Static and Floquet topological defects:
Topological modes and higher-dimensional topological invariants

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Outline

- Line defect as one-way fiber of electromagnetic waves (protected by the second Chern number)
- Topological invariants of Floquet systems: General formulation, and applications to Floquet topological defects

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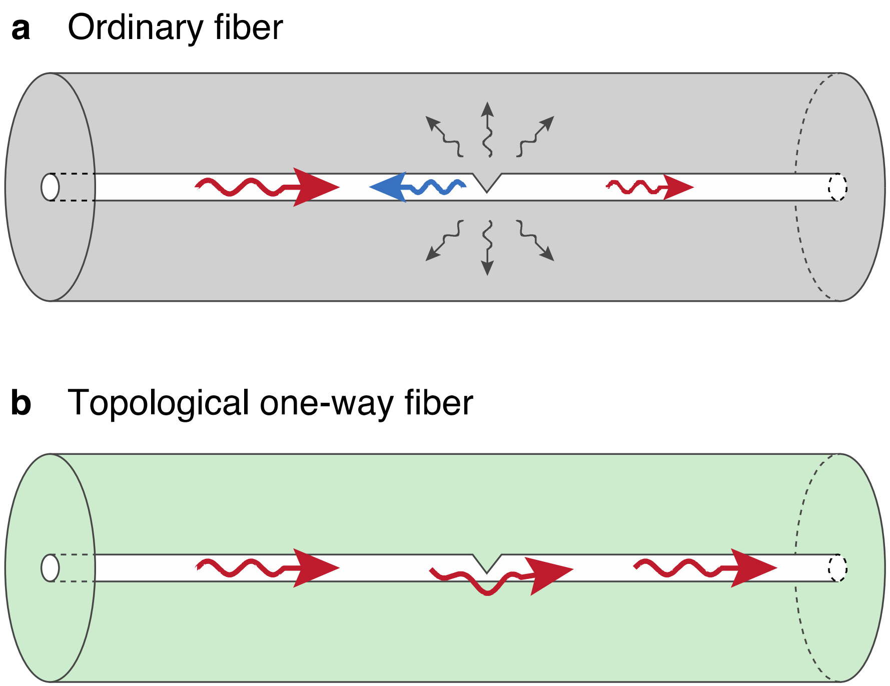
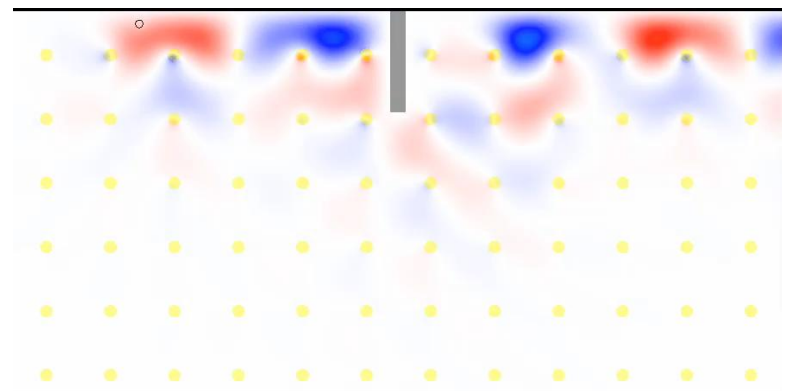
- Topological invariants of Floquet systems: Formulation, and applications to Floquet topological defects

Ling Lu (Institute of Physics, CAS)





Edge of 2d system

Theory: Haldane&Raghu

Experiment: Zheng Wang et al (MIT)

3d systems??
(without interface)

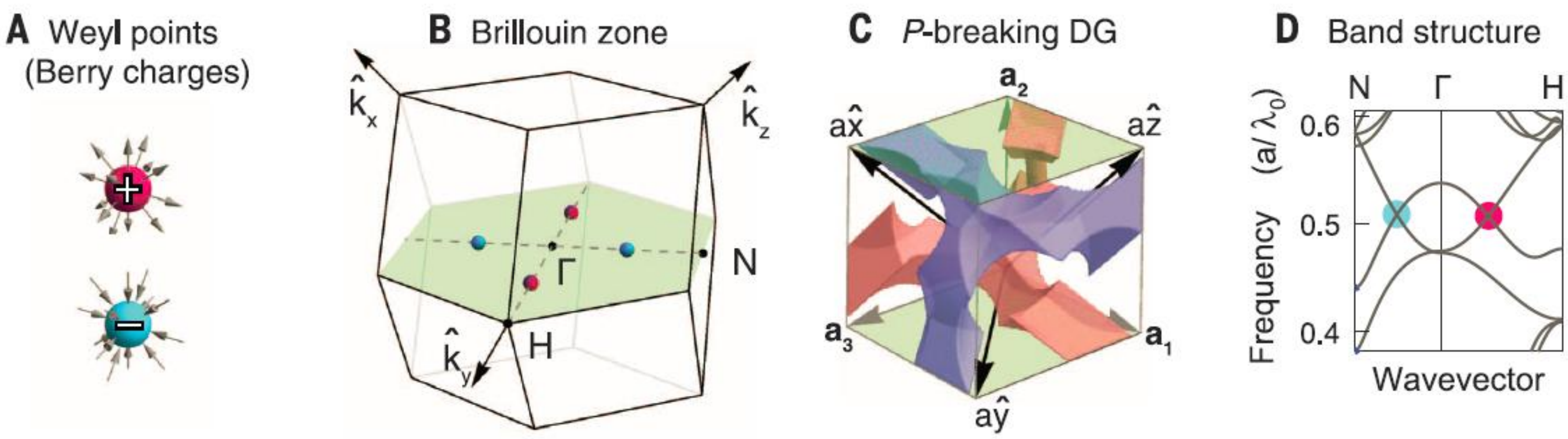
In other words:

Line defect carrying one-way (chiral) modes in a 3d photonic crystal

Try and error? Sounds not good…

Our approach: Obtain one-way fiber from photonic Weyl crystals by spatial modulation.

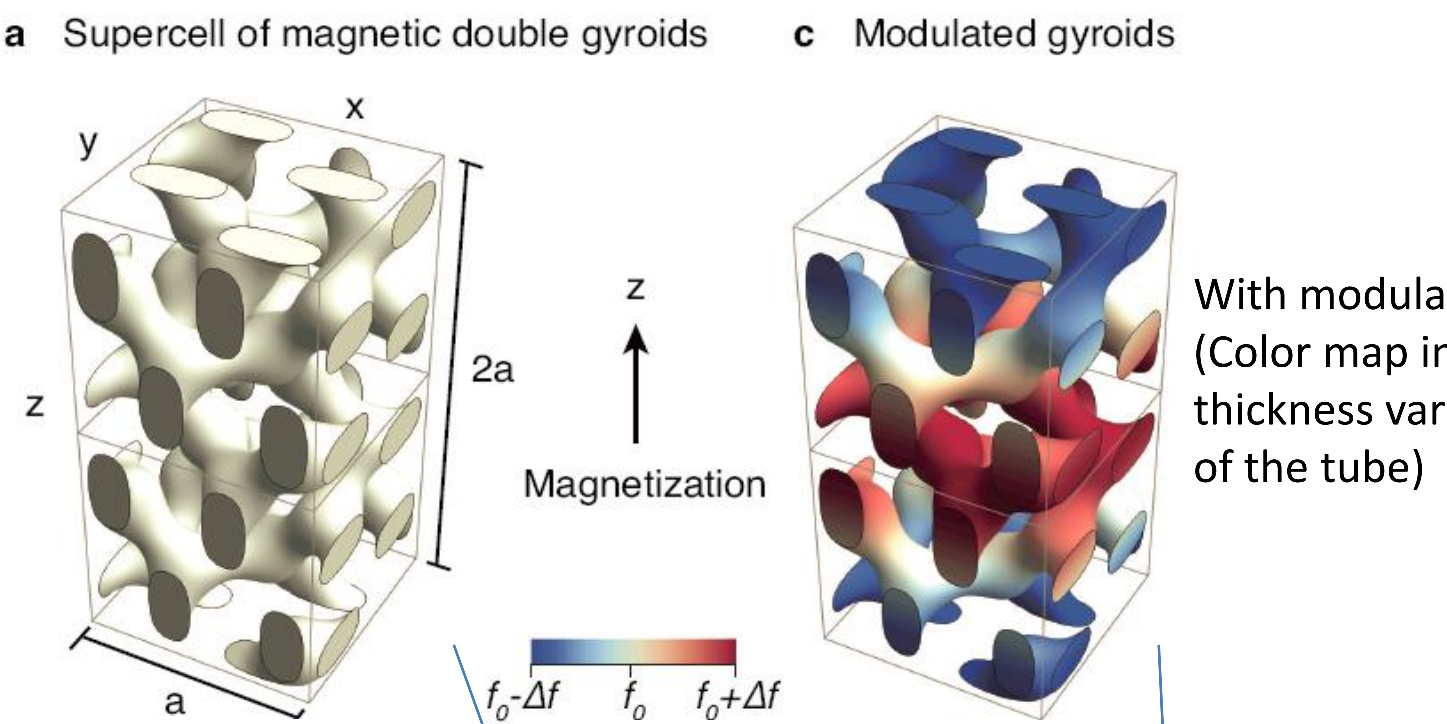
A prerequisite, photonic Weyl crystal, has already been experimentally realized (Ling Lu, *et al*, Science, 2015):



