Topology and many-body physics in synthetic lattices

Alessio Celi
Synthetic Hofstadter strips as minimal quantum Hall experimental systems

Alessio Celi

Synthetic dimensions workshop, Zurich 20-23/11/17
Plan

- Synthetic lattice (*Extradimension*)
- Synthetic strip as minimal integer quantum Hall systems
  - Edge states in narrow strips
  - Topological response in narrow strips
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- Dimerized interacting ladder
  - Meissner/Vortex phase (in analogy to type II superconductors)

Effect of the dimerization:

- Reverse of chiral current (single particle)
- Commensurate-Incommensurate transition (strong interactions)
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• Prospects and philosophical view
Simulating an extra dimension
[Boada, AC, Latorre, Lewenstein, PRL 108, 133001 (2012)]

In optical lattices 1D-3D Hubbard model by tuning optical potential

And > 3D? In a lattice Dimensionality $\equiv$ Connectivity

Atoms in D+1-lattice = coherently coupled atomic states in D-lattices

Not only spin states
Momentum states
Trap modes...

Not only atoms
Cold molecules,
Photonic crystal,
Ring resonators...
Synthetic gauge fields in synthetic dimension

[AC et al PRL 112, 043001 (2014)]

1d-lattice loaded e.g. with $^{87}\text{Rb}$ (F=1, m=-1,0,1) +
Raman dressing

Constant magnetic flux $\phi$!

Minimal instance of a quantum Hall system!
Synthetic gauge fields in synthetic dimension
[AC et al PRL 112, 043001 (2014)]

1d-lattice loaded e.g. with $^{87}\text{Rb}$ (F=1, m=-1,0,1) +
Raman dressing

$\begin{align*}
&\text{Synthetic dim.} \\
&\text{Constant magnetic flux } \phi!
\end{align*}$

Sharp Boundaries $\rightarrow$ Edge currents (hard to get in real 2d lattice)
signal of Topological nature of quantum Hall
(bulk-boundary correspondence)
"Genuine" **Edge states** for small $J'/J$:
- live in the gap,
- have linear dispersion
- have well defined spin
Synthetic gauge fields in synthetic dimension
[AC et al PRL 112, 043001 (2014)]

Experimental Realizations:

1) Bosons: NIST Spielman group $^{87}\text{Rb}$ [Science (2015)]

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Spectrum

Experimental Realizations:

I) Bosons: NIST Spielman group $^{87}\text{Rb}$ [Science (2015)]

II) Fermions: LENS Fallani group $^{173}\text{Yb}$ [Science (2015)]

Also with clock states (ladder)
LENS: Livi et al. PRL 117, 220401 (2016)

"Genuine" Edge states for small $J'/J$:
- live in the gap,
- have linear dispersion
- have well defined spin
Topology in narrow strips

Narrow Hofstadter strips have edge states

What about the “bulk”? 
Topology in narrow strips

Narrow Hofstadter strips have edge states

What about the “bulk”? 

How big should it be to display topological properties?

Is there some reminiscence of open/closed boundary correspondence?

Is Chern number defined? / Can we measure it?
Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument
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- Large (periodic) system, a lowest band state well localized in $y$ and spread in $x$
- Apply a force along $x$
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- Large (periodic) system, a lowest band state well localized in $y$ and spread in $x$
- Apply a force along $x$
- After a Bloch oscillation observe the displacement

Displacement in $y$ due to anomalous velocity!
Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

\textit{In formulae}: semiclassical approach

\begin{align*}
\mathbf{k}(t) &= \mathbf{k}_0 + \frac{t}{\hbar d} F_x \mathbf{e}_x \\
\mathbf{v}(k) &= \frac{1}{\hbar} \partial_k E(k) + \frac{F_x}{\hbar d} \mathcal{F}(k) \mathbf{e}_y
\end{align*}

\[|\psi(k)|^2 \sim \frac{1}{A_{BZ}} \delta(k_x - k_x(t))\]

Wave packet:

\[\langle \mathbf{r}(T) - \mathbf{r}(0) \rangle = \int_0^T \langle \mathbf{v}(t) \rangle = \frac{\hbar d}{|F_x|} \int_{BZ} \mathbf{v}(k) = \text{sgn}(F_x) C d \mathbf{e}_y\]

State easy to prepare if the coupling \( J_y \ll J_x \)

Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

In formulae: semiclassical approach

\[ k(t) = k_0 + \frac{t}{\hbar d} F_x e_x \quad \rightarrow \quad v(k) = \frac{1}{\hbar} \partial_k E(k) + \frac{F_x}{\hbar d} \mathcal{F}(k)e_y \]

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State easy to prepare if the coupling \( J_y \ll J_x \)

Applicable also to strips until we don’t reach the boundary...

