf{r}} + e^{iB_{yw}y} c^\dagger_{\mathbf{r}+\hat{e}_w} c_{\mathbf{r}} + \text{h.c.} \right)}$$

Applying perturbing fields δE_y and δB_{zw}, current in x direction is

$$\boxed{j^x = \frac{\nu_2}{(2\pi)^2} \delta E_y \delta B_{zw}}$$

Extracted 2nd Chern number = -0.98



Price, Zilberberg, TO, Carusotto, & Goldman, PRL **115**, 195303 (2015)

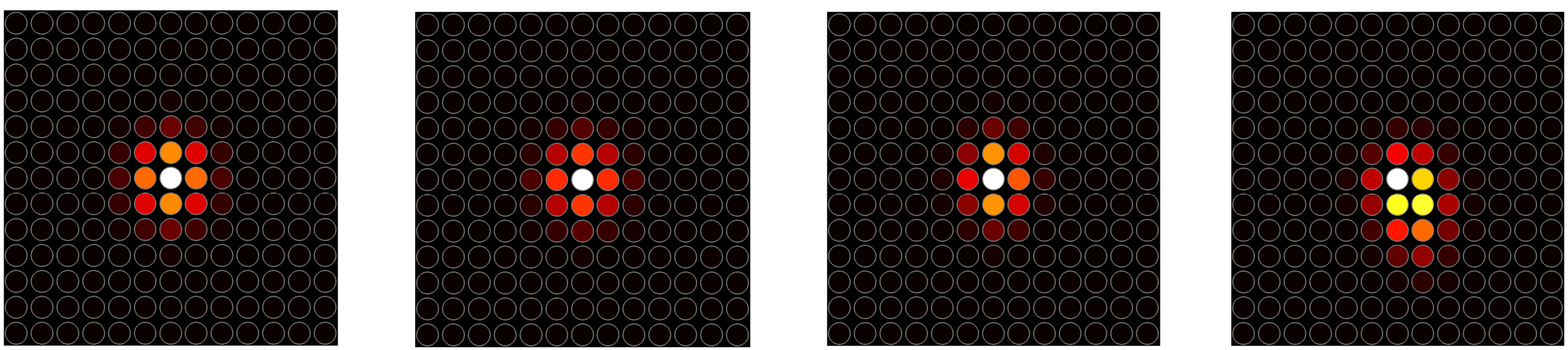
4D quantum Hall effect in photonics

In driven-dissipative systems, the Hall current is proportional to the center-of-mass shift of photonic fields

$$\langle y \rangle \approx \frac{(2\pi)^4}{\gamma \underbrace{A_{\mathrm{BZ}}}_{\text{BZ volume}}} \underbrace{j^y}_{\text{Hall current}} = -\frac{(2\pi)^2}{\underbrace{\gamma}_{\text{Loss}} A_{\mathrm{BZ}}} \underbrace{\nu_2}_{\text{2nd Chern number}} \overbrace{\delta E_x \delta B_{zw}}^{\text{external electromagnetic field}}$$

Numerical simulation pumping the center: projection onto x-y plane

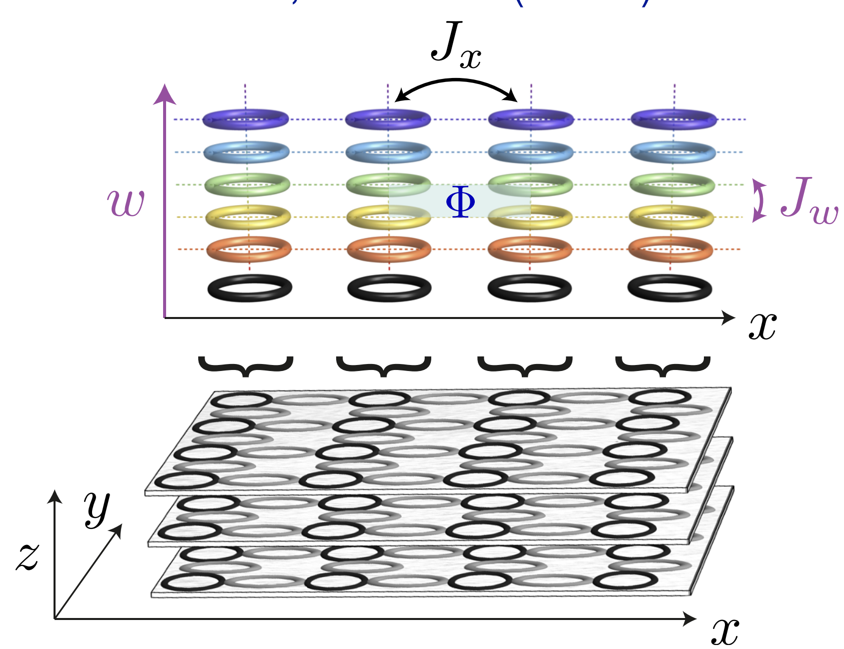
$\delta E_x = \delta B_{zw} = 0$ $\quad$ $\delta E_x = 0,\ \delta B_{zw} \neq 0$ $\quad$ $\delta E_x \neq 0,\ \delta B_{zw} = 0$ $\quad$ $\delta E_x \neq 0,\ \delta B_{zw} \neq 0$



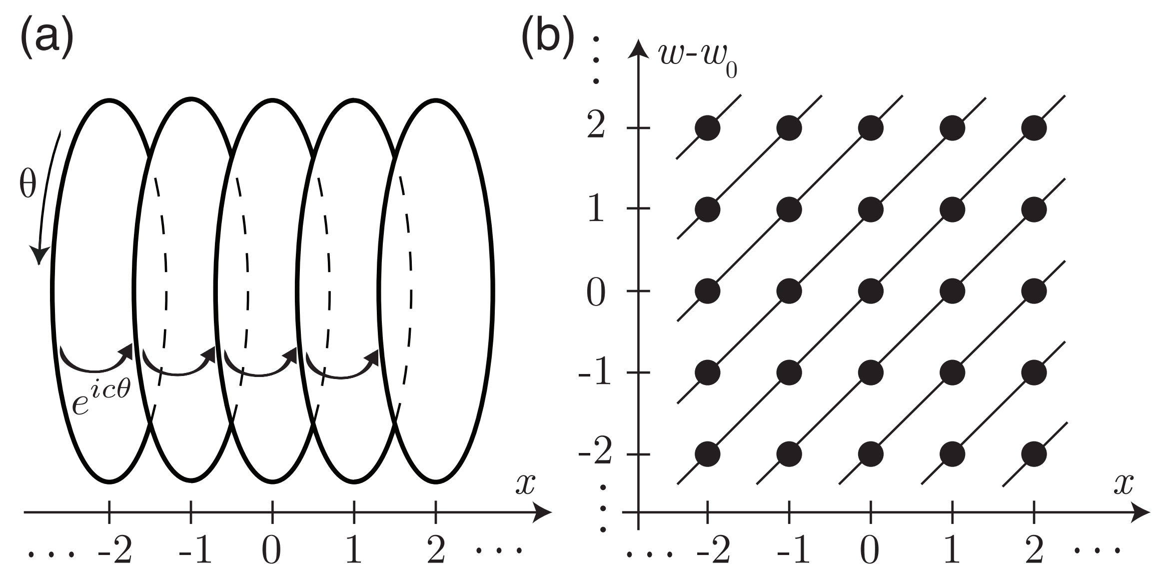
TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)

Summary

- Synthetic frequency dimension
  TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)



- Angular coordinate as a dimension
  TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)



- Harmonic oscillator states as a dimension
  Price, TO, & Goldman, PRA **95**, 023607 (2017)