$$H_{\mathrm{Weyl}} = \pm\vec{\sigma}\cdot(\vec{p}-\vec{K}_{\pm})$$

The first step is to gap out the Weyl points

Double gyroid Weyl crystal

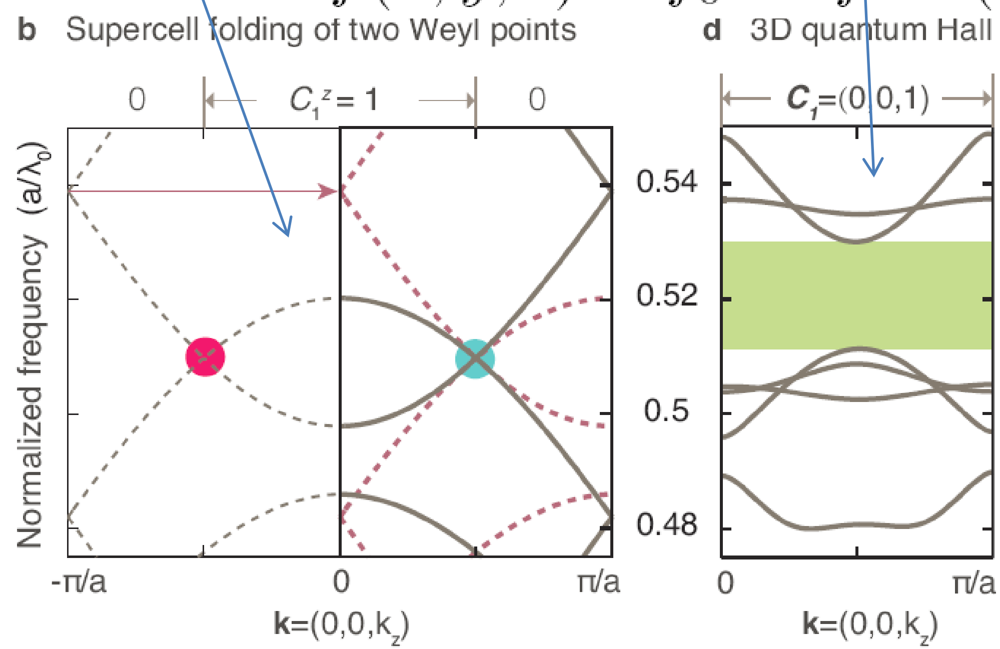


With modulation (Color map indicates thickness variation of the tube)

$$f(x,y,z) > f_0 = 0.4$$

$$f(x,y,z) > f_0 + \Delta f \cos(\pi z/a)$$

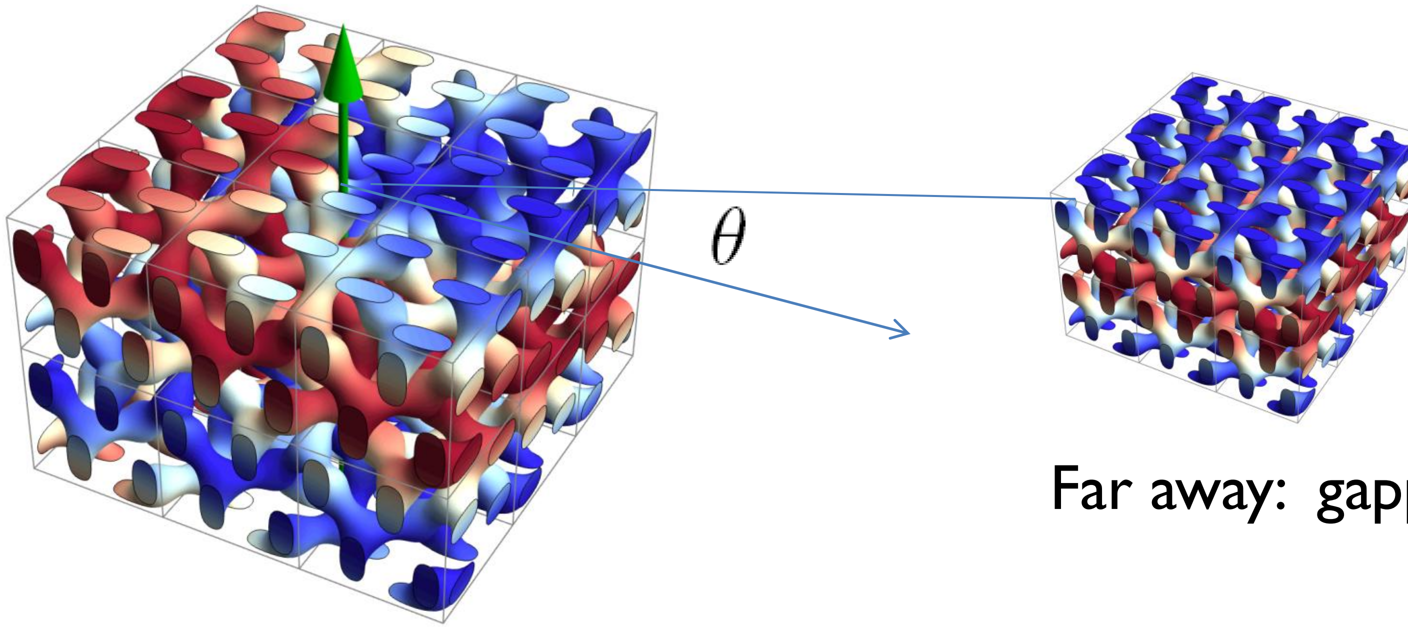
$$f(x,y,z) = \sin(2\pi\frac{x}{a})\sin(2\pi\frac{y}{a})\cos\left(2\pi\frac{z}{a}\right) + \sin(2\pi\frac{y}{a})\sin(2\pi\frac{z}{a})\cos(2\pi\frac{x}{a}) + \sin(2\pi\frac{z}{a})\sin(2\pi\frac{x}{a})\cos(2\pi\frac{y}{a})$$



L. Lu, Z. Wang
[arXiv:1611.01998 (2016)]

No room for the one-way mode…

The system should be made gapless somewhere…….