Scheme:

Results: \( J_y = \frac{1}{5} J_x \), \( \Phi = \frac{2\pi}{3} \), \( N_y = 3 \)
Why does it work? **Perturbative argument** as for the edge states:

- Gap linear in $J_y / J_x$
- Hybridization spin states (spreading in $y$) quadratic in $J_y / J_x$

![Graph showing quadratic degradation of the measurement](image)

Quadratic degradation of the measurement

Scheme:

Results: \( J_y = \frac{1}{5} J_x, \quad \Phi = \frac{2\pi}{3}, \quad N_y = 3 \)
Higher $C$ possible for $N_y \geq C + 2$

Ex: $\Phi = \frac{4\pi}{5} \rightarrow C_1 = -2$


Robust to disorder (< gap) and typical harmonic confinement

Interactions? Gap small (although may hold thought adiabatic argument, see later)
Synthetic lattices in interaction

- Interesting route to interaction -> Fractional QH effect?! -> many-body localization?!

No heating expected

- Peculiarity: Interactions are naturally long range in the synthetic dimension

- Quasi 1D approach to 2D interesting both theoretically & practically
Synthetic lattices in interaction

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  -> many-body localization?!

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Many studies: Meissner-vortex and commensurable incommensurable transitions, Fractional pumping, Laughlin like states, pseudo Majorana...

Here: effect of dimerization on synthetic Hofstadter ladder
Meissner/Vortex phase in flux ladder

No interactions: real = synthetic ladder

Weak interleg (Raman) coupling:

2 minima, \( k_m \sim \pm \frac{\phi}{2} \)

Strong interleg (Raman) coupling:

1 minimum, \( k_m = 0 \)

\( J_{\perp} \ll J \)

\( J_{\perp} \gtrsim J \)

[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions
Real ladder experiment [Atala et, Nature Phys. 2014]
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1 minimum, $k_m = 0$

Observables

$J_c(j,m) = i \langle \hat{a}_{j+1,m} \hat{a}_{j,m} \rangle + H.c.$

$J_\perp(j) = i \langle \hat{a}_{j,1/2} \hat{a}_{j,-1/2} \rangle + H.c.$

$J_\perp \ll J$

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Effect of interactions: suppress vortex phase

Real ladder: vortex phase survives in the hard-core limit for $\phi$ large

more phases at $U \neq \infty$  

see [Petrescu, Le Hur, PRL 2013]  
[Piraud et al, PRB 2015]

Synthetic ladder: vortex phase disappears in the hard-core limit

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Synthetic ladder: vortex phase disappears in the hard-core limit

more phases at $U \neq \infty$

Idea: enhance vortex phase by dimerizing the lattice (“easy” exp. handle)
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M. Lewenstein, AC in progress

$$J = \frac{J_O + J_E}{2}$$

Effect of dimerization: new handle  \( \Delta = \frac{J_O - J_E}{J_O + J_E} \)
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\[ J = \frac{J_O + J_E}{2} \]

Effect of dimerization: new handle \[ \Delta = \frac{J_O - J_E}{J_O + J_E} \]

No interactions: 4 bands

\[ \Delta = 0.5 \]

Bands deform & mix

\[ J_\perp \ll J \]

Minima separate: dimerization enhances vortex phase!
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Effect of dimerization: new handle \[ \Delta = \frac{J_O - J_E}{J_O + J_E} \]

No interactions: Reverse of chiral current

Current behavior confirms vortex enhancement!
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Interactions: \( U \rightarrow \infty \) 3 states per rung
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Interactions:  \( U \to \infty \)  3 states per rung

\[ J_E \ll J_O \]  9 states per plaquette

1 \( n=0 \),  4 \( n=1 \),  4 \( n=2 \)

