Exercise 1. Ising model: Finite size scaling analysis

Goal: In numerical simulations we are only able to tackle relatively small system sizes whereas real physical systems are usually much larger. Finite size scaling analysis *is a technique which allows us to get good approximations for the thermodynamic limit.*

Task 1: *Use your program of the first exercise sheet to perform simulations of the 3D Ising system for different system sizes to determine the critical exponents γ and ν .*

Hint: Use the finite size scaling relation of the magnetic susceptibility and the fact that the critical temperature is given by $T_c \approx 4.51$.

You might find the following points useful:

- *You can get a first estimate for the ratio γ/ν by plotting χ_{max} as a function of the system size.*
- *Vary γ/ν and $1/\nu$ until you get the best possible data collapse. Judge the quality of the data collapse "by eye".*

Task 2 (OPTIONAL): *Repeat the same process for the specific heat.*

Solution. The goal is to find the critical exponents γ and ν for the 3D Ising model. At first we simulate the system for $L = 10$, $L = 12$ and $L = 14$ using last week's program. The plots for the magnetic susceptibility are shown in Fig. 1.

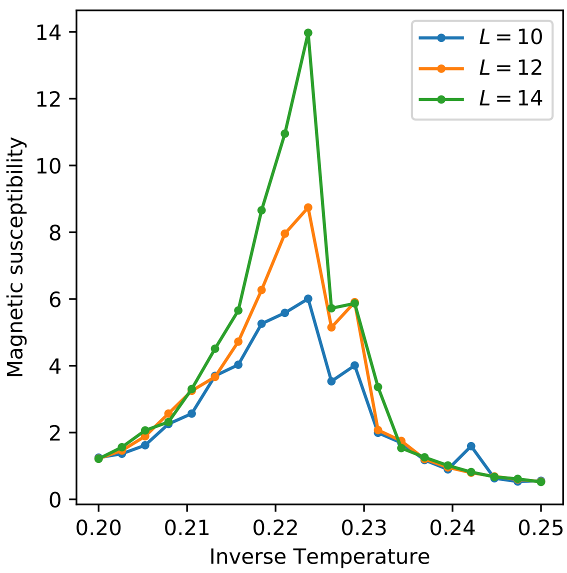

Figure 1: Magnetic susceptibility χ as a function of inverse temperature β for $100L^3$ thermalization sweeps and 3000 samples ($3L^3$ subsweeps).

Once the data is generated we determine the critical exponents by using the finite size scaling relation of the magnetic susceptibility

$$\chi(T, L) = L^{\gamma/\nu} \mathcal{F}_\chi \left[(T - T_c) L^{1/\nu} \right].$$

At T_c we find

$$\chi_{\max}(L) = \chi(T = T_c, L) \sim L^{\gamma/\nu}.$$

Plotting $\chi_{\max}$ as a function of L in a loglog-plot gives us a first estimate for the ratio γ/ν (slope of the curve).

By varying γ/ν and $1/\nu$ we observe the desired data collapse (Fig. 2).

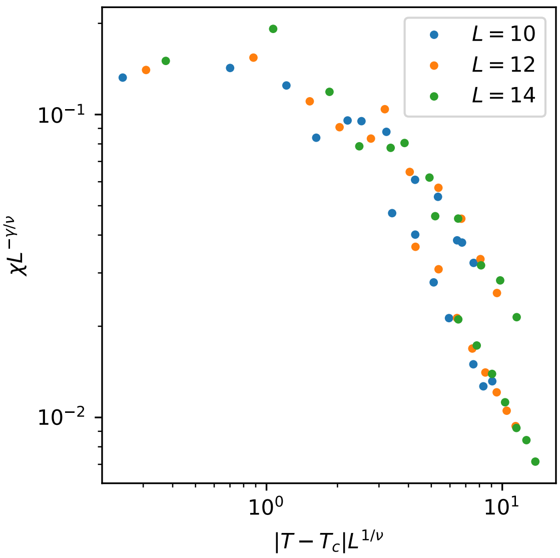

Figure 2: Data collapse for the literature values of γ and ν.

The literature values are $\gamma \approx 1.24$, $\nu \approx 0.63$.