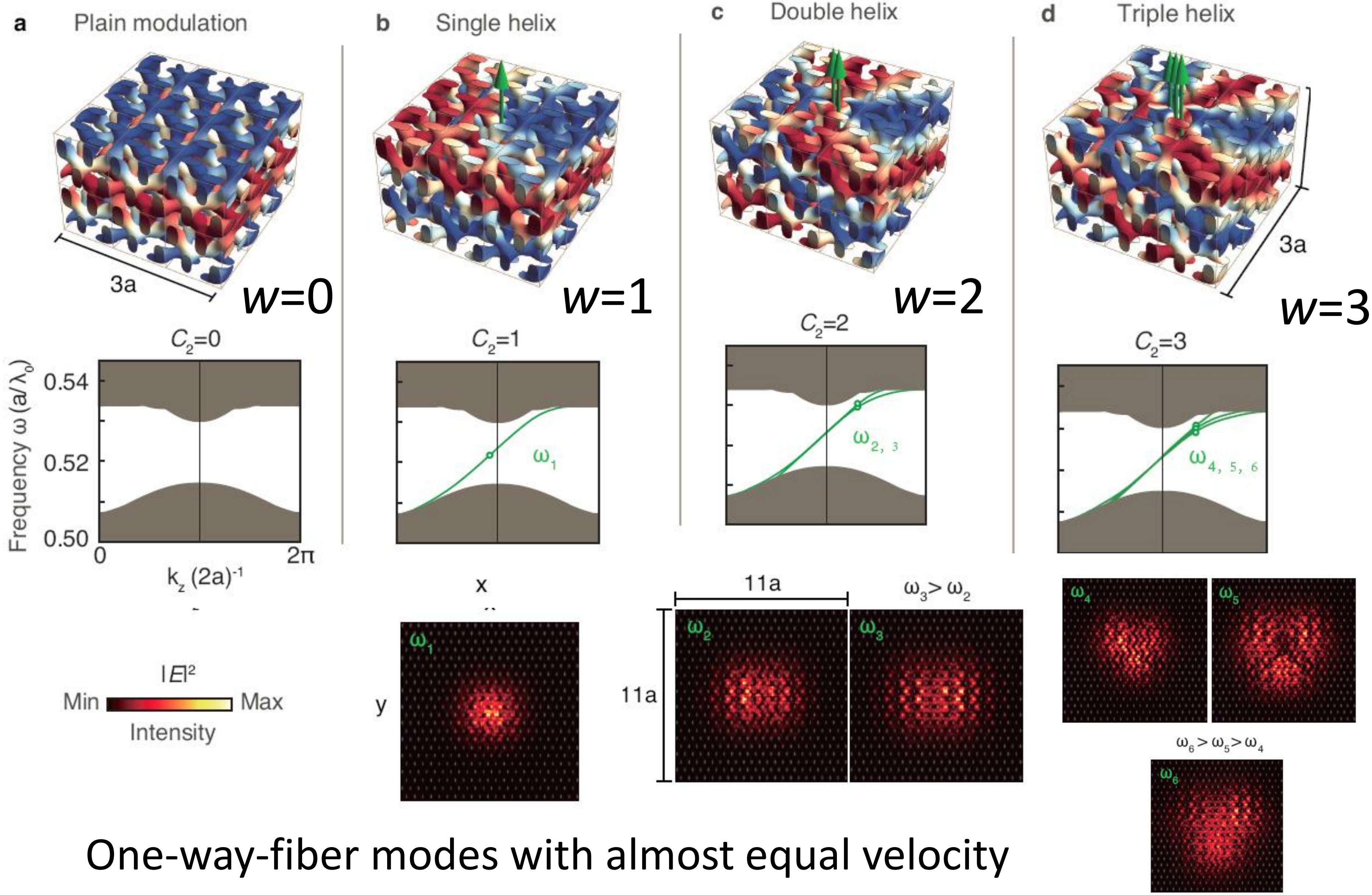


Far away: gapped

A helix-shape modulation

Interesting thing can happen around the core of this line defect

$$f(x,y,z) > f_0 + \Delta f \cos(\pi z/a) \Longrightarrow f(x,y,z) > f_0 + \Delta f \cos(\pi z/a + w\theta)$$



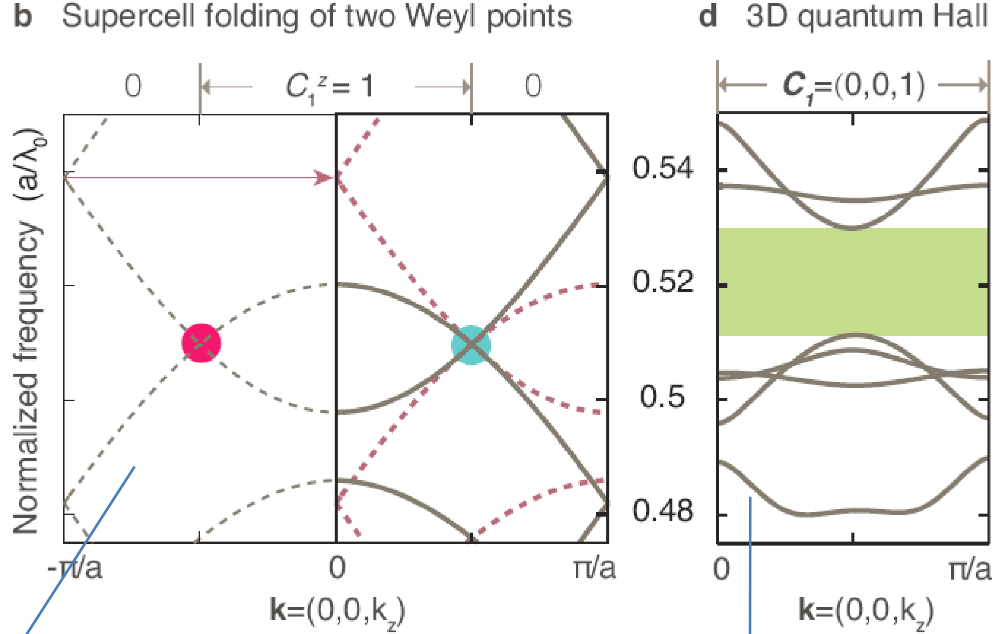
One-way-fiber modes with almost equal velocity

[Lu, Wang arXiv:1611.01998 (2016)](https://arxiv.org/abs/1611.01998)

Why does it work?

$$f(x,y,z) > f_0 = 0.4$$

$$f(x,y,z) > f_0 + \Delta f\cos(\pi z/a)$$



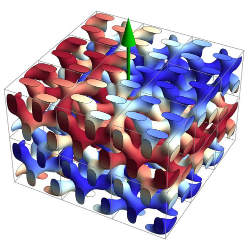
Dirac Hamiltonian

$$H_{\mathrm{D}} = -iv(\sigma_x\partial_x + \sigma_y\partial_y + \sigma_z\partial_z)\tau_z$$

with Dirac mass

$$m_1\tau_x + m_2\tau_y = m\tau_+ + m^*\tau_-,$$

$$H_{\mathrm{eff}} = -iv(\sigma_x\partial_x + \sigma_y\partial_y + \sigma_z\partial_z)\tau_z + \boxed{m\tau_+ + m^*\tau_-}$$

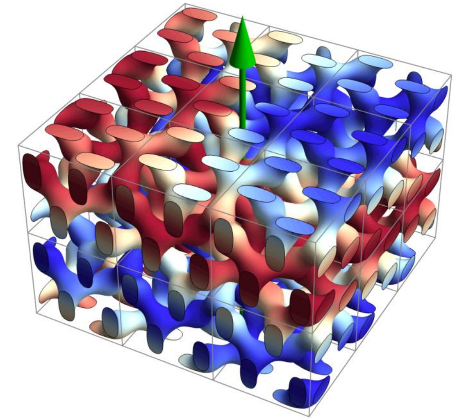


``Phason'' of the modulation = Phase of Dirac mass

L. Lu, Z. Wang, arXiv:1611.01998; Z. Wang, S.-C. Zhang, PRB, 87, 161107(2013)

$$f(x,y,z) > f_0 + \Delta f\cos(\pi z/a + w\theta) \Longrightarrow m(\theta) = m_0\exp(iw\theta)$$

= vortex line of Dirac mass



$$H_{\text{eff}} = -iv(\sigma_x\partial_x + \sigma_y\partial_y + \sigma_z\partial_z)\tau_z + m\tau_+ + m^*\tau_-.$$

$$\Downarrow$$

$$H_{\text{eff}} = \begin{bmatrix} vk_z, & -ive^{-i\theta}(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}), & m_0e^{iw\theta}, & 0 \\ -ive^{i\theta}(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}), & -vk_z & 0, & m_0e^{iw\theta} \\ m_0e^{-iw\theta}, & 0, & -vk_z, & ive^{-i\theta}(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}), \\ 0, & m_0e^{-iw\theta}, & ive^{i\theta}(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}) & vk_z \end{bmatrix}$$

If we remove the k_z term, it resembles the 2d problems:

Jackiw &Rossi, Nucl.Phys. B, 1981: Zero modes of vortex-fermion system;

Hou, Chamon &Mudry, 2006: Zero mode in graphenelike systems

There are *w* one-way modes

e.g., for *w*=1:

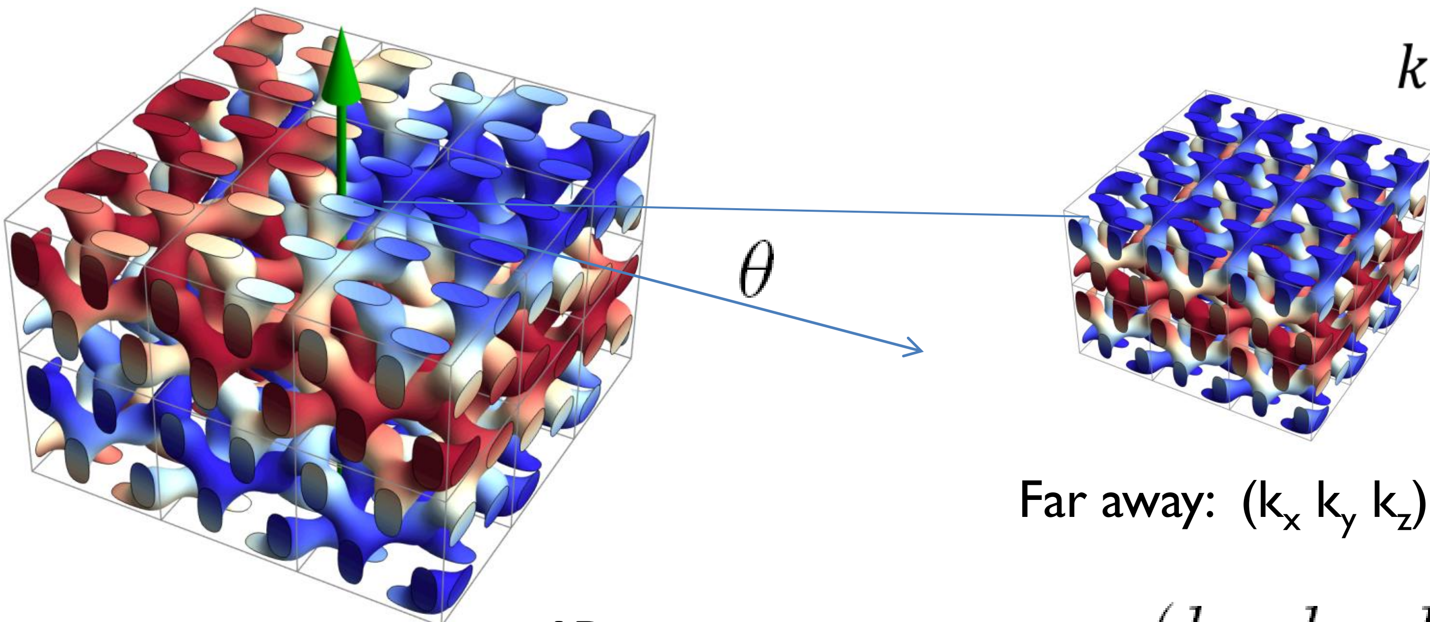
$$|\psi_{w=1}\rangle = \begin{pmatrix} e^{i\pi/4} \\ 0 \\ 0 \\ e^{-i\pi/4} \end{pmatrix} e^{-\frac{m_0}{v}r}.$$

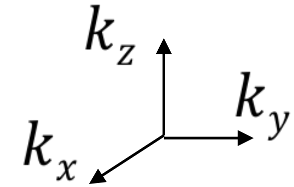
$$E(k_z) = vk_z$$

Identical velocity *v* for *w*>1.