Spectrum plaquette

\[ \pm J_\perp \sqrt{1 + \left( \frac{J_O}{J_\perp} \right)^2 - 2 \cos \frac{\phi}{2} \left( \frac{J_O}{J_\perp} \right)} \quad \pm J_\perp \sqrt{1 + \left( \frac{J_O}{J_\perp} \right)^2 + 2 \cos \frac{\phi}{2} \left( \frac{J_O}{J_\perp} \right)} \]

\[ \pm 2J_\perp \quad \pm 0 \]
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Spectrum plaquette

\[ 0 \]

\[ \pm J_\perp \sqrt{1 + \left( \frac{J_O}{J_\perp} \right)^2} - 2 \cos \frac{\phi}{2} \left( \frac{J_O}{J_\perp} \right) \]
\[ \pm J_\perp \sqrt{1 + \left( \frac{J_O}{J_\perp} \right)^2} + 2 \cos \frac{\phi}{2} \left( \frac{J_O}{J_\perp} \right) \]

\[ J_\perp \geq J_O \] Plaquette in \( n=2 \) \hspace{1cm} Band insulator

\[ J_\perp < J_O \] Plaquette in \( n=1 \) \hspace{1cm} Imprinted vortex
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M. Lewenstein, AC in progress

DMRG calculations confirm perturbative expectations

Phase diagram through calculation of currents and structure factors
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M. Lewenstein, AC in progress

\[ \phi = \frac{3\pi}{4} \]

Ex.

\[ \phi = \frac{3\pi}{4} \times J_E = 0.7 \]

0 \[\rightarrow\] \[J_E\] \[\rightarrow\] \[J_O\]

Imprinted vortex \[\rightarrow\] Melted vortex \[\rightarrow\] Meissner charge density wave \[\rightarrow\] Meissner

Similar to attractive interleg interaction, [Orignac et al, PRB 96, 014518 (2017)]

Evidence of commensurate-incommensurate transition
Further steps

• No hard-core boson limit: bosons different from fermions

• Study the accessible experimental parameters

• Search for “visible” Laughlin-like states in such regimes
  cf. [Calvanese et al, PRX 7, 021033 (2017)], [Petrescu et al, PRB 96, 014524 (2017)]

• ...Toy model for many-body localization?
Alternative route to synthetic interacting ladders... long range interactions!

PHYSICAL REVIEW A 91, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,¹ Christine Muschik,¹,²,³ Alessio Celi,¹ Ravindra W. Chhajlany,¹,⁴ and Maciej Lewenstein¹,⁵

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How? E.g. ion analogue simulation
Alternative route to synthetic interacting ladders... long range interactions!

Driven Dicke model with “fluxes” seems robust to photon heating!

\[ H_0(t) = \sum_m \hbar \omega_m a_m^\dagger a_m + \sum_{i,m} \hbar \Omega_i \eta_{i,m} (a_m + a_m^\dagger) \sigma_i^x \sin(\omega t) + \sum_i B_i(t) \sigma_i^z \]


Deviation from the effective spin model are suppressed also for strong driving in presence of fluxes! Topological protection?
Summary

• Synthetic edge state in synthetic Hofstadter strips

• “Bulk topology” in synthetic Hofstadter strips

• Effect of dimerization in synthetic Hofstadter ladder w/o interactions

• Synthetic fluxes in driven ion chain
A bit of philosophy...

Synthetic lattices: practical tool for reshaping degrees of freedom in experimentally convenient way...

Same Hamiltonian = Same model

Useful approach for questioning conventions, ease detection, suggest new models...
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Synthetic lattices: practical tool for reshaping degrees of freedom in experimentally convenient way...

Same Hamiltonian = Same model

Useful approach for questioning conventions, ease detection, suggest new models...

Pushing it further...

Same time evolution = Same model

Synthetic lattices naturally combine with the Floquet approach...

Realization of quantum simulation paradigm
“Extradimensional” collaborators

J.I. Latorre

O. Boada

M. Lewenstein
“Extradimensional” collaborators

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