[Lu, Wang arXiv:1611.01998 (2016)](https://arxiv.org/abs/1611.01998)

The topological invariant?





Far away: (k_x k_y k_z) well defined

4D parameter space: (k_x, k_y, k_z, θ)

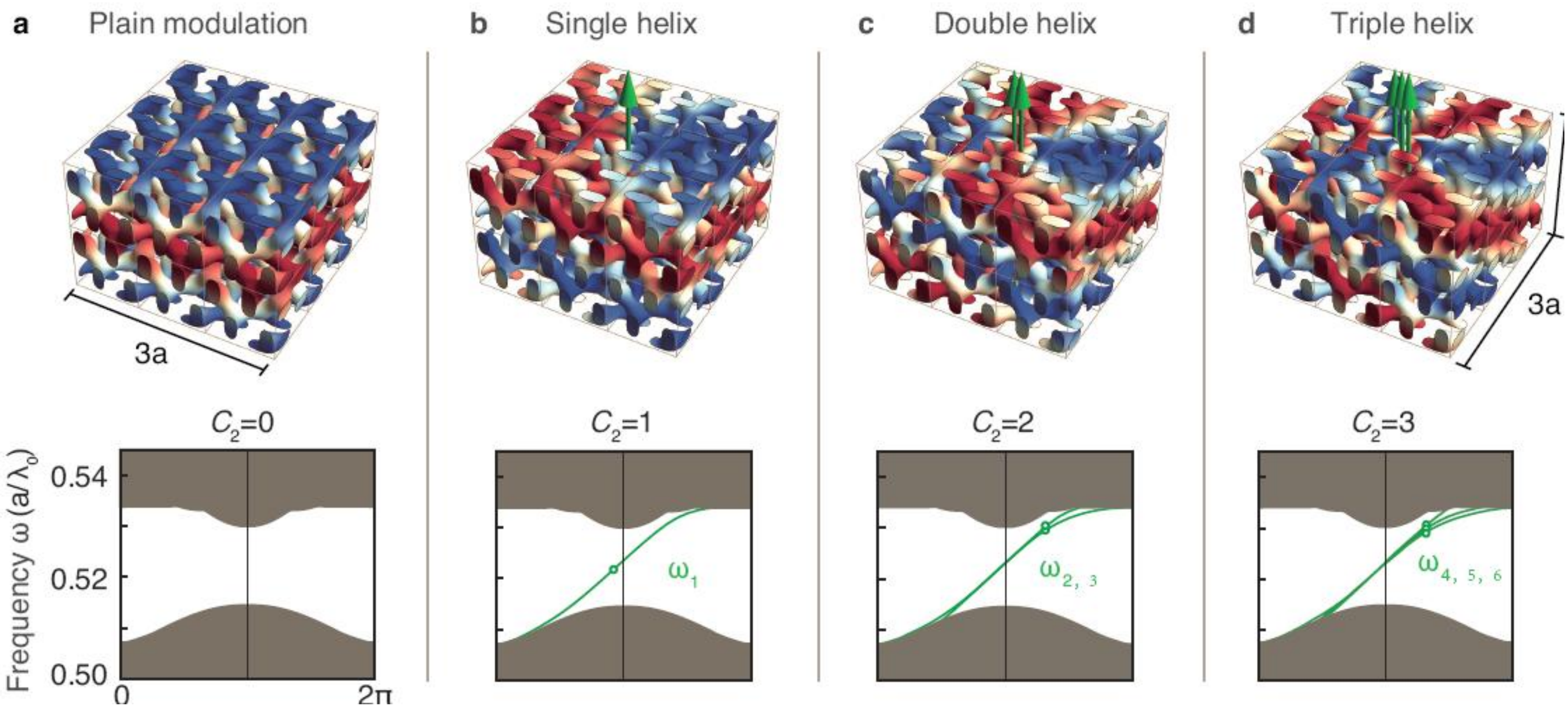
Second Chern number:

(Qi-Hughes-Zhang, PRB, 2008; Teo-Kane, PRB, 2010)

$$C_2 = \frac{1}{4\pi^2} \int d^3k d\theta \mathrm{Tr} \left[\mathcal{F}_{\mathrm{xy}} \mathcal{F}_{\mathrm{z}\theta} + \mathcal{F}_{\mathrm{yz}} \mathcal{F}_{\mathrm{x}\theta} + \mathcal{F}_{\mathrm{zx}} \mathcal{F}_{\mathrm{y}\theta} \right]$$

4D topology:

S.-C. Zhang and J. Hu, Science **294**, 823 (2001).
Y. E. Kraus, Z. Ringel, and O. Zilberberg, Physical review letters **111**, 226401 (2013).
H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, and N. Goldman, Physical review letters **115**, 195303 (2015).
T. Ozawa, H. M. Price, N. Goldman, O. Zilberberg, and I. Carusotto, Physical Review A **93**, 043827 (2016).



C_2 =0 C_2 =1 C_2 =2 C_2 =3

Lu, Wang arXiv:1611.01998 (2016)

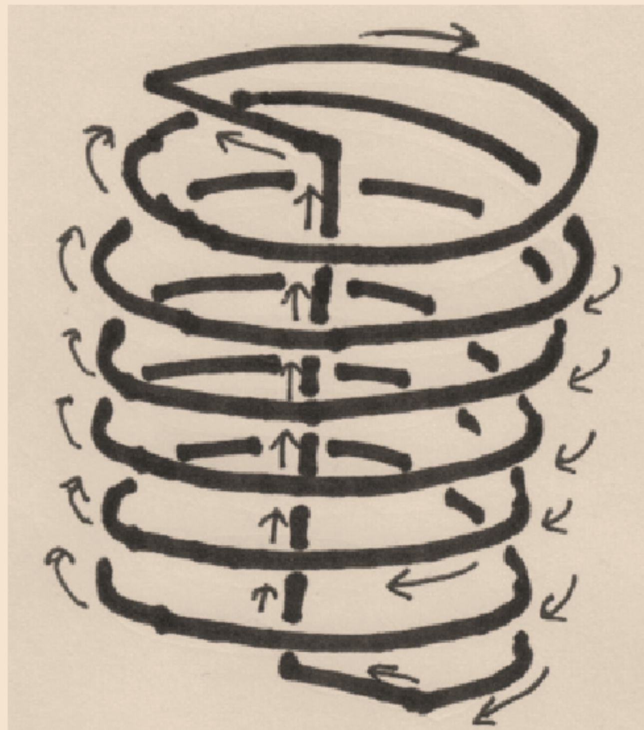
Features of this design of topological one-way fiber:

1) Protected by the second Chern number (C_2)
2) C_2 can be tune from - ∞ to + ∞
3) All the one-way modes have almost the same velocity

Lattice dislocation (Haldane 2010, MRS)

edge states on a screw dislocation in a 3D "photonic Chern crystal"

- one-way photon propagation along the core of the dislocation
- protected provided the radius of the "optical fiber" is large compared to the evanescence length inside the 3D photonic crystal



1) Characterized by the first Chern number

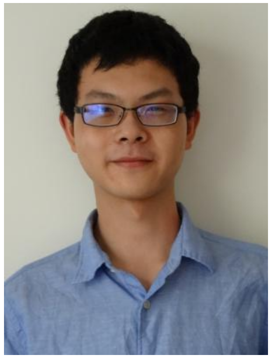
2) Don't work well in our system

Outline

- Line defect as one-way fiber of electromagnetic waves (protected by the second Chern number)
- Topological invariants of Floquet systems: General formulation, and applications to Floquet topological defects

  S. Yao, Z. Yan, Z. Wang, PRB, 96, 195303 (2017);

  R. Bi, Z. Yan, L. Lu, Z. Wang, PRB, 95, 161115 (2017)



Zhongbo Yan, Shunyu Yao, (Tsinghua) Ren Bi

Ling Lu (Institute of Physics, CAS)

*Can one create one-way modes purely by periodic driving , even if the 3d system is defect-free?

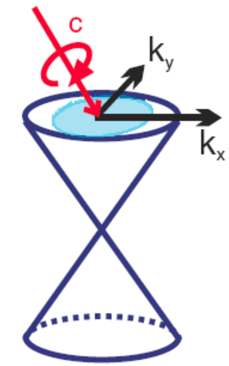
*If ``yes'', what is its topological invariant?

Periodically driven (Floquet) topological systems

$$\hat{H}(t) = \hat{H}(t+T)$$

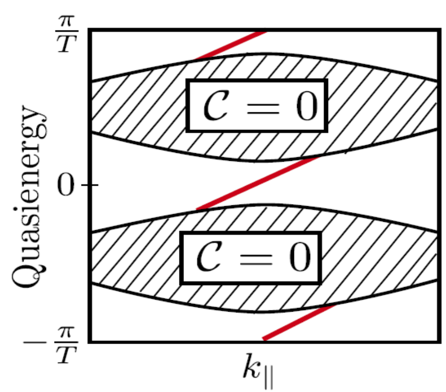
Physical realizations:

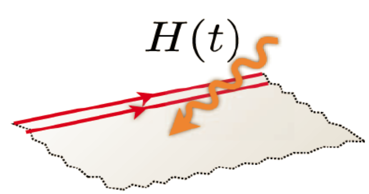
1) Light-matter interaction: $H(\mathbf{k}) \rightarrow H[\mathbf{k} + e\mathbf{A}(t)].$
2) Shaking optical lattices
3) Modulated photonic systems
4) ……



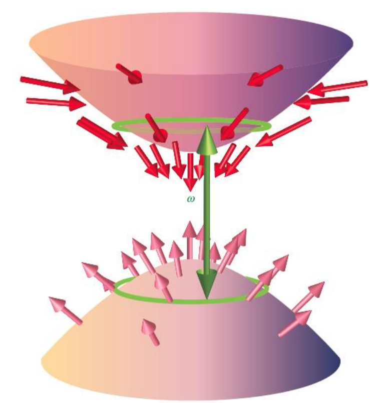
Motivations (from topological-state perspective)

1) To make more tunable topological materials
2) To create fundamentally new topological states without static counterpart





Chiral edge modes for $\mathcal{C} = 0$ bands

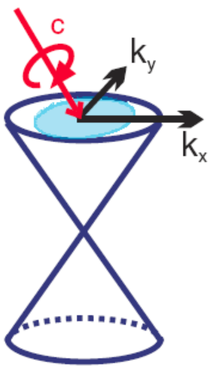


Floquet topological insulators (Lindner et al, Nat. Phys. 2011)

Graphene + circularly polarized light

Graphene + circularly polarized light:

$$H(\mathbf{k},t) = v_F[\sigma_x\tau_z(q_x + eA_x(t)) + \sigma_y(q_y + eA_y(t))]$$
$$= H_0(\mathbf{k}) + \sum_{m\neq 0} H_m e^{im\omega t}$$

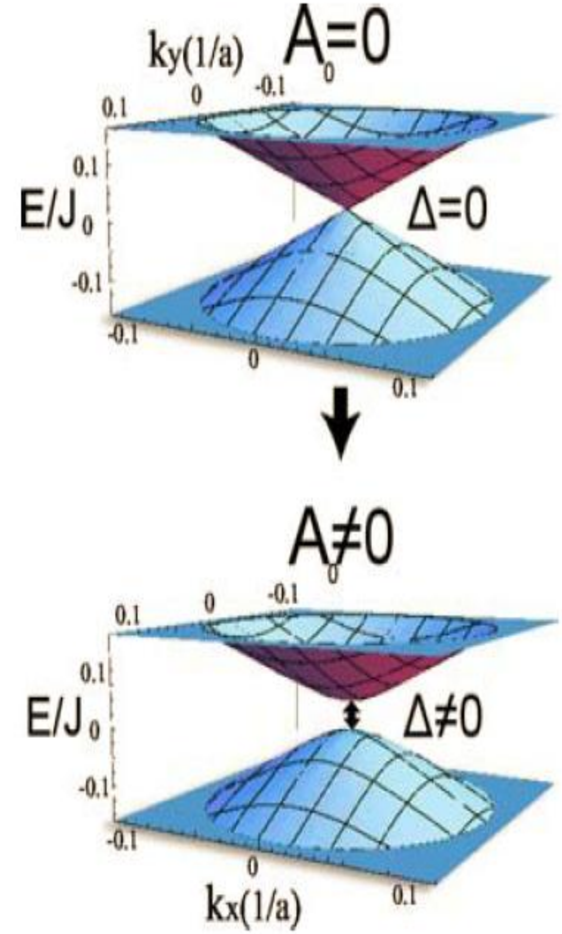


$$\boldsymbol{A} = -A_0\left(\sin(\omega t)\,\boldsymbol{e}_x + \sin(\omega t - \varphi)\,\boldsymbol{e}_y\right)$$

$$\Rightarrow \quad H^{\text{eff}} = H_0(\mathbf{k}) + \sum_{m>0} [H_m, H_{-m}]/\omega$$
$$= H_0(\mathbf{k}) - (eA_0 v_F)^2/\omega \boxed{\sin\varphi\,\sigma_z\tau_z}$$

(see Goldman &Dalibard, PRX, 2014)

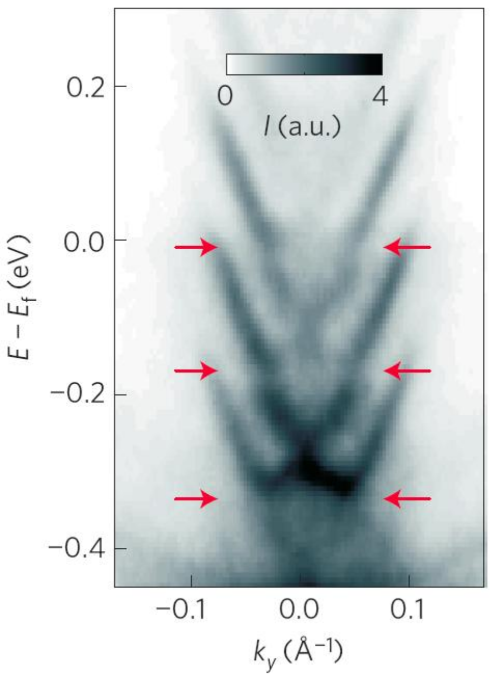
Haldane mass term

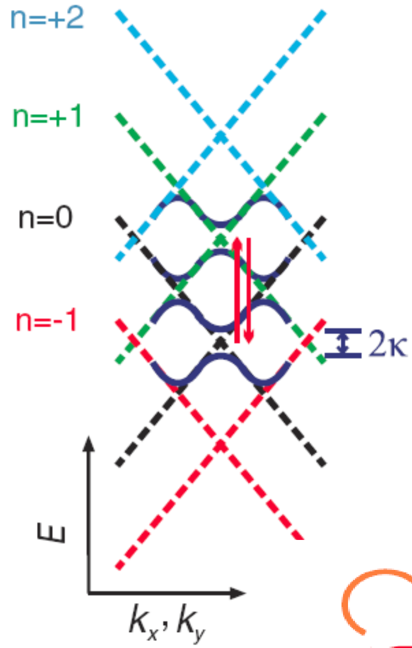


[Oka, Aoki, PRB, 2009]

→ Nonzero band Chern numbers

Floquet bands oberved in pump-probe ARPES



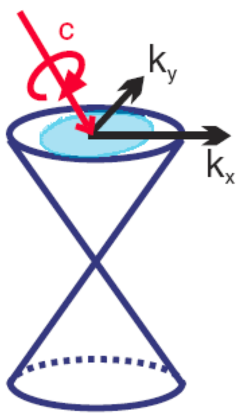


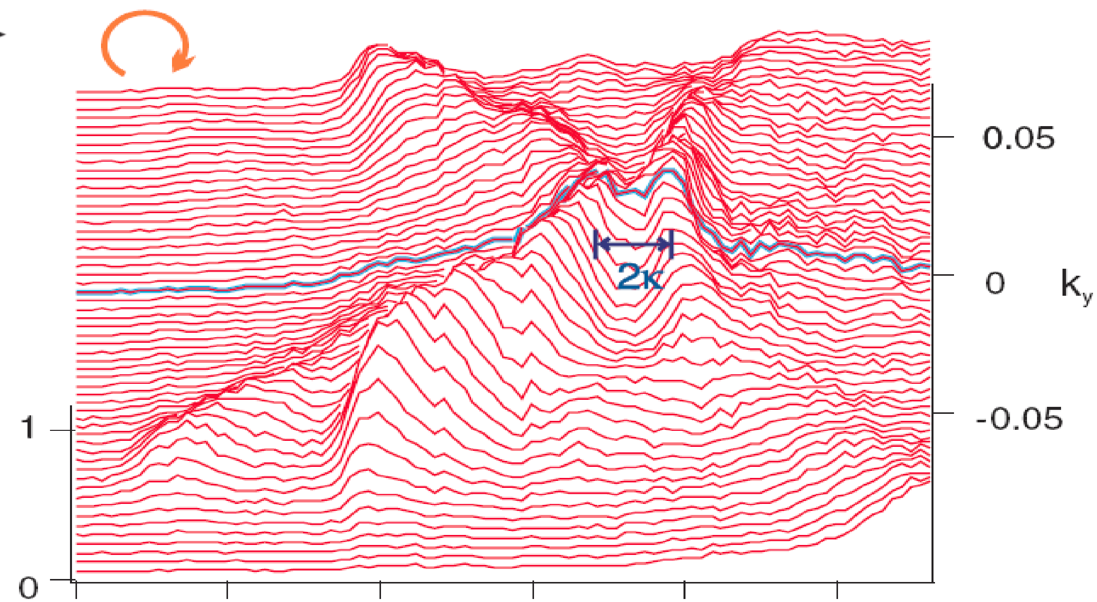
Observation of Floquet-Bloch States on the Surface of a Topological Insulator

Y. H. Wang,* H. Steinberg, P. Jarillo-Herrero, N. Gedik†

The unique electronic properties of the surface electrons in a topological insulator are protected by time-reversal symmetry. Circularly polarized light naturally breaks time-reversal symmetry, which may lead to an exotic surface quantum Hall state. Using time- and angle-resolved photoemission spectroscopy, we show that an intense ultrashort midinfrared pulse with energy below the bulk band gap hybridizes with the surface Dirac fermions of a topological insulator to form Floquet-Bloch bands. These photon-dressed surface bands exhibit polarization-dependent

[Wang et al, Science 342,453 (2013) (Gedik group)]





Shaking optical lattice & Photonic systems

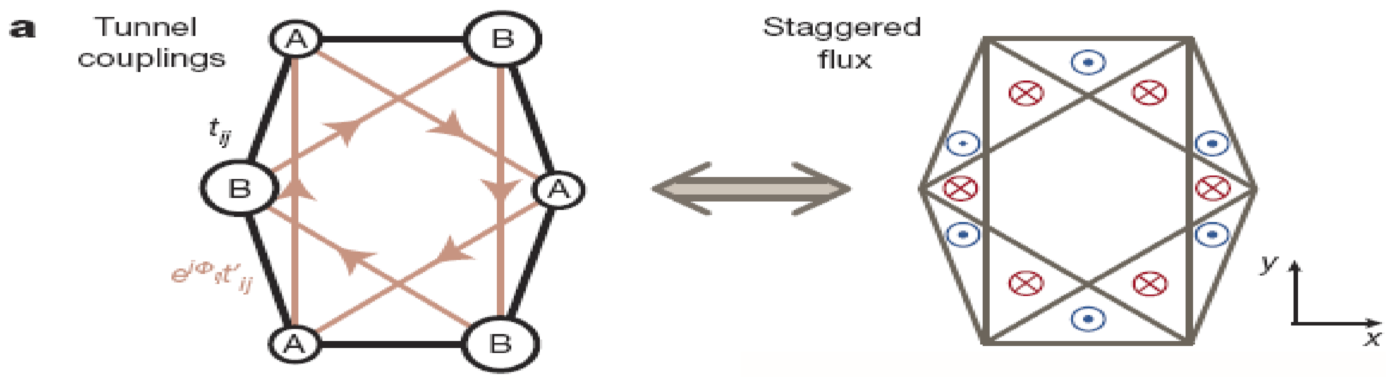
Shaking optical lattice

doi:10.1038/nature13915

Experimental realization of the topological Haldane model with ultracold fermions

Gregor Jotzu[1], Michael Messer[1], Rémi Desbuquois[1], Martin Lebrat[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

The Haldane model on
of a Hamiltonian feat

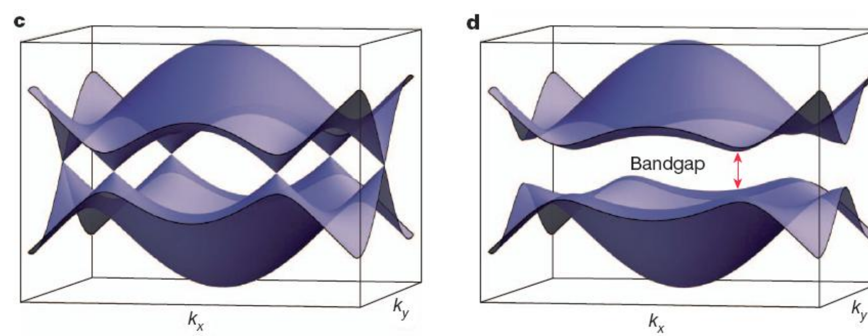


Modulated photonic/acoustic systems

Photonic Floquet topological insulators

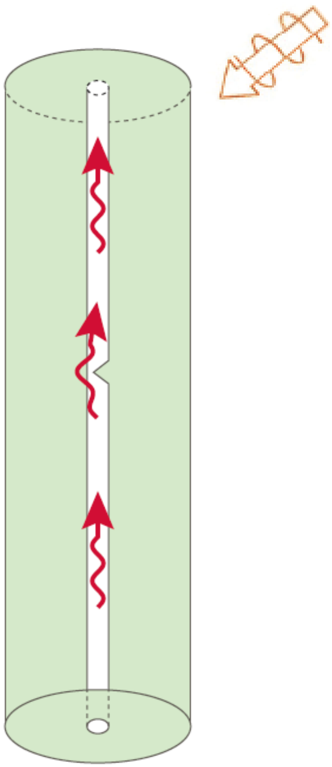
Mikael C. Rechtsman[1]*, Julia M. Zeuner[2]*, Yonatan Plotnik[1]*, Yaakov Lumer[1], Daniel Podolsky[1], F
Mordechai Segev[1] & Alexander Szameit[2]

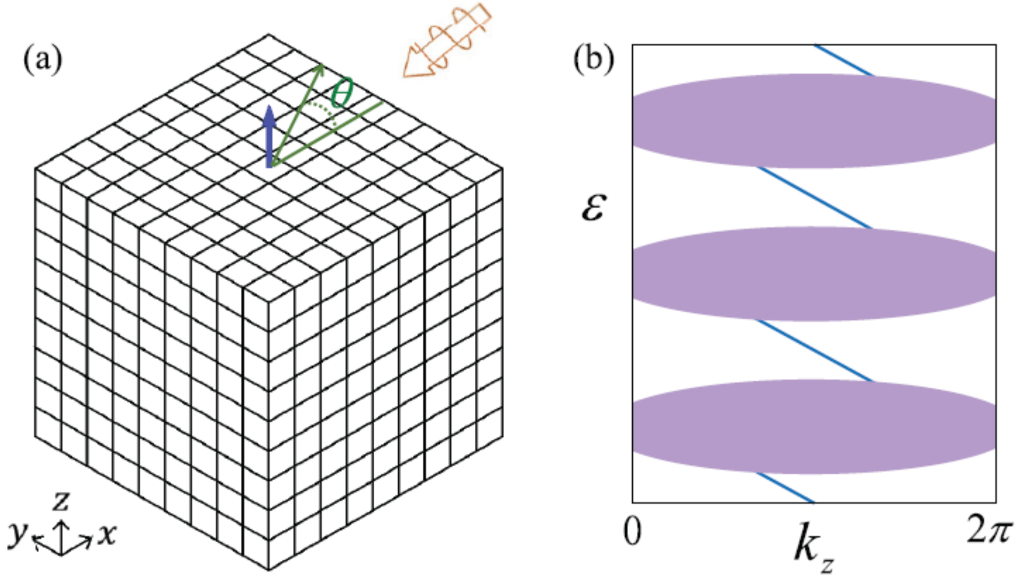
Rechtsman et al 2013



Back to the one-way mode

Flouqet one-way modes in driven (defect-free) Dirac semimetals





* Chiral modes can be generated even if the static system is defect-free.

R. Bi, Z. Yan, L. Lu, Z. Wang, PRB, 95, 161115(2017)

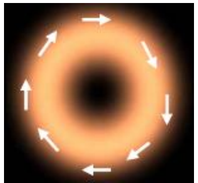
$$H(\mathbf{k},t) = H_0(\mathbf{k}) + H_d(t),$$

$$H_0(\mathbf{k}) = (2t_x \sin k_x \sigma_x + 2t_y \sin k_y \sigma_y + 2t_z \sin k_z \sigma_z) \otimes \tau_z + m(\mathbf{k})\sigma_0 \otimes \tau_x$$

$$H_d(t) = 2D\cos(\omega t)\sigma_0 \otimes (\tau_x \cos\theta + \tau_y \sin\theta),$$

Driving is angle-dependent

Topological invariant?



(Cylindrical vector beam of light)

Let us not be restricted to this specific question,
but consider a more general problem:

General topological invariants of Floquet topological insulators and Floquet topological defects?

A few insightful studies of topological invariants of Floquet Systems (incomplete list) :

Class A in 2d

Class AIII in 1d

M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Anomalous Edge States and the Bulk-Edge Correspondence for Periodically Driven Two-Dimensional Systems, Phys. Rev. X **3**, 031005 (2013).

D. Carpentier, P. Delplace, M. Fruchart, and K. Gawedzki, Topological Index for Periodically Driven Time-Reversal Invariant 2D Systems, Phys. Rev. Lett. **114**, 106806 (2015).

J. K. Asbóth, B. Tarasinski, and P. Delplace, Chiral symmetry and bulk-boundary correspondence in periodically driven one-dimensional systems, Phys. Rev. B **90**, 125143 (2014).

M. Fruchart, Complex classes of periodically driven topological lattice systems, Phys. Rev. B **93**, 115429 (2016).

Class AII in 2d

Class A and class AIII

1) Not applicable to topological defects;
2) Not sufficiently general to include all the ``tenfold-way'' Altland-Zirnbauer symmetry classes

We want to give a general formulation of topological invariants:

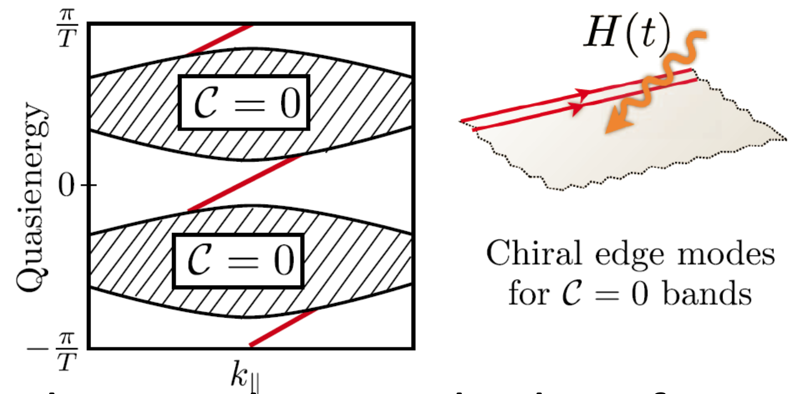
- Applicable to both Floquet topological insulators and topological defects
- For all the tenfold-way symmetry classes
- For all spatial dimensions

Review: Topological invariants of 2D Floquet systems in Class A

$$U(\mathbf{k},t) = \mathcal{T}\exp\left(-i\int_0^t dt' H(\mathbf{k},t')\right),$$

$$H_{\text{eff}}(\mathbf{k}) = \frac{i}{T}\log U(\mathbf{k},T)$$

(see Goldman &Dalibard, PRX, 2014)



*Effective Hamiltonian fails to capture the topology in the low-frequency regime

*Periodized time evolution operator (``micromotion'' operator):

$$U_\varepsilon(\mathbf{k},t) = U(\mathbf{k},t)\exp[iH_\varepsilon^{\text{eff}}(\mathbf{k})t] \qquad U_\varepsilon(t) = U_\varepsilon(t+T)$$

branch cut at ε

$$W(U_\varepsilon) = \frac{1}{24\pi^2}\int_{T^2\times S^1} d^2k\,dt\,\mathrm{Tr}\{\epsilon^{\alpha_1\alpha_2\alpha_3}[U_\varepsilon^{-1}\partial_{\alpha_1}U_\varepsilon][U_\varepsilon^{-1}\partial_{\alpha_2}U_\varepsilon][U_\varepsilon^{-1}\partial_{\alpha_3}U_\varepsilon]\}$$

Rudner, Lindner, Berg, Levin, PRX, 2013

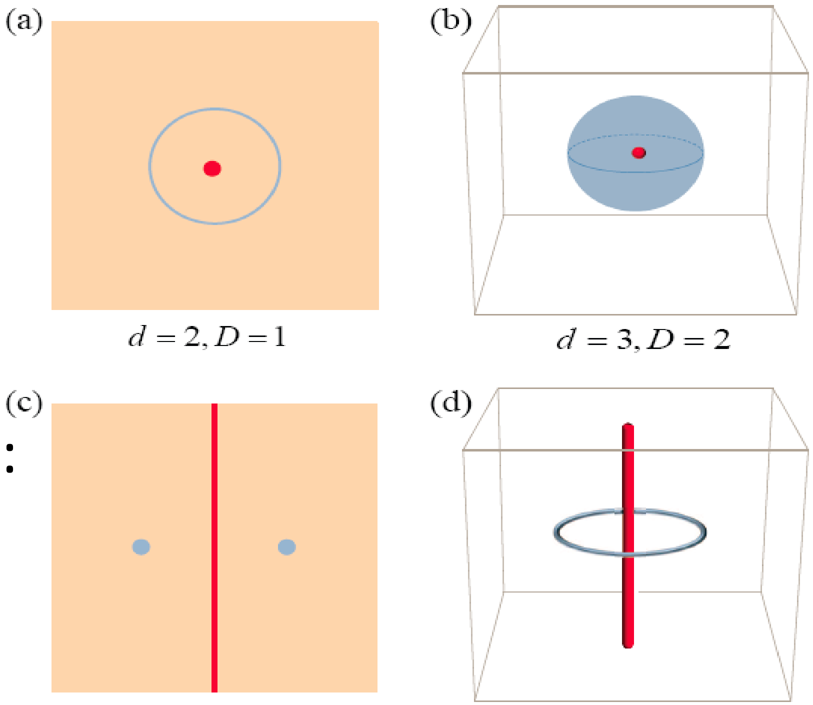
Generalization to 2d time-reversal-invariant systems: Carpentier, *et al*, PRL 2015

Topological defects

(For static defects, see Teo-Kane, PRB, 2010)

*Dimension of the space: *d*

*Dimension of the surrounding sphere: *D*



Parameter space of Floquet defects:

$(k_1, k_2, \ldots, k_d, \; r_1, r_2, \ldots, r_D, t)$

The simplest case: Class A

(Without symmetry)

$$K_{d+D+1} = \frac{(-1)^{\frac{d+D}{2}}\left(\frac{d+D}{2}\right)!}{(d+D+1)!}\left(\frac{i}{2\pi}\right)^{\frac{d+D}{2}+1}$$

$$W(U_\varepsilon(\mathbf{k},\mathbf{r},t)) = K_{d+D+1}\int_{T^d\times S^D\times S^1} d^d k d^D r dt \times \mathrm{Tr}\left[\epsilon^{\alpha_1\alpha_2\cdots\alpha_{d+D+1}}\left(U_\varepsilon^{-1}\partial_{\alpha_1}U_\varepsilon\right)\cdots\left(U_\varepsilon^{-1}\partial_{\alpha_{d+D+1}}U_\varepsilon\right)\right]$$

$$U_\varepsilon(\mathbf{k},\mathbf{r},t) = U(\mathbf{k},\mathbf{r},t)\exp\left[iH_\varepsilon^{\mathrm{eff}}(\mathbf{k},\mathbf{r})t\right]$$

Yao, Yan, Wang, PRB, 96, 195303 (2017)

Include the ``tenfold way'' symmetries:

$$C^{-1}H(\mathbf{k},\mathbf{r},t)C = -H^*(-\mathbf{k},\mathbf{r},t)$$

$$T^{-1}H(\mathbf{k},\mathbf{r},t)T = H^*(-\mathbf{k},\mathbf{r},-t).$$

$$S^{-1}H(\mathbf{k},\mathbf{r},t)S = -H(\mathbf{k},\mathbf{r},-t).$$

$$\Longrightarrow$$

$$C^{-1}U_\varepsilon(\mathbf{k},\mathbf{r},t)C = U^*_{-\varepsilon}(-\mathbf{k},\mathbf{r},t)\exp\left(i\frac{2\pi t}{\tau}\right)$$

$$T^{-1}U_\varepsilon(\mathbf{k},\mathbf{r},t)T = U^*_\varepsilon(-\mathbf{k},\mathbf{r},-t),$$

$$S^{-1}U_\varepsilon(\mathbf{k},\mathbf{r},t)S = U_{-\varepsilon}(\mathbf{k},\mathbf{r},-t)\exp\left(i\frac{2\pi t}{\tau}\right)$$

* Depending on the symmetries, **Z** topological invariants are defined as winding numbers in either the (**k,r,**t) or the (**k,r**) space [with chiral symmetry];
* **Z$_2$** topological invariants are Wess-Zumino-Witten terms:

$$W(U_\varepsilon(\mathbf{k},\mathbf{r},t,\lambda)) = K_{d+D+2}\int_{T^{d+1}\times S^D\times S^1} d^dk\,d^Dr\,dt\,d\lambda \times \mathrm{Tr}\left[\epsilon^{\alpha_1\alpha_2\cdots\alpha_{d+D+2}}\left(U_\varepsilon^{-1}\partial_{\alpha_1}U_\varepsilon\right)\cdots\left(U_\varepsilon^{-1}\partial_{\alpha_{d+D+2}}U_\varepsilon\right)\right],$$

extra dimension

parameter space: $(\mathbf{k},\mathbf{r},\lambda,t)$

Yao, Yan, Wang, PRB, 96, 195303 (2017)

An application of topological invariants:

Topological classification of Floquet topological insulators and topological defects

Time-reversal symmetry:

$$T^{-1}U_\varepsilon(\mathbf{k},\mathbf{r},t)T = U_\varepsilon^*(-\mathbf{k},\mathbf{r},-t),$$

Winding number in the (**k**,**r**,t) space:

$$W(U_\varepsilon(\mathbf{k},\mathbf{r},t)) = K_{d+D+1}\int_{T^d\times S^D\times S^1} d^dk d^Dr dt$$

$$\times \mathrm{Tr}[\epsilon^{\alpha_1\alpha_2\cdots\alpha_{d+D+1}}(U_\varepsilon^{-1}\partial_{\alpha_1}U_\varepsilon)\cdots(U_\varepsilon^{-1}\partial_{\alpha_{d+D+1}}U_\varepsilon)].$$

$$K_{d+D+1} = \frac{(-1)^{\frac{d+D}{2}}(\frac{d+D}{2})!}{(d+D+1)!}\left(\frac{i}{2\pi}\right)^{\frac{d+D}{2}+1}$$

complex conjugation

$$(-1)^{\frac{d+D}{2}+1}(-1)^d(-1)^1 = (-1)^{2d+2-\delta/2} = (-1)^{2-\delta/2}$$

Time-reversal symmetry $\quad\quad \delta = d - D$

$$\Downarrow$$

$$w(U_\varepsilon)(\mathbf{k},\mathbf{r},t) = w(U_\varepsilon)(-\mathbf{k},\mathbf{r},-t)(-1)^{2-\delta/2}$$

* It depends only on the combination δ = d - D

*With TRS, winding numbers can be nonzero only when

$\delta = 0, 4, 8, 12, \ldots\ldots$

S. Yao, Z. Yan, Z. Wang, PRB, 96, 195303 (2017)

Other symmetries can be done in a similar way….

(Z_2 cases are more subtle; see: Yao, Yan, Wang, PRB, 96, 195303 (2017))

Periodic table of topological defects

(Readily obtained from the topological invariants)

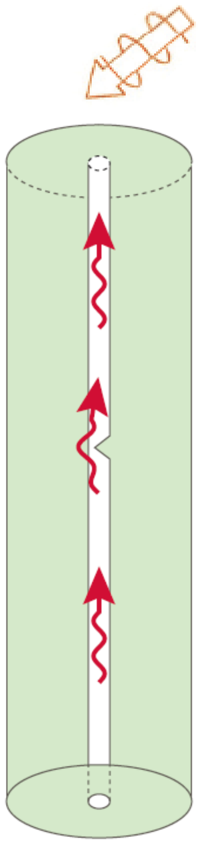
Only depends on $\delta = d - D$.

Symmetry					$\delta = d - D$							
s	AZ	T	C	S	0	1	2	3	4	5	6	7
0	A	0	0	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0	$\mathbb{Z}^n$	0
1	AIII	0	0	1	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$	0	$\mathbb{Z}^2$
0	AI	+1	0	0	$\mathbb{Z}^n$	0	0	0	$2\mathbb{Z}^n$	0	$\mathbb{Z}_2^n$	$\mathbb{Z}_2^n$
1	BDI	+1	+1	1	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$2\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$
2	D	0	+1	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$2\mathbb{Z}^2$	0
3	DIII	−1	+1	1	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0	0	$2\mathbb{Z}^2$
4	AII	−1	0	0	$2\mathbb{Z}^n$	0	$\mathbb{Z}_2^n$	$\mathbb{Z}_2^n$	$\mathbb{Z}^n$	0	0	0
5	CII	−1	−1	1	0	$2\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0	0
6	C	0	−1	0	0	0	$2\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$	0
7	CI	+1	−1	1	0	0	0	$2\mathbb{Z}^2$	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}^2$

S. Yao, Z. Yan, Z. Wang, PRB, 96, 195303 (2017)

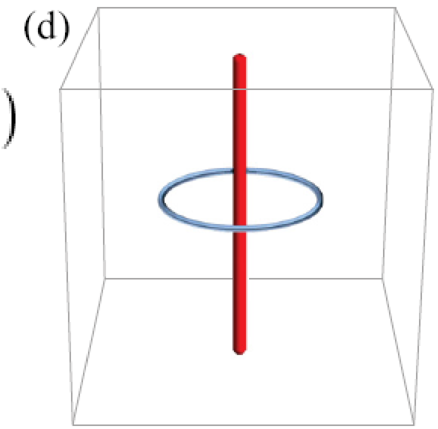
- Taking the special case D=0 gives the periodic table of Floquet topological insulators, which can be found in Roy, Harper, PRB, 2017, where it is obtained using the efficient and more formal K-theory (without explicit topological invariants)
- Many-body invariants are not discussed here. See e.g., Po, Fidkowski, Morimoto, Potter, Vishwanath, PRX, 6, 041070 (2016); Fidkowski , Po, Potter, Vishwanath, 1703.07360, ...

An application: Line defect (class A) in 3d space



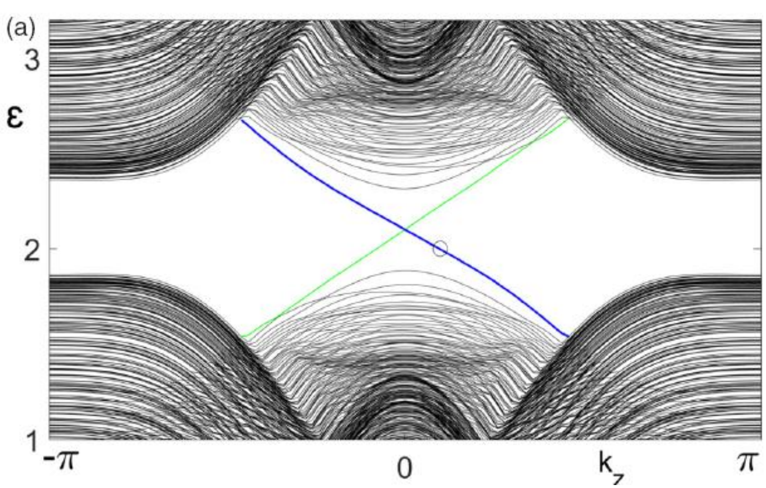
polar angle

$$W(\varepsilon) = \frac{i}{480\pi^3} \int dt d\theta d^3k \, \mathrm{Tr}[\epsilon^{\mu\nu\rho\sigma\tau} (U_\varepsilon^{-1}\partial_\mu U_\varepsilon)(U_\varepsilon^{-1}\partial_\nu U_\varepsilon) \times (U_\varepsilon^{-1}\partial_\rho U_\varepsilon)(U_\varepsilon^{-1}\partial_\sigma U_\varepsilon)(U_\varepsilon^{-1}\partial_\tau U_\varepsilon)],$$

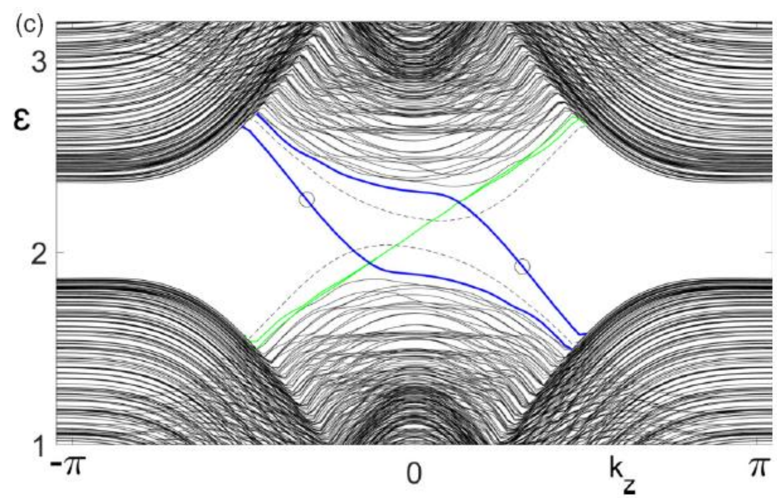


$d = 3, D = 1$

Application to the lattice model of driven Dirac semimetals:



W= - 1



W= - 2

(A proof-of-principle study; not yet at the level of concrete material)

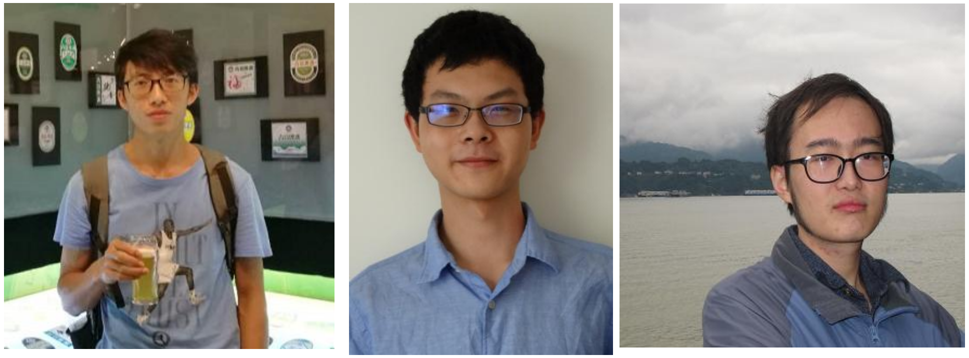
Summary

- One-way fiber of electromagnetic waves; protected by the second Chern number [Lu, Wang arXiv:1611.01998 (2016)]
- Topological invariants of Floquet systems; their applications to topological defects, including a simple derivation of the periodic table of Floquet topological insulators and topological defects

[S. Yao, Z. Yan, Z. Wang, PRB, 96, 195303 (2017)]

Thanks!

Ling Lu (Institute of Physics, CAS